

TOOLKIT FOR HEALTHCARE PROFESSIONALS: A COLORED PETRI NETS BASED APPROACH FOR MODELING AND SIMULATION OF HEALTHCARE WORKFLOWS

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ABSTRACT

In 2017, Academic Emergency Medicine convened a consensus conference on *Catalyzing System Change through Health Care Simulation: Systems, Competency, and Outcomes* to assess the impact of simulation on various aspects of healthcare delivery. One focus area was the role that computer modeling and simulation can and should play in the research and development of emergency care delivery systems. In this paper, we illustrate the use and application of Colored Petri Nets (CPNs) for modeling and simulating healthcare workflow processes. Specifically, we detail our approach by modeling patient flow to an operating room. The model accounts for various resources and their utilization. We use hierarchical Colored Petri Nets with modules and the associated graphical integrated development environment called CPN Tools for creating and simulating the model. The hierarchy and module concepts of CPNs allow the modeling of large and complex systems in an incremental and top-down manner.

1 INTRODUCTION

In 2009, the US Government passed the Health Information Technology for Economic and Clinical Health (HITECH) Act, which included incentives to accelerate the adoption of health information technology (HIT) by the healthcare industry. Given that healthcare information technology can dramatically improve healthcare services delivery, reduce cost, improve care efficiency, and patient safety, under a government mandate, hospitals and medical care providers were required to adopt/introduce electronic systems for the management and delivery of healthcare services.

The adoption of EHR has resulted in a large amount of healthcare data in electronic form that can be computationally processed. Several healthcare organizations are utilizing data mining, machine learning, and related approaches to analyze healthcare data and improve the quality of care. However, data analysis alone cannot give insights into the underlying process. For example, the efficacy of clinical interventions identified by data analysis cannot be evaluated unless the underlying cause and effects are modeled. A

report by the US Institute of Medicine emphasizes that many serious errors result from systems and their interactions rather than individual failures (Reid et al. 2005). Thus, to effect changes to improve healthcare and to design and deploy better systems for improving human health, we also need to adopt tools and techniques for process modeling, simulation, and analysis.

Although modeling and simulation are widely used in many sectors, their adoption in healthcare has been challenging. A study, reported in (Jahangirian et al. 2012), investigates modeling and simulation in healthcare against a context of defense and manufacturing industries. The authors report limited evidence of modeling and simulation being used to drive change in the healthcare delivery system. In addition to the complexities of a healthcare system, both (Brailsford et al. 2009) and (Eldabi 2009) identify stakeholder issues as a barrier to the successful and widespread use of simulation in healthcare. Results of a relatively recent survey dealing with modeling and simulation in healthcare are reported in (Tako and Robinson 2015). The key summary of the survey is that modeling in healthcare is perceived to be different and more difficult across a range of factors. Young et al. (2009) highlight three challenges for health modeling: first, how good is good enough, that is, what level of details should be included in models; second, clearly understanding how modeling is linked to decision making; and third, dealing with the cultural barriers to adoption of modeling and simulation in the health sector.

In 2017, Academic Emergency Medicine convened a consensus conference on *Catalyzing System Change through Health Care Simulation: Systems, Competency, and Outcomes* to assess the impact of simulation on various aspects of healthcare delivery. The work reported in (Laker et al. 2018) is the summary of a breakout session on *understanding complex interactions through systems modeling*. Specifically, it explores the role that computer modeling and simulation can and should play in the research and development of emergency care delivery systems. The authors note that “One underutilized approach to addressing problems in health care quality and value, particularly in emergency care, is through the use of computer simulation modeling.” Furthermore, they emphasize that “Not unlike high-fidelity patient simulation for training clinicians in clinical care through the use of mannequins, computer simulation provides a platform to inform decision making prior to implementation in the real world.”

Ample data is confirming that the number of emergency visits in the US is going up whereas the number of emergency departments providing such services is on the decline. Furthermore, COVID-19 forced many hospitals to re-evaluate and re-engineer their workflows. For example, recently a healthcare facility had to transform from a traditional model of care to a virtual model of care in orthopaedic surgery (King et al. 2020). Laker et al. (2018) note that “Computer simulation should be viewed as a necessary first step prior to implementation of a change in procedure or practice.”

As noted above, stakeholder issues appear to be a barrier. However, in our own experience, part of the issue is the perceived learning curve associated with the simulation language (notation) and the lack of user-friendliness of associated tools. Even though stakeholders are not directly involved with actual model development, they need to be convinced that the adopted approach is user-friendly and, in particular, the adopted notation is understandable. This is where we see the strengths of a Colored Petri Nets (CPNs) based approach and the underlying CPN Tools software (Gehlot 2019; Jensen and Kristensen 2015). The basic graphical/visual vocabulary of CPNs is small and intuitive, which renders them an attractive choice for modeling and simulation in healthcare.

The remainder of this paper is organized as follows. In Section 2, we review some related work in the area of healthcare modeling and simulation. Section 3 contains a hospital workflow example as described in (Barkaoui et al. 2002). We use this example to build our hierarchical CPN model, which we describe in Section 5. Before it, in Section 4, we give an overview of CPN and introduce the vocabulary of the CPN modeling language utilizing a simple example. Section 6 contains details of our simulation data collection and results. Finally, in Section 7, we present our conclusions and describe the future work.

2 RELATED WORK

Early work on engineering medical processes and improving safety through the use of process modeling and analysis is reported in (Christov et al. 2008; Osterweil et al. 2007). Our prior work focused on the use of Colored Petri Net models in planning, design, and simulation of intelligent wireless medical device networks for safe and flexible hospital capacity management (Gehlot and Sloane 2006; Sloane and Gehlot 2007). Work reported in (Eldabi 2009) highlights the value of process modeling in healthcare and discusses barriers to implementing model-driven healthcare. According to the authors “...healthcare systems and operations would work better if they are driven by model-informed decisions.”

