A RISK MINIMIZING METHOD OF LARGE GROUP DECISION AND ITS APPLICATION IN ENGINEERING CONSTRUCTION ACCIDENT RESCUE

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ABSTRACT

Based on the decision risk caused by both the ambiguity of emergency information and the large group preference conflict, a risk minimizing method of large group decision is proposed. First, the decision group is clustered by preferences to form aggregation preference matrix. Second, the interval-valued intuitionistic fuzzy (IVIF) distance is proposed in form of intuitionistic fuzzy (IF) number in order to reduce the loss of preference information. And generalized IVIF number is also defined. Combining with prospect theory, the IF prospect matrix of different cluster is obtained by conversion. Then, a optimization model of large group decision fuzzy conflict entropy is constructed. Prospect matrix and attribute’s weight are aggregated to figure out the comprehensive prospect values which decide the ranking of alternatives. Finally, a case analysis of Coal-mine Engineering Water-penetration Accident Rescue and comparison are used to illustrate the rationality and effectiveness of above method.

1 INTRODUCTION

In recent years, with the increasing development of China’s economic strength, the number of projects shows the prominent upward trend. The frequent events highlight the modern society in a desperate need of emergency decision-making (Zhong et al. 2012). It is an urgent problem to be solved to decrease or avoid the major loss of people injury, harm and even death as early as possible under the pressure of limited information and time.

The scale of experts from different fields participating in decision-making is usually so large that this kind of emergency decision is featured with large group decision-making (decision expert numbers is not less than 11 (Song et al. 2000)). In the emergency decision-making process, the preference information cannot usually be accurately expressed, there is a certain degree of hesitation, often manifested as IF form, for affecting factors such as the uncertainty of the external environment and so on. The concept of IF set (Atanassov 1986) and the IVIF set (Atanassov 1989) extending IF set to the interval numbers on [0,1] are proposed in this context to express the fuzziness and uncertainty of preference information of decision makers. However, the traditional distance of the IVIF numbers is mostly in the form of real numbers (Xu and Chen 2011; Zhang et al. 2014; Dugenci 2016), which is bound to result in the loss of information and cannot reflect the fuzziness of the initial information. Besides, there is much research by using multiple attribute decision making method, prospect theory etc. to study emergency decision problems (Li et al. 2013; Fan et al. 2012; Xu et al. 2015). The researches provide reference for the emergency decision problem, but there are few researches having the comprehensive consideration of large group conflicts under uncertain information in the background of engineering.

Therefore, this paper proposes a new method of risky large group emergency decision-making based on fuzzy-conflict entropy, combining large group decision making with risky emergency decision making.
by introducing the decision risk into the emergency decision the background of engineering. Then, in order to make full use of the fuzzy and uncertain preference information and reduce the loss of information, we put forward an IVIF distance with IF form, while defining generalized IF numbers and its operators. Combining with prospect theory, the generalized IF prospect matrix of different cluster is obtained, and then the IF prospect matrix of different cluster can be obtained by conversion. Furthermore, large group fuzzy-conflict entropy emergency decision-making model is constructed of which the goal is to minimize the decision-making risk in the process of emergency decision-making. According to this optimization model, we can get every attribute’s weight. Finally, prospect matrix and attribute’s weight are aggregated to figure out the comprehensive prospect values which decide the ranking of alternatives. With the ranking results, quick response should be made to prevent major loss under the pressure of limited information and time before things get worse. It will provide a support for project management under the emergency situation.

2 PROBLEM DESCRIPTION AND PRELIMINARIES

2.1 Problem Description

In the process of risky large group emergency decision, \( E = \{e_1, e_2, \ldots, e_M\} \) stands for the set of M DMs; \( X = \{x_1, x_2, \ldots, x_p\} \) stands for the set of P alternatives; \( F = \{f_1, f_2, \ldots, f_N\} \) indicates the set of N attributes. The vector \( \mathbf{w} = (w_1, w_2, \ldots, w_N)^T \) denotes the attribute weights and some attribute weights are known; \( w_j \geq 0, \sum_{j=1}^{N} w_j = 1 \); the set \( Z \) denote the known information, let \( \Theta \) stands for the set of H scenarios and \( \Theta = \{\theta_1, \theta_2, \ldots, \theta_H\} \); let \( \Phi = \{p_{lh}\}_{P \times H} \) stands for the scenario probability matrix, where \( p_{lh}(l = 1, 2, \ldots, P; h = 1, 2, \ldots, H) \) denotes the probability of being controlled of scenario \( \theta_h \) when takes alternative \( x_i \) and satisfies conditions of \( 0 \leq p_{lh} \leq 1 \) and \( \sum_{h=1}^{H} p_{lh} = 1 \); let \( A^i = \{\tilde{a}_{ij}\}_{P \times N} \) stands for the decision matrix of DM \( e_i \), where \( \tilde{a}_{ij} \) denotes the preference value of alternative \( x_i \) given by DM \( e_i \) with respect to the attribute \( f_j \); \( \tilde{a}_{ij} \) stands for the set of ordered pairs, that is \( \tilde{a}_{ij} = \{(\tilde{a}_{ij}^1, p_{1h}), (\tilde{a}_{ij}^2, p_{2h}), \ldots, (\tilde{a}_{ij}^{wh}, p_{wh})\} \), where \( \tilde{a}_{ij}^{wh} \) is in the form of an IVIF number and denotes the result of scenario \( \theta_h \).

2.2 IVIF Set

**Definition 1** (Atanassov 1989) Let \( X \) be a non-empty real number set. Define \( A = \{x, \tilde{u}_A(x), \tilde{v}_A(x) \mid x \in X\} \) as the IVIF set, where \( \tilde{u}_A(x) = [u_A^-(x), u_A^+(x)] \subset [0, 1] \) and \( \tilde{v}_A(x) = [v_A^-(x), v_A^+(x)] \subset [0, 1] \) denote the membership degree interval and the non-membership degree interval, respectively, of the element \( x \) in \( X \) mapped to \( A \). In addition, \( u_A^-(x) + v_A^+(x) \leq 1 \) is true for every \( x \in X \).

Let \( \tilde{u}_A(x) = 1 - \tilde{v}_A(x) - \tilde{v}_A(x) \), that is \( \pi_A(x) = 1 - u_A^-(x) - v_A^+(x), \pi_A(x) = 1 - u_A^-(x) - v_A^+(x) \), which is called the hesitant degree interval of the element \( x \) in \( X \) mapped to \( A \).

For convenience, we denote \( \tilde{a} = ([a, b], [c, d]) \) as an IVIF number, where the membership degree interval \([a, b] \subset [0, 1]\), the non-membership degree interval \([c, d] \subset [0, 1]\), and \( b + d \leq 1 \). And the hesitant degree interval \([e, f] = [1 - b - d, 1 - a - c]\).

**Definition 2** (Xu 2007) Let \( \tilde{a}_j = ([a_j, b_j], [c_j, d_j]) (j = 1, 2) \) be two IVIF numbers, then
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\[ d(\tilde{a}_1, \tilde{a}_2) = (a_1 + a_2 - a_1 a_2, b_1 + b_2 - b_1 b_2, c_1 c_2, d_1 d_2)\]
\[ \lambda \tilde{a}_1 = (1 - (1 - a_1)^\lambda, 1 - (1 - b_1)^\lambda, 1 - (1 - c_1)^\lambda, 1 - (1 - d_1)^\lambda)\]
\[ \tilde{a}^2 = (a^2, b^2, c^2, d^2)\]

**Definition 3 (Xu 2007)** Let \( \tilde{a}_j = ([a_j, b_j], [c_j, d_j]) \) be a series of IVIF numbers, then the IVIF weighted average (IFWA) operator can be denoted as follows:

\[ \text{IFWA}_w(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = ((1 - \prod_{j=1}^{n} (1 - a_j))^{-w_j}, 1 - \prod_{j=1}^{n} (1 - b_j))^{-w_j}, \prod_{j=1}^{n} (1 - c_j))^{-w_j}, \prod_{j=1}^{n} (1 - d_j)^{-w_j}) \]

where \( w = (w_1, w_2, ..., w_n) \) is the weight vector of \( \tilde{a}_j (j = 1, 2, ..., n) \) and \( w_j \in [0, 1] \), \( \sum w_j = 1 \).

**Definition 4 (Xu 2007)** Let \( \tilde{a}_j = ([a_j, b_j], [c_j, d_j]) \) be two IVIF numbers, \( s(\tilde{a}_j) = \frac{a_j - c_j + b_j - d_j}{2} \)

and \( h(\tilde{a}_j) = \frac{a_j + b_j + c_j + d_j}{2} \) denote the score function and accuracy function, respectively.