Cho et al. (2017) use queuing theory to analyze changes in outpatients waiting times before and after the introduction of Electronic Medical Record (EMR) systems. Another work, reported in (Creemers and Lambrecht 2007), evaluates orthopaedic department of a Belgian hospital focusing on the impact of outages on the effective utilization of resources and the flow-time of patients. Queuing networks analysis, in general, provides insights into steady-state behavior, and depending on the complexity of the underlying network, a closed-form solution may not be feasible always. Creemers and Lambrecht (2011) draw a distinction between healthcare systems and manufacturing systems and how to construct a queuing network of a general class of healthcare systems. For analysis, they decompose the network into a set of single queuing systems.

Several published works focus on simulation-based, including Petri nets and their derivatives, approaches to modeling in healthcare. Centeno et al. (2010) examine historical records and present a simulation study to increase the throughput at an endoscopy center. A discrete event simulation (DES) study to reduce nurse overtime and improve patient flow time at a hospital endoscopy unit is presented in (Taheri et al. 2012). A non-hierarchical CPN model dealing with patients’ workflow in heart clinics is presented in (Zeinalnezhad et al. 2020).

Several researchers have explored the relationships between the Petri nets-based formalisms and Discrete Event Simulation (DES) approaches. For example, the work reported in (Simon et al. 2018) investigates the suitability and relevance of Discrete-Event Simulation (DES) software for Petri net modeling in the context of manufacturing systems. Giua and Silva (2017) provide a Petri net perspective towards the modeling and analysis of Discrete Event Systems (DES). In particular, the paper reviews the development of Petri net research within the area of DES and establishes the relevance of Petri nets for modeling control systems in a broad sense.

3 EMERGENCY WORKFLOW EXAMPLE

To illustrate our Colored Petri Nets-based approach, in this paper we provide details of a CPN model of the emergency workflow described in (Barkaoui et al. 2002). The workflow, as described in the paper, is shown in Figure 1. As depicted in this figure, there are two separate paths that a patient may take. The one on the left is taken by emergency patients whereas the one on the right is for elective surgeries where patients are initially hospitalized.

As part of the patient flow, the diagram explicitly depicts various resources that are needed at different stages of the flow. The aforementioned paper focuses on and distinguishes two types of resources: rooms (physical) and hospital staff (human). The various labels and their descriptions given in (Barkaoui et al. 2002) are as follows:

- Activity: reception (AA), transfer (AT), induction (AI), surgical operation (AO), and recovery (AR).
- Staff: nurse for reception (RI), anesthesiology staff for induction and operation (MSI), surgical staff for elective surgeries (MSH), surgical staff for emergency surgeries (MSU), nurse assistant (RAS), anesthesiology staff for recovery (MSR).
- Rooms: reception room (MA), induction room for elective surgery (MIH), induction room for emergency surgery (MIU), operating room for elective surgery (AOH), operating room for emergency surgery (AOU), recovery room (MR1).

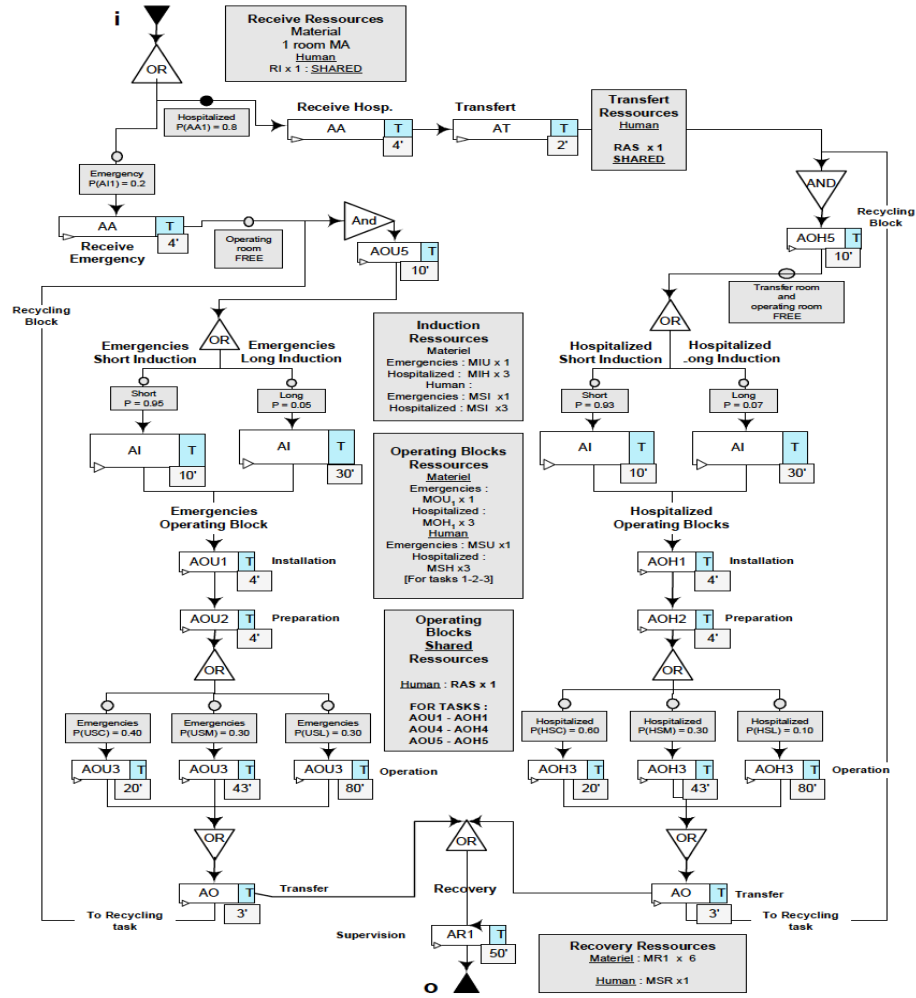


Figure 1: The emergency workflow as described using a Workflow Management Systems (WFMS) notation in (Barkaoui et al. 2002). It describes the overall patient workflow in a health care system focusing on two different paths to OR, namely, Emergency workflow and Elective workflow.