**Definition 5 (Dong et al. 2009)** Let \( \tilde{a}_j = ([a_j, b_j], [c_j, d_j]) \) be two IVIF numbers, then the similarity degree \( K_{IFNS}(\tilde{a}_1, \tilde{a}_2) \) can be denoted as follows:

\[ K_{IFNS}(\tilde{a}_1, \tilde{a}_2) = \frac{a_1 b_2 + b_1 c_2 + c_1 d_2 + d_1 a_2 + e_1 e_2 + f_1 f_2}{\sqrt{(a_1)^2 + (b_1)^2 + (c_1)^2 + (d_1)^2 + (e_1)^2 + (f_1)^2 + (a_2)^2 + (b_2)^2} + (c_2)^2 + (d_2)^2 + (e_2)^2 + (f_2)^2} \]

\( K_{IFNS}(\tilde{a}_1, \tilde{a}_2) \) satisfies the property in literature (Dong et al. 2009).

### 2.3 The New Distance of IVIF Numbers

The traditional interval valued intuitionistic fuzzy distance with the real number form is bound to result in the loss of information. Therefore, this paper defines a new distance of an intuitionistic fuzzy form reflecting hesitation degree and simplifying the operation.

**Definition 6** Let \( \tilde{a}_j = ([a_j, b_j], [c_j, d_j]) \) be two IVIF numbers, then the distance between \( \tilde{a}_1 \) and \( \tilde{a}_2 \) can be denoted as follows:

\[ d(\tilde{a}_1, \tilde{a}_2) = \left( \frac{R_{\min} + S_{\min}}{2}, 1 - \frac{R_{\max} + S_{\max}}{2} \right) \]

where \( R_{\min} = \min(|u_1 - u_2|, |u_1^+ - u_2^+|), R_{\max} = \max(|u_1 - u_2|, |u_1^+ - u_2^+|), S_{\min} = \min(|v_1 - v_2|, |v_1^+ - v_2^+|), S_{\max} = \max(|v_1 - v_2|, |v_1^+ - v_2^+|) \).

### 2.4 Prospect Theory

Prospect value function of prospect theory is \( V = \sum_{i=1}^{n} w^+(p_i) \cdot v(x_i) + \sum_{p_i \in \mathbb{R}^n \geq 0} w^-(p_i) \cdot v(x_i) \) . In this function, \( V \) is the prospect value; \( w^+(p_i) \) and \( w^-(p_i) \) are respectively the gain and loss of the decision weight function; \( v(x_i) \) is the value function. Decision weight function is \( w^+(p_i) = \frac{p_i^{\gamma_1}}{(p_i^{\gamma_1} + (1 - p_i)^{\gamma_1})^{\gamma_1}}, \)

\( w^-(p_i) = \frac{p_i^{\gamma_2}}{(p_i^{\gamma_2} + (1 - p_i)^{\gamma_2})^{\gamma_2}}, \) where \( \gamma_1 \) and \( \gamma_2 \) is the risk attitude coefficients for gains and losses, respectively.

Value function is \( v(x_i) = \begin{cases} d(x_i, x_0)^\alpha, x_i \geq x_0, \\ -\eta \cdot d(x_i, x_0)^\beta, x_i < x_0 \end{cases} \), where \( x_0 \) is the reference point, \( d(x_i, x_0) \) is the distance between the reference point \( x_0 \) and the preference \( x_i \) of decision schemes, \( \alpha \) and \( \beta \) are respectively the risk
2.5 Generalized IVIF Numbers

We define the generalized IVIF number based on IVIF number.

**Definition 7** Let X be a non-empty real number set. Define \( \tilde{\alpha}_x = ([\hat{u}_A(x), \bar{u}_A(x)], [\hat{v}_A(x), \bar{v}_A(x)]) \) as a generalized IVIF number, where \( \hat{u}_A(x), \hat{v}_A(x), \bar{u}_A(x), \bar{v}_A(x) \) are real numbers. Its corresponding set \( A = \{ < x, \hat{u}_A(x), \bar{v}_A(x) | x \in X \} \) is called the generalized IVIF set.

**Definition 8** Let \( \tilde{\alpha}_j = ([a_j, b_j], [c_j, d_j]) \) \((j = 1, 2)\) be two IVIF numbers, then

1. \( \tilde{\alpha}_1 + \tilde{\alpha}_2 = ([a_1 + a_2, b_1 + b_2], [c_1 + c_2, d_1 + d_2]) \);  
2. \( \lambda \tilde{\alpha}_1 = (\lambda [a_1, b_1], \lambda [c_1, d_1]), \lambda \geq 0 \);  
3. \( \lambda \tilde{\alpha}_1 = (\lambda [b_1, a_1], \lambda [d_1, c_1]), \lambda < 0 \).

From the above definition, the generalized IF numbers can be regarded as a special form of generalized IVIF numbers, so all of the above definitions also apply to generalized IF numbers, it will not be explained here.

3 THE PRINCIPLE OF METHOD

3.1 Preference Aggregation

We can cluster \( M \) DMs by means of Eq.(2), which is used as a similarity model(Xu 2007). We assume that the decision group \( E \) formed \( K(1 \leq K \leq m) \) clusters after clustering. Large-scale clusters should be assigned larger weight because they contain the majority’s opinion. Therefore, let \( \lambda_k = \frac{n_k}{M} \) be the weight of the cluster \( C^k \), where \( k = 1, 2, \ldots, K \) and \( n_k \) is the number of the cluster \( C^k \). Then the cluster weight vector can be denoted as \( \Lambda = (\lambda_1, \lambda_2, \ldots, \lambda_K)^T \).

We can obtain the IVIF preference matrix of cluster \( C^i \) by means of Definition 3, and the matrix is

\[
G^k = \left[ g_{ij}^k \right]_{p \times n} = \left[ \frac{1}{n_k} \sum_{i=1}^{n_k} \tilde{\alpha}_{ij} \right]_{p \times n}.
\]

3.2 Determine the IF Prospect Matrix of Clusters

According to the definition of IVIF numbers and the score function, let \( g_0 = ([0.5, 0.5], [0.5, 0.5]) \) be the reference point. The prospect value of each alternative about each attribute in cluster \( C^i \) can be calculated as follows:

\[
q_{ij}^k = \sum_{h=1, y(h) \geq 0}^{H} w^+ (p_{ih}) \cdot v(g_{ij}^{kh}) + \sum_{h=1, y(h) < 0}^{H} w^- (p_{ih}) \cdot v(g_{ij}^{kh})
\]

(4)

The calculation of prospect value will involve the linear combination of IF numbers, therefore, we use Definition 8 to calculate the prospect value, consider IF numbers as the special form of generalized IF numbers, then the generalized IF prospect matrix \( \hat{Q}_i^k = [\hat{q}_{ij}^k]_{p \times N} \) of cluster \( C^k \) can be obtained, where \( \hat{q}_{ij}^k \) is a generalized IF number.

According to literature(Kahneman and Tversky 1979; Langer and Weber 2001), let \( \alpha = 0.89, \beta = 0.92, \eta = 2.25, \gamma_1 = 0.61, \gamma_2 = 0.69 \).

Before the next step, we first convert the generalized IF number into the IF number, thus we can use algorithm of IF numbers in the subsequent steps. The transformation method is as follows:
1. Let \( \zeta_j = \min\{ u_{ij}^k, v_{ij}^k \} \), \( l = 1,2,\ldots, P, k = 1,2,\ldots, N \), \( \hat{q}_{ij}^k = (\hat{u}_{ij}^k, \hat{v}_{ij}^k) = (u_{ij}^k - \zeta_j, v_{ij}^k - \zeta_j) \), the transformed value matrix is denoted as \( \hat{Q}^k = [\hat{q}_{ij}^k]_{P \times N} \).

2. Let \( \mathcal{G} = \max\{ \hat{u}_{ij}^k + \hat{v}_{ij}^k \} \), \( l = 1,2,\ldots, P, j = 1,2,\ldots, N, k = 1,2,\ldots, K \), \( q_{ij}^k = (u_{ij}^k, v_{ij}^k) = (\hat{u}_{ij}^k / \mathcal{G}, \hat{v}_{ij}^k / \mathcal{G}) \), the transformed value matrix is denoted as \( Q^k = [q_{ij}^k]_{P \times N} \), where \( q_{ij}^k \) is an IF number. If \( \mathcal{G} \leq 1 \), it is no necessary to carry out \( 2 \).

3.3 Determine the Weight of the Attribute

Combining the prospect decision matrix \( Q^k = [q_{ij}^k]_{P \times N} \) with the weight vector \( \lambda = (\lambda_1, \lambda_2, \ldots, \lambda_K)^T \) of clusters, we can obtain the prospect decision matrix \( Q = [q_{ij}]_{P \times N} = \left[ \sum_{k=1}^{K} \lambda_k q_{ij}^k \right]_{P \times N} \) of the large group.