The shown diagram also gives delays in minutes for various activities as well as the probability of various choices. For example, the probability of a patient needing short induction on the emergency side is specified as 0.95 whereas the probability of short induction on the elective side is given as 0.93. In building our model, we use the same label and values where possible. For the benefit of the reader, before going into the details of our model, we give a brief introduction to the CPN vocabulary and modeling approach next.

4 COLORED PETRI NETS

Colored Petri Nets (CPNs) provide a graphical (visual) modeling notation well suited for concurrent and distributed systems in which communication, synchronization, and resource-sharing play an important role. A key aspect of the CPN vocabulary is the ability to express a cause and its effect, which allows one to capture a workflow in a natural manner. In terms of depiction, a CPN consists of *places* (depicted as circles or ovals), *transitions* (depicted as rectangles), and *arcs* (depicted as arrows) that connect a place to a transition or a transition to a place.

Places are containers of *tokens*. Depending on the context, tokens may represent a state, or a data value, or a resource, or some other entity. Transitions represent (abstraction of) actions. The cause and effect

dynamics of a CPN are defined using the *firing rule*, whereby tokens are removed from input places of a transition and deposited in the output places of a transition. Thereby, recording the fact that the associated action has occurred. The distribution of tokens across places in a net is called a *marking* and describes the global state of the system being modeled. As mentioned above, another crucial aspect of the CPN notation is its ability to express sharing of resources and associated constraints, which are also inherent to healthcare workflows. For example, the availability of an operating room or an infusion pump is a resource constraint that would be part of the flow of care in a hospital dealing with trauma patients.

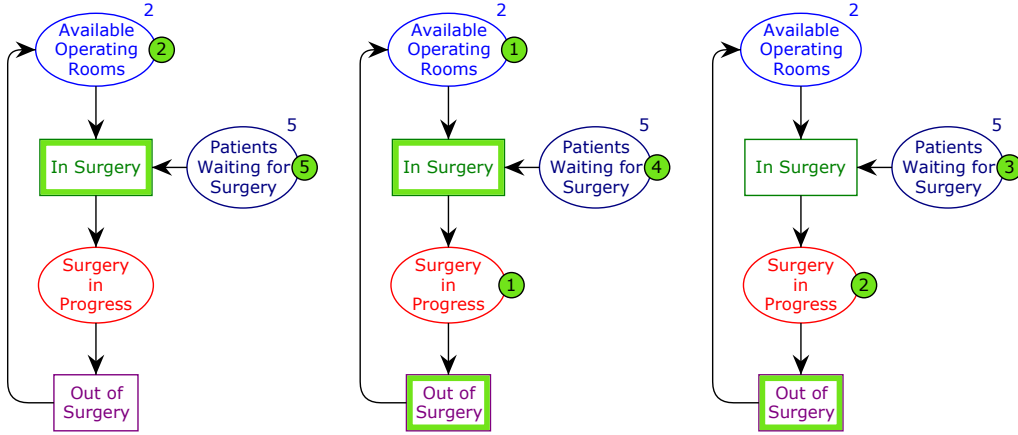


Figure 2: A CPN model of a very simplified operating room workflow taking into account just the room availability. The net on the left shows initially we have 2 operating rooms and 5 patients waiting. The net in the middle shows 1 surgery in progress with 1 room available. The net on the right shows the state where 2 active surgeries are in progress and we cannot take any more patients since the transition *In Surgery* is not enabled (highlighted in green).

To explain the basic CPN notation and its capability, we consider a concrete example of a very simple workflow where patients waiting for surgery can be taken in for surgery only if there is an operating room available. For this example, we are ignoring other resources, such as surgical staff, surgical instruments, patient monitoring devices. The net on the left in Figure 2 captures this basic workflow. In this net, the active tokens are shown in small green circles. In this initial state, there are 2 *Available Operating Rooms*, as depicted by the associated token, and 5 *Patients Waiting for Surgery* as indicated by the associated token. The transition *In Surgery* can fire only if a patient is waiting (at least one token in the place named *Patients Waiting for Surgery*) and an operating room is available (at least one token in the place named *Available Operating Rooms*). The net in the middle is a snapshot of the next simulation step showing the state where one surgery is in progress (one token in the place named *Surgery in Progress*) and only one operating room is available, that is, the token count of *Available Operating Rooms* is now down to 1. At this stage, either another waiting patient can be taken in the surgery, or the current in surgery patient can be out of surgery or both since in the depicted net, both *In Surgery* and *Out of Surgery* transitions are simultaneously enabled (highlighted in green) and can fire. The net on the right depicts the state where we have two patients in active surgery and we cannot take the next patient in since there is no token in *Available Operating Rooms* thereby disabling the *In Surgery* transition (not highlighted in green) even though we have three more patients waiting. Once one of the currently active surgeries is done, a token representing room availability will be deposited in *Available Operating Rooms* via the arc connecting the transition *Out of Surgery* to *Available Operating Rooms*.

With this given background, we are now ready to describe the details of our CPN model. Readers interested in more details of CPN, including formal definitions and theoretical foundations, may refer to (Jensen 1981), (Jensen 1994), and (Jensen and Kristensen 2009).

5 CPN MODEL DETAILS

The original work reported in (Barkaoui et al. 2002) provides a non-hierarchical model. From a practical application point of view, CPNs support a mechanism of modules that allows one to construct models of large systems in a hierarchical manner. The hierarchy and module concept of CPNs allow the modeling of levels of abstraction that are inherent in most systems in practice. We give details of our hierarchical CPN model that captures the details of the workflow shown in Figure 1. The creation of hierarchical nets is based on the simple idea that any transition can be replaced or substituted by a (sub) net that details the activities underlying it. Such transitions are called *substitution transitions* (or *modules*) in the CPN parlance. Pictorially, a substitution transition is drawn with double rectangles.

The (hierarchical) net on the left in Figure 3 shows the overall patient workflow starting with the entry of a patient from reception to the exit from the recovery system. The shown patient workflow net consists of four modules, namely, *Patient Entry*, *Emergency Workflow*, *Elective Workflow*, and *Recovery*, and 5 places namely *To Emergency*, *To Elective*, *From Emergency*, *From Elective*, and *Discharge*. The diagram on the right in Figure 3 shows the module hierarchy, that is, the various sub-modules and their nesting structure that comprises our hierarchical model.

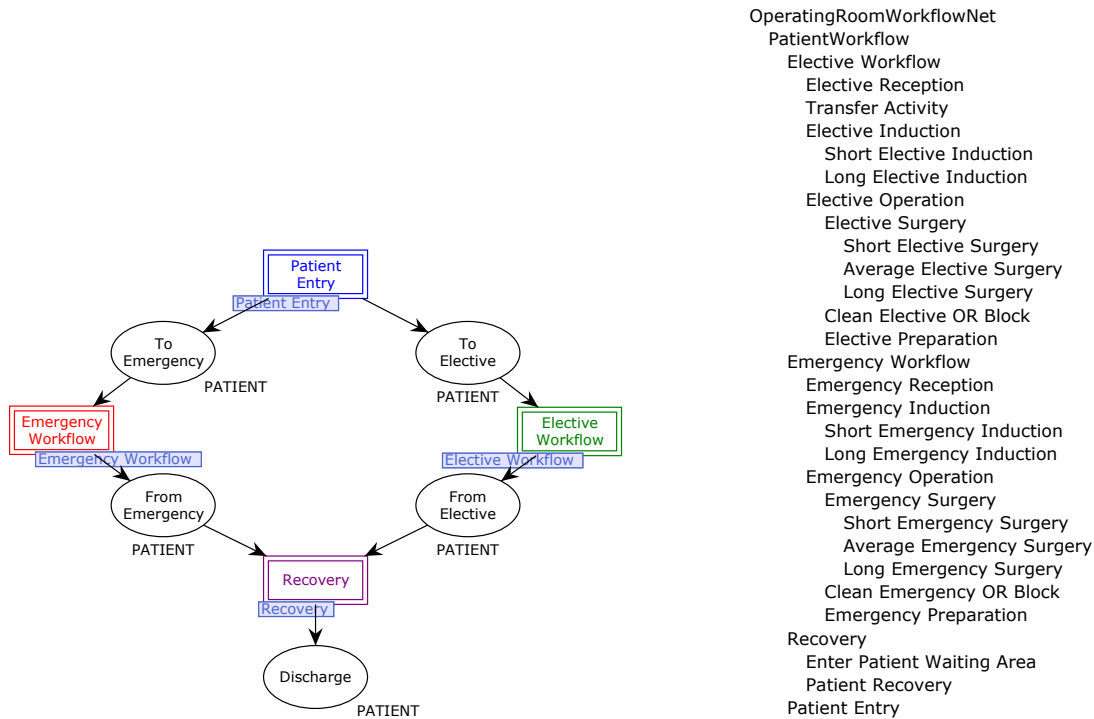


Figure 3: The top-level net showing the overall workflow on the left and the associated module hierarchy of the CPN model on the right. The module hierarchy gives the various sub-modules and their nesting structure that comprises our hierarchical model.

The tokens in the basic model in Figure 2 do not carry any information. For a detailed analysis, we may want to carry additional information in tokens. For example, we may want to distinguish different types of operating rooms or patients with different conditions. CPNs provide an enhanced vocabulary to create tokens of different data types (or *colorsets* in CPN parlance) and utilize the full functionality of the underlying inscription language CPN ML, which is built on top of the functional programming language SML. Before going into details of some of the sub-modules, we give a brief description of key colorsets used in this model below:

```
(* Model colset declarations *)
colset PTYPE = with EM | EL;
colset PID = INT;
colset PID_T = PID timed;
colset AT = INT;
colset PATIENT = product PTYPE * PID * AT;
colset PATIENTS = list PATIENT;
colset ROOM = with MA | MIU | MIH | AOU | AOH | MRI | WR;
colset ROOMS = list ROOM;
colset HR = with RI | MSI | MSH | MSU | RAS | MSR;
colset STAFF = list HR;
colset PSTAT = product PATIENT * ROOMS * STAFF;
colset PSTAT_T = PSTAT timed;
colset HRACT = product HR * ROOM timed;
```

These types are used to carry the following information, which is used in the model description, creation, and simulation:

- PTYPE or patient type allows us to distinguish emergency EM from elective (EL). In general, a more complex type may be associated that will allow other patient or application-specific attributes.
- PID is patient ID and PID_T is the associated timed version. The latter allows the creation of the timed tokens to account for various delays and processing times.
- PATIENT is a compound type consisting of patient type, patient ID, and patient's arrival time. PATIENTS is a list of patients useful in describing a queue.
- ROOM is a room type based on the workflow described above and ROOMS is used to represent a set of rooms.
- HR is a human resource type per the workflow described above and STAFF is a list of those.
- PSTAT is a compound type that captures the status of a patient in terms of assigned rooms and assigned staff. PSTAT_T is its associated timed version for performance metrics. HRACT is a compound type denoting which human resource is active (or assigned to) in which room. It is a timed colorset for performance metrics.