This paper uses the fuzzy entropy and the conflict entropy to describe the preference vagueness of the large group and the uncertainty caused by conflict, respectively.

3.3.1 Calculate the Fuzzy Entropy of the Alternative

This paper describes this uncertainty by fuzzy entropy. We can get the comprehensive prospect value \( q_i = \sum_{j=1}^{N} w_j q_{ij} \) of the alternative \( x_i \) by the prospect matrix \( Q = [q_{ij}]_{P \times N} \) of the large group, then we can get the fuzzy entropy \( H(q_i) = H(\sum_{j=1}^{N} w_j q_{ij}) = \frac{1}{\lambda} | u_j - v_j | + \pi_l \) of the alternative \( x_i \).

According to the concept of the fuzzy entropy, if the fuzzy entropy of the alternative is greater, the uncertainty of the decision result will be greater, and so will be the risk of decision-making caused by the uncertainty of preference information.

3.3.2 Calculate the Conflict Entropy of the Cluster

The deviation between the large group’s preferences and those of cluster \( C^k \) for the alternative \( x_i \) under the attribute \( f_j \) can be expressed as follows:

\( \delta_{ij}^k = \max\{| u_{ij}^k - u_{ij} |, | v_{ij}^k - v_{ij} |, | \pi_{ij}^k - \pi_{ij} | \} \).

The total deviation of preferences between the cluster \( C^k \) and the large group can be expressed as follows:

\( \sigma_k = \sum_{l=1}^{P} \sum_{j=1}^{N} w_j \delta_{ij}^k \)

In this paper, the uncertainty is described by using the conflict entropy. Let variable \( \psi_k \) be the proportion of the difference \( \sigma_k \) between \( C^k \) and large group preference in total difference \( \sum_{k=1}^{K} \sigma_k \), i.e. \( \psi_k = \frac{\lambda_k \sigma_k}{\sum_{k=1}^{K} \lambda_k \sigma_k} \).

The uncertainty of the preference difference between the cluster and the large group can be called the conflict entropy, and then conflict entropy of the cluster \( C_k \) can be defined as
According to the definition of entropy, the greater the entropy is, the smaller the preference difference between the corresponding cluster and the large group is, i.e. the preference between the cluster and large group is more consistent.

3.3.3 Construct Risky-Minimizing Model of Large Group Emergency Decision Based on Fuzzy and Conflict Entropy

Based on the above analysis, we combine fuzzy entropy with conflict entropy, and then regard the minimization of decision risk as a target. Afterwards, the attribute weights partially known are regarded as constraint conditions to construct risky large group emergency decision-making model based on fuzzy-conflict entropy. Finally we can get weight of every attribute.

\[
\begin{align*}
(\text{M1}) & : & \min H_f &= 1 \frac{1}{P} \sum_{l=1}^{P} H(q_l) = 1 \frac{1}{P} \sum_{j=1}^{N} H(w_j q_l) \\
& & \max H_c &= -\sum_{k=1}^{K} \varepsilon_k \ln \varepsilon_k \\
& & \text{s.t. } w_j \in Z, \sum_{j=1}^{N} w_j = 1, 0 \leq w_j \leq 1; j = 1,2,\ldots, N
\end{align*}
\]

Because the dimensions of the objective functions \( H_f \) and \( H_c \) are different, it is essential to carry out no-dimensional disposal to these two objective functions. The multi-objective optimization problem can be transformed into a single objective optimization model:

\[
(\text{M2}) : \quad \min Z = \delta_1 \frac{H_f - H_{f_{\min}}}{H_{f_{\max}} - H_{f_{\min}}} + \delta_2 \frac{H_c - H_{c_{\min}}}{H_{c_{\max}} - H_{c_{\min}}}
\]

\[
\text{s.t. } w_j \in Z, \sum_{j=1}^{N} w_j = 1, 0 \leq w_j \leq 1; j = 1,2,\ldots, N
\]

where \( \delta_1 \) and \( \delta_2 \) denote the importance of the two targets respectively. They satisfy conditions of \( 0 \leq \delta_1, \delta_2 \leq 1 \) and \( \delta_1 + \delta_2 = 1 \). If no special preference is set for the target, take \( \delta_1 = \delta_2 = 1/2 \) generally.

Based on the above model, the weight vector \( w = (w_1, w_2, \ldots, w_N)^T \) of the attribute can be obtained on the basis of minimizing the decision risk.

4 CASE APPLICATION

4.1 Case Background

The effectiveness of the proposed method is illustrated by the alternative selection of Datong coal mine engineering project major water-penetration accident of Shanxi Province of China in 19 April 2015. At that time, the working surface and two crossheading were submerged under the water in a length of about 550 meters. 67 people were trapped underground when the accident occurred. In order to rescue the trapped people, 20 emergency decision-making experts from fire, safety analysis and other related departments gathered to analyze the real situations. Emergency alternatives, evaluation criteria and possible scenarios is presented in Table 1.
Table 1: Emergency alternatives, evaluation criteria and possible scenarios.

<table>
<thead>
<tr>
<th>Measures</th>
<th>Criteria and Weights</th>
<th>Possible Scenarios</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>$f_1$: saving lives; 0.55 ≤ $w_1$ ≤ 0.65</td>
<td>$\theta_1$: there is still a large area of water penetration in the coal mine;</td>
</tr>
<tr>
<td></td>
<td>$f_2$: reducing costs; 0.05 ≤ $w_2$ ≤ 0.15</td>
<td>$\theta_2$: there is still a small area of water penetration in the coal mine;</td>
</tr>
<tr>
<td>$x_2$</td>
<td>$f_3$: shortening the rescue time; 0.35 ≤ $w_3$ ≤ 0.45.</td>
<td>$\theta_3$: the water penetration situation has been effectively curbed;</td>
</tr>
<tr>
<td>$x_3$</td>
<td>$f_4$: evacuation of small batch burst</td>
<td></td>
</tr>
<tr>
<td>$x_4$</td>
<td>$f_5$: evacuation of large batch burst</td>
<td></td>
</tr>
</tbody>
</table>

The probability of being controlled of the scenarios is given by the experts who are experienced in coal mine water penetration accident and combined with historical data. It is presented in Table 2.

Table 2: The probability of being controlled of the scenarios when different alternatives are chosen.

<table>
<thead>
<tr>
<th></th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>0.45</td>
<td>0.30</td>
<td>0.25</td>
</tr>
<tr>
<td>$x_2$</td>
<td>0.50</td>
<td>0.20</td>
<td>0.30</td>
</tr>
<tr>
<td>$x_3$</td>
<td>0.55</td>
<td>0.27</td>
<td>0.18</td>
</tr>
<tr>
<td>$x_4$</td>
<td>0.48</td>
<td>0.20</td>
<td>0.32</td>
</tr>
</tbody>
</table>

DMs give the preference of alternatives with respect to attributes by IVIF numbers, and then the original decision matrix $A^t = [\tilde{a}_{ij}^t]_{p \times N}$ can be constructed.

4.2 Decision-Making Steps

Step 1. Classify 20 experts by the algorithm proposed by literature (Xu et al. 2008) and we can obtain 4 clusters, that is $C_1 = \{e_1, e_3, e_5, e_7, e_{10}, e_{19}\}$, $C_2 = \{e_2, e_5, e_6, e_{10}, e_{13}, e_{15}, e_{17}, e_{20}\}$, $C_3 = \{e_4, e_5, e_{11}, e_{13}\}$, $C_4 = \{e_{12}, e_{14}\}$. And then we can calculate the weight of clusters, $\lambda = (0.3,0.4,0.1,0.2)^T$. The clustering results of cluster $C_1$ are shown in Table 3.

Table 3: The clustering results of cluster $C_1$.

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>(0.12,0.22), (0.04,0.67), (0.03,0.60), (0.20,0.50), (0.04,0.50), (0.38,0.42), (0.10,0.14), (0.10,0.40), (0.12,0.67), (0.46,0.50)</td>
<td>(0.20,0.23)</td>
<td>(0.20,0.38)</td>
<td>(0.20,0.40)</td>
</tr>
<tr>
<td>$x_2$</td>
<td>(0.05,0.15), (0.05,0.60), (0.05,0.50), (0.10,0.15), (0.10,0.10), (0.12,0.19), (0.20,0.48), (0.38,0.40), (0.20,0.40), (0.40,0.80)</td>
<td>(0.20,0.40)</td>
<td>(0.40,0.66)</td>
<td>(0.40,0.65)</td>
</tr>
<tr>
<td>$x_3$</td>
<td>(0.15,0.57), (0.20,0.62), (0.20,0.23), (0.10,0.10), (0.10,0.40), (0.40,0.78), (0.10,0.12), (0.20,0.20), (0.10,0.10), (0.30,0.40)</td>
<td>(0.13,0.25)</td>
<td>(0.30,0.62)</td>
<td>(0.10,0.30)</td>
</tr>
<tr>
<td>$x_4$</td>
<td>(0.28,0.37), (0.22,0.66), (0.10,0.40), (0.30,0.30), (0.50,0.60), (0.20,0.60), (0.20,0.40), (0.20,0.50), (0.10,0.40), (0.30,0.48)</td>
<td>(0.14,0.20)</td>
<td>(0.17,0.58)</td>
<td>(0.10,0.10)</td>
</tr>
</tbody>
</table>