We start with the *Patient Entry* module. This module is responsible for generating patients that either go for elective or emergency surgery. In this paper, we have assumed the inter-arrival time to be exponentially distributed. Internally, this module utilizes the type PID_T to generate a timed token with the next patient ID and arrival time. Based on this information, a token of type PATIENT is generated, which will move either to *To Emergency* or *To Elective* depending on the PTYPE value of the token.

After this, the patient (or token) will follow the *Emergency Workflow* module or the *Elective Workflow* module of the net shown in Figure 3 on the left. The two workflows essentially differ in terms of the *Transfer Activity* module as given by the module hierarchy diagram on the right in Figure 3. We, therefore, focus mainly on the details of the *Emergency Workflow* module. Specifically, we present details of the following sub-modules: *Emergency Induction* and its sub-module *Long Emergency Induction*; *Emergency Operation* and two of its sub-modules, namely, *Emergency Preparation* and *Long Emergency Surgery*; and finally the *Patient Recovery* module.

The next two modules, namely *Emergency Induction* and its sub-module *Long Emergency Induction* are shown in Figure 4 on the left and right, respectively. As shown in the figure, when the transition *Add to Queue* fires, the incoming patient token will be added to the *Emergency Induction Queue*. The next patient in the queue enters the induction room only if *OR Block for Urgencies* and *Induction Room for Urgencies* is available. We are using the term Urgency instead of Emergency per the original paper. Additionally, it requires the availability of an *Anaesthesiologist Staff*. All these resource constraints are captured in a very simple and visual manner by the incoming arcs of the *Enter Induction Room* transition in the figure.

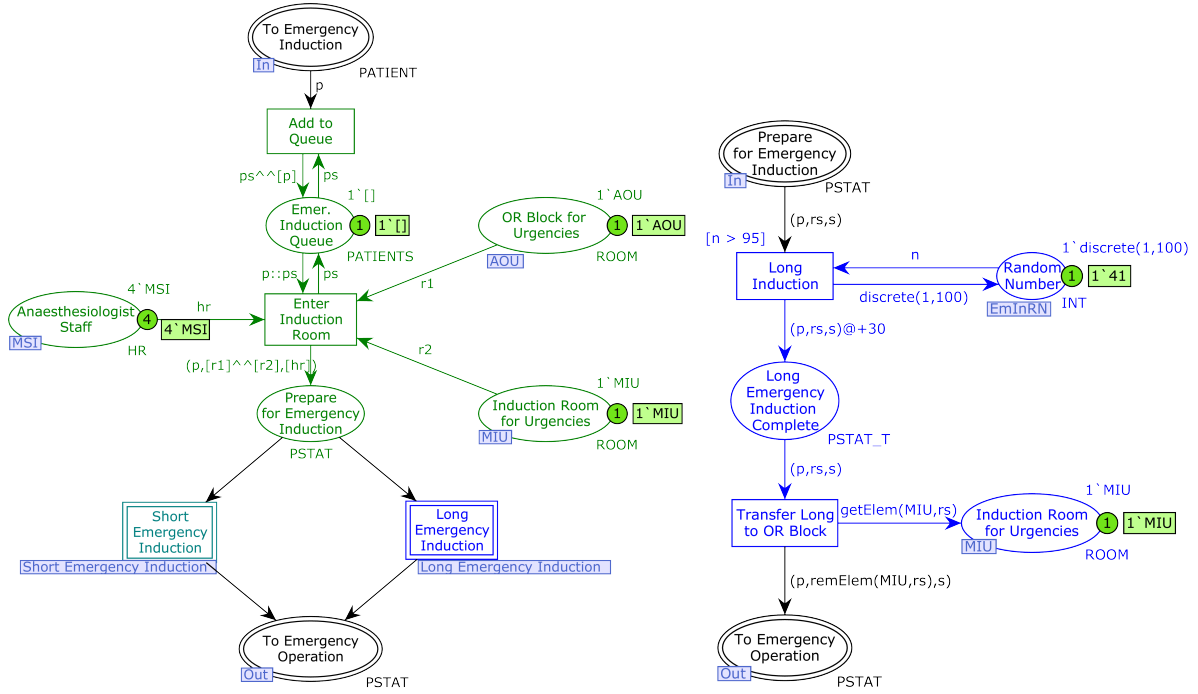


Figure 4: Left to right: the *Emergency Induction* module and its sub-module *Long Emergency Induction* as shown in the module hierarchy of *Emergency Workflow* in Figure 3.

The net on the right in Figure 4 shows the *Long Emergency Induction* module. The boolean condition $[n > 95]$ on the transition *Long Induction* and the random number in the connecting place *Random Number* guarantee the probability of long induction to be 0.05, as specified in Figure 1. An associated timed token in *Long Emergency Induction Complete* determines the time for long induction.

After induction, a patient moves to *Emergency Operation*, which itself consists of two sub-modules: *Emergency Preparation* and *Emergency Surgery*. As depicted in Figure 1, emergency surgeries can be either of short duration or average duration or long duration. We only include the *Long Emergency Surgery* module here since the other two are similar. Figure 5 shows the two sub-modules *Emergency Preparation* and *Long Emergency Surgery* from left to right. As shown in the associated net, Patient Installation requires the availability of *Medical Staff for Urgencies* and *Nurse Assistants*. Once *Patient Preparation* is finished, the *Nurse Assistant* becomes available for other patients as captured by the outgoing arc from *Patient Preparation* to *Nurse Assistant*. At this stage, the human resource *Medical Staff for Urgencies* is considered still in use, that is, busy. The prepared patient then enters *Emergency Surgery*. A patient requiring long surgery will follow the net depicted on the right in Figure 5. The boolean condition $[n > 7]$ on the transition *Long Emergency Surgery* and the random number in the connecting place *Random Number* guarantee the probability of long surgery to be 0.30, as specified in Figure 1. An associated timed token in *Patient in Long Emergency Surgery* determines the time for surgery. When done, that is, the transition *Complete Long Surgery* fires, both human resources, namely, *Anaesthesiologist Staff* from induction stage and *Medical Staff for Urgencies* the patient preparation stage are returned to their respective free pools.