Step 2. Compute the IF prospect matrix of the clusters

The computing results are shown in Table 4 and Table 5 according to the Eq.(3), Eq.(4) and the transformation method in Section 3.2.
of the cluster,

$$\lambda = (0.3, 0.4, 0.2, 0.1)^T$$

of the cluster, the comprehensive prospect value of the large group is obtained. The results are shown in Table 6.
Table 6: Comprehensive prospect value of the large group.

<table>
<thead>
<tr>
<th></th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>(0.41,0.39)</td>
<td>(0.42,0.45)</td>
<td>(0.38,0.26)</td>
</tr>
<tr>
<td>$x_2$</td>
<td>(0.41,0.37)</td>
<td>(0.39,0.28)</td>
<td>(0.40,0.34)</td>
</tr>
<tr>
<td>$x_3$</td>
<td>(0.39,0.31)</td>
<td>(0.37,0.23)</td>
<td>(0.35,0.18)</td>
</tr>
<tr>
<td>$x_4$</td>
<td>(0.39,0.30)</td>
<td>(0.41,0.34)</td>
<td>(0.39,0.29)</td>
</tr>
</tbody>
</table>

We can obtain the attribute weight $w = (0.55,0.05,0.40)^T$ through model (M2).

**Step 4.** According to the Table 6, the comprehensive prospect value of the alternatives can be obtained, and values are $q_1 = (0.40,0.34), q_2 = (0.40,0.35), q_3 = (0.38,0.25), q_4 = (0.39,0.30)$. Then scores are $s_1 = 0.06, s_2 = 0.05, s_3 = 0.13, s_4 = 0.09$ got by Definition 4. The ranking of the alternative is $x_3 > x_4 > x_1 > x_2$, so we select alternative $x_3$ as a quick response to the coal mine accident.

**4.3 The Simulation of Parameters $\delta_1$ and $\delta_2$ in Objective Optimization Model**

In the above steps, parameters $\delta_1, \delta_2$ are given to the value of 0.5 the objective optimization model to obtain the attribute weight $w$. Next, we simulate different values of $\delta_1$ and $\delta_2$ to observe the influence on the values of attribute weight. And then discuss the effect of different values of $\delta_1$ and $\delta_2$ on the ranking of the alternative.

Table 7: Simulation of parameters $\delta_1, \delta_2$ and the value of attribute weight $w$.

<table>
<thead>
<tr>
<th>$\delta_1$</th>
<th>$\delta_2$</th>
<th>$w$</th>
<th>$\delta_1$</th>
<th>$\delta_2$</th>
<th>$w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>(0.55,0.05,0.4)$^T$</td>
<td>0.4</td>
<td>0.6</td>
<td>(0.55,0.045)$^T$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>(0.55,0.05,0.4)$^T$</td>
<td>0.4</td>
<td>0.6</td>
<td>(0.55,0.045)$^T$</td>
</tr>
<tr>
<td>0.9</td>
<td>0.1</td>
<td>(0.55,0.05,0.4)$^T$</td>
<td>0.3</td>
<td>0.7</td>
<td>(0.55,0.045)$^T$</td>
</tr>
<tr>
<td>0.9</td>
<td>0.1</td>
<td>(0.55,0.05,0.4)$^T$</td>
<td>0.3</td>
<td>0.7</td>
<td>(0.55,0.045)$^T$</td>
</tr>
<tr>
<td>0.9</td>
<td>0.1</td>
<td>(0.55,0.05,0.4)$^T$</td>
<td>0.3</td>
<td>0.7</td>
<td>(0.55,0.045)$^T$</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2</td>
<td>(0.55,0.05,0.4)$^T$</td>
<td>0.2</td>
<td>0.8</td>
<td>(0.55,0.045)$^T$</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2</td>
<td>(0.55,0.05,0.4)$^T$</td>
<td>0.2</td>
<td>0.8</td>
<td>(0.55,0.045)$^T$</td>
</tr>
<tr>
<td>0.7</td>
<td>0.3</td>
<td>(0.55,0.05,0.4)$^T$</td>
<td>0.1</td>
<td>0.9</td>
<td>(0.6,0.05,0.35)$^T$</td>
</tr>
<tr>
<td>0.7</td>
<td>0.3</td>
<td>(0.55,0.05,0.4)$^T$</td>
<td>0.1</td>
<td>0.9</td>
<td>(0.6,0.05,0.35)$^T$</td>
</tr>
<tr>
<td>0.6</td>
<td>0.4</td>
<td>(0.55,0.05,0.4)$^T$</td>
<td>0</td>
<td>1</td>
<td>(0.6,0.05,0.35)$^T$</td>
</tr>
<tr>
<td>0.6</td>
<td>0.4</td>
<td>(0.55,0.05,0.4)$^T$</td>
<td>0</td>
<td>1</td>
<td>(0.6,0.05,0.35)$^T$</td>
</tr>
</tbody>
</table>