The final stage is patient recovery. The associated *Patient Recovery* sub-module is shown in Figure 6. As depicted in the associated net, Transfer to Recovery Room requires availability in the *Recovery Room* and an available *Anaesthesiologist Staff for Recovery*. At this stage, the *Nurse Assistant* and the *Waiting Room* from the previous stage are returned to their respective free pools. An associated timed token in *Enter Recovery Room* determines the time for recovery. Once the recovery is complete, that is, the model

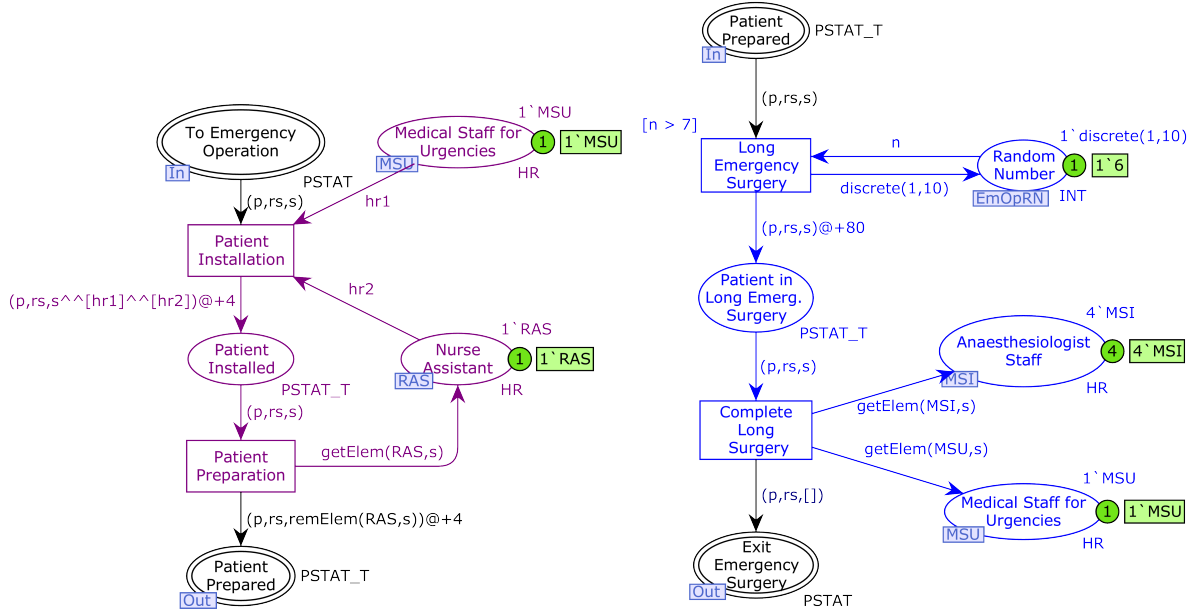


Figure 5: Left to right: *Emergency Preparation* and *Long Emergency Surgery* sub-modules as shown in the module hierarchy of *Emergency Workflow* in Figure 3.

time reaches the time stamp on the timed token, and the *Recovery* transition fires, the room and the staff are returned to their respective free pools, and the patient is moved to *Discharge*.

This completes the discussion of our hierarchical CPN model. Next, we briefly describe the monitoring faculties of CPN Tools we utilized to collect data and generate performance reports.

6 DATA COLLECTION AND RESULTS

CPN Tools provide a monitoring facility to conduct performance analysis of a system (Wells 2006). Monitors are used to extract relevant data during a simulation run. Monitors can be associated with any subnet of interest. Different types of monitors can be defined for a net. For example, a *simulation breakpoint monitor* can be used to stop a simulation run based on a specified condition. A *data collector monitor* is used to extract numerical data from a model during a simulation and to calculate statistics for the extracted data. Once monitors have been created, the built-in function `CPN' Replications.nreplikations` can be used to run any number of simulation replications, collect data, and calculate, among other values, 90%, 95%, and 99% confidence intervals for averages. It also auto-generates a *performance report* containing statistics, including confidence intervals, that are calculated for the *independent and identically distributed* (IID) data values in the replication output log files.

We set a breakpoint monitor for a 24-hour period and ran simulation replications with a medium traffic flow with an average inter-arrival of one hour and another with intense traffic flow with an average inter-arrival of ten minutes. Following the recommendations by Law (2015), we set the replication count to be five. Table 1 contains some data from the first replication run. Our results show that the utilization rates of both the anaesthesiologist staff and recovery rooms were low, highlighting a potential area to save resources. Furthermore, while the nurse assistant maintained a comfortably high utilization rate, the rate of the reception nurse was much lower, showing the potential of reclassifying them into a shared resource.

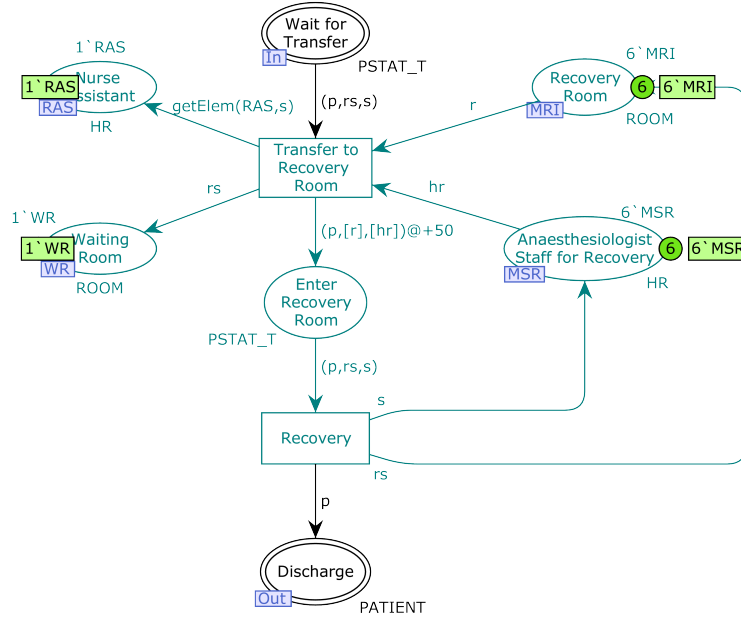


Figure 6: The *Patient Recovery* sub-module as shown in the module hierarchy of *Emergency Workflow* in Figure 3.

7 CONCLUSION AND FUTURE WORK

Adoption of modeling and simulation in healthcare continues to be a challenging issue. One key barrier is buy-in from the stakeholders. Certainly, as noted by Laker et al. (2018), simulation-based approaches can help improve patient safety and help better manage resources in a costly and constrained system like healthcare. Of particular importance is emergency care since there is data confirming that the number of emergency visits in the US is going up whereas the number of emergency departments providing such services is on the decline. Furthermore, COVID-19 forced many hospitals to re-evaluate and re-engineer their workflows but in absence of any simulation-based tools, there is no simple way to evaluate the impact of such changes. In our own experience, we have found a Colored Petri Nets-based approach to be less of a barrier for the stakeholders owing to a simple and visual graphical representation of the net model and its associated intuitive semantics. Furthermore, the free CPN Tools software with its visual editing and simulation capabilities renders it a very user-friendly environment for model development and analysis. We illustrated our approach by employing an operating room workflow and taking into account a variety of resources and constraints (room and staff availability) in a natural manner using the hierarchical CPN notation. The modular approach offered by the hierarchical CPNs allows a model to be constructed incrementally and, therefore, supports a very agile approach. We presented details of data collection and summarized our results.

In absence of other details in (Barkaoui et al. 2002), our model currently assumes the same patient flow and availability of resources throughout the day. However, in reality, these details vary by the time of the day. Furthermore, there could be other multiple points of entry into the system in the light of virtual care. Our current project is to extend this model to accommodate the virtual model of care in orthopaedic surgery as described in (King et al. 2020) and then validate the model against a set of real data as described in (Sargent 2013). In the long-term, we want to focus on the whole continuum of care by factoring in other performance indicators such as re-hospitalization and patient satisfaction.

Table 1: Sample data from the auto-generated performance report after five simulation replications are run for a set of input parameters.

Name	Avrg	90% Half Len	95% Half Len	99% Half Len	StD	Min	Max
Emerg_MWT.Before.Induction							
count_iid	4.00	1.34	1.75	2.91	1.41	2	6
max_iid	15.60	15.38	20.02	33.21	16.13	0	41
min_iid	0.00	0.00	0.00	0.00	0.00	0	0
sum_iid	16.20	15.67	20.40	33.84	16.43	0	41
avrg_iid	3.36	2.80	3.64	6.04	2.93	0.00	6.83
RAS.Util_Rate							
count_iid	191.60	30.99	40.35	66.93	32.50	140	230
max_iid	1.00	0.00	0.00	0.00	0.00	1	1
min_iid	0.00	0.00	0.00	0.00	0.00	0	0
avrg_iid	0.31	0.05	0.06	0.11	0.05	0.23	0.38
RI.Util_Rate							
count_iid	52.60	9.66	12.58	20.87	10.13	37	65
max_iid	1.00	0.00	0.00	0.00	0.00	1	1
min_iid	0.00	0.00	0.00	0.00	0.00	0	0
avrg_iid	0.07	0.01	0.01	0.02	0.01	0.04	0.08