From the above simulation analysis in Table 7, we can see that when $\delta_1$ and $\delta_2$ are given different values, the weight of the attribute will change. There are three different values of attribute weight appear, namely, (0.55,0.05,0.4)$^T$, (0.55,0.0,45)$^T$, (0.6,0.05,0.35)$^T$. In next step, according to Definition 4, we obtain the scores and rankings under different attribute weights, as shown in Table 8.
Combin and

\[ w \cdot q_1 + w \cdot q_2 + q_3 + q_4 \]

...\( \sum_{j=1}^{N} w_j = 1, 0 \leq w_j \leq 1; \ j = 1, 2, 3 \). We can get the weight \( w = (0.55, 0.10, 0.35)^T \). Combining the prospect values of alternatives, we can obtain the comprehensive prospect values are \( q_1 = 0.07, q_2 = -0.08, q_3 = -0.080, q_4 = -0.002 \). Then the ranking of the alternatives is \( x_1 > x_4 > x_2 > x_3 \), \( x_1 \) is selected as a quick response to the coal mine accident.

It is clear that the two methods have different ranking with consider the loss of preference information and the decision-making risk or not.

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**Cao, Xu, and Pan**

Table 8: Scores and ranking of alternative under different attribute weights.

<table>
<thead>
<tr>
<th>( w )</th>
<th>( q_1 )</th>
<th>( q_2 )</th>
<th>( q_3 )</th>
<th>( q_4 )</th>
<th>Scores</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (0.55, 0.5, 0.4)^T )</td>
<td>( (0.40, 0.34) )</td>
<td>( (0.40, 0.35) )</td>
<td>( (0.38, 0.25) )</td>
<td>( (0.39, 0.30) )</td>
<td>( s_1 = 0.06, s_2 = 0.05, s_3 = 0.13, s_4 = 0.09 )</td>
<td>( x_3 &gt; x_4 &gt; x_1 &gt; x_2 ).</td>
</tr>
<tr>
<td>( (0.55, 0.45)^T )</td>
<td>( (0.40, 0.33) )</td>
<td>( (0.41, 0.36) )</td>
<td>( (0.37, 0.25) )</td>
<td>( (0.39, 0.30) )</td>
<td>( s_1 = 0.07, s_2 = 0.05, s_3 = 0.12, s_4 = 0.09 )</td>
<td>( x_3 &gt; x_4 &gt; x_1 &gt; x_2 ).</td>
</tr>
<tr>
<td>( (0.6, 0.05, 0.35)^T )</td>
<td>( (0.40, 0.35) )</td>
<td>( (0.41, 0.36) )</td>
<td>( (0.38, 0.26) )</td>
<td>( (0.39, 0.30) )</td>
<td>( s_1 = 0.05, s_2 = 0.05, s_3 = 0.11, s_4 = 0.09 )</td>
<td>( x_3 &gt; x_4 &gt; x_1 &gt; x_2 ).</td>
</tr>
</tbody>
</table>

From Table 8, we can see that under different attribute weights, the scores of the alternatives have not changed greatly. The score of the alternative \( x_1 \) has not even changed. Correspondingly, the ranking has not changed, it’s still \( x_3 > x_4 > x_1 > x_2 \). That means that within the scope of attribute weight, different parameter values of \( \delta_i \) and \( \delta_j \) in objective optimization model has no effect on the ranking of alternatives.

4.4 The Comparison of the Method and Discuss

In order to ensure the comparability of the two methods, situation 2 (does not take the loss of preference