REFERENCES

- Barkaoui, K., P. Dechambre, and R. Hachicha. 2002. "Verification and optimisation of an operating room workflow". In *Proceedings of the 35th Annual Hawaii International Conference on System Sciences*, 2581–2590.
- Brailsford, S. C., T. Bolt, C. Connell, J. H. Klein, and B. Patel. 2009. "Stakeholder engagement in health care simulation". In *Proceedings of the 2009 Winter Simulation Conference*, edited by M. D. Rossetti, R. R. Hill, B. Johansson, A. Dunkin, and R. G. Ingalls, 1840–1849. Piscataway, New Jersey: Institute of Electrical and Electronics Engineers, Inc.
- Centeno, M. A., H. R. Dodd, M. Aranda, and Y. Sanchez. 2010. "A simulation study to increase throughput in an endoscopy center". In *Proceedings of the 2010 Winter Simulation Conference*, 2462–2473.
- Cho, K. W., S. M. Kim, Y. M. Chae, and Y. U. Song. 2017. "Application of Queueing Theory to the Analysis of Changes in Outpatients' Waiting Times in Hospitals Introducing EMR". *Healthcare Informatics Research* 23(1):35–42.
- Christov, S., B. Chen, G. S. Avrunin, L. A. Clarke, L. J. Osterweil, D. Brown, L. Cassells, and W. Mertens. 2008. "Rigorously Defining and Analyzing Medical Processes: An Experience Report". In *Models in Software Engineering*, edited by H. Giese, 118–131. Berlin, Heidelberg: Springer Berlin Heidelberg.
- Creemers, S., and M. Lambrecht. 2011. *Modeling a Hospital Queueing Network*, 767–798. Boston, MA: Springer US.
- Creemers, S., and M. R. Lambrecht. 2007. "Modeling a healthcare system as a queueing network: The case of a Belgian hospital". Technical report, Department of Decision Sciences and Information Management, Katholieke Universiteit Leuven, Belgium.
- Eldabi, T. 2009. "Implementation issues of modeling healthcare problems: Misconceptions and lessons". In *Proceedings of the 2009 Winter Simulation Conference*, edited by M. D. Rossetti, R. R. Hill, B. Johansson, A. Dunkin, and R. G. Ingalls, 1831–1839. Piscataway, New Jersey: Institute of Electrical and Electronics Engineers, Inc.
- Gehlot, V. 2019. "From Petri Nets to Colored Petri Nets: A Tutorial Introduction to Nets Based Formalism for Modeling and Simulation". In *Proceedings of the 2019 Winter Simulation Conference*, edited by N. Mustafee, K.-H. G. Bae, S. Lazarova-Molnar, M. Rabe, C. Szabo, P. Haas, and Y.-J. Son, 1519–1533. Piscataway, New Jersey: Institute of Electrical and Electronics Engineers, Inc.
- Gehlot, V., and E. B. Sloane. 2006. "Ensuring patient safety in wireless medical device networks". *Computer* 39(4):54–60.
- Giua, A., and M. Silva. 2017. "Modeling, analysis and control of Discrete Event Systems: a Petri net perspective". *IFAC-PapersOnLine* 50(1):1772–1783. 20th IFAC World Congress.
- Jahangirian, M., A. Naseer, L. Stergioulas, T. Young, T. Eldabi, S. Brailsford, B. Patel, and P. Harper. 2012. "Simulation in health-care: lessons from other sectors". *Operational Research* 12:45–55.
- Jensen, K. 1981. "Coloured Petri Nets and the Invariant Method". *Theoretical Computer Science* 14(3):317–336.
- Jensen, K. 1994. "An Introduction to the Theoretical Aspects of Coloured Petri Nets". In *A Decade of Concurrency*, edited by J. W. de Bakker, W.-P. de Roever, and G. Rozenberg, Volume 803 of *Lecture Notes in Computer Science*, 230–272. Berlin-Heidelberg: Springer-Verlag.
- Jensen, K., and L. M. Kristensen. 2009. *Coloured Petri Nets. Modelling and Validation of Concurrent Systems*. Berlin-Heidelberg: Springer-Verlag.
- Jensen, K., and L. M. Kristensen. 2015. "Colored Petri Nets: A Graphical Language for Formal Modeling and Validation of Concurrent Systems". *Communications of the ACM* 58(6):61–70.

- King, D., A. K. Emara, M. K. Ng, P. J. Evans, K. Estes, K. P. Spindler, T. Mroz, B. M. Patterson, V. E. Krebs, S. Pinney, and N. S. P. and J. L. Schaffer. 2020. "Transformation from a traditional model to a virtual model of care in orthopaedic surgery: COVID-19 experience and beyond". *Bone & Joint Open* 1(6):272–280.
- Laker, L. F., E. Torabi, D. J. France, C. M. Froehle, E. J. Goldlust, N. R. Hoot, P. Kasaie, M. S. Lyons, L. H. BargWalkow, M. J. Ward, and R. L. Wears. 2018. "Understanding Emergency Care Delivery Through Computer Simulation Modeling". *Acad Emerg Med*. 25(2):116–127.
- Law, A. M. 2015. *Simulation Modeling and Analysis*. 5th ed. New York: McGraw-Hill.
- Osterweil, L. J., G. S. Avrunin, B. Chen, L. A. Clarke, R. Cobleigh, E. A. Henneman, and P. L. Henneman. 2007. "Engineering Medical Processes to Improve Their Safety". In *Situational Method Engineering: Fundamentals and Experiences*, edited by J. Ralyté, S. Brinkkemper, and B. Henderson-Sellers, 267–282. Boston, MA: Springer US.
- Reid, P. P., W. D. Compton, J. H. Grossman, and G. Fanjiang. 2005. "Building a Better Delivery System: A New Engineering/Healthcare Partnership". Technical report, National Academies Press, Washington D.C.
- Sargent, R. G. 2013. "Verification and validation of simulation models". *Journal of Simulation* 7(1):12–24.
- Simon, E., J. Oyekun, W. Hutabarat, A. Tiwari, and C. J. Turner. 2018. "Adapting Petri Nets to Discrete Event Simulation for the Stochastic Modelling of Manufacturing Systems". *International Journal of Simulation Modelling* 17(1):5–17.
- Sloane, E. B., and V. Gehlot. 2007. "Use of Coloured Petri Net models in planning, design, and simulation of intelligent wireless medical device networks for safe and flexible hospital capacity management". *International Journal of Networking and Virtual Organisations* 4(2):118–129.
- Taheri, J., Z. Gellad, D. Burchfield, and K. Cooper. 2012. "A simulation study to reduce nurse overtime and improve patient flow time at a hospital endoscopy unit". In *Proceedings of the 2012 Winter Simulation Conference (WSC)*, 1–12.
- Tako, A. A., and S. Robinson. 2015. "Is simulation in health different?". *Journal of the Operational Research Society* 66(4):602–614.
- Wells, L. 2006. "Performance analysis using CPN tools". In *Proceedings of the 1st International ICST Conference on Performance Evaluation Methodologies and Tools*, 59–68.
- Young, T., J. Eatock, M. Jahangirian, A. Naseer, and R. Lilford. 2009. "Three critical challenges for modeling and simulation in healthcare". In *Proceedings of the 2009 Winter Simulation Conference*, edited by M. D. Rossetti, R. R. Hill, B. Johansson, A. Dunkin, and R. G. Ingalls, 1823–1830. Piscataway, New Jersey: Institute of Electrical and Electronics Engineers, Inc.
- Zeinalnezhad, M., A. Chofreh, F. Goni, J. Klemes, and E. Sari. 2020. "Simulation and improvement of patients' workflow in heart clinics during covid-19 pandemic using timed coloured Petri nets". *International Journal of Environmental Research and Public Health* 17(22):1–18.

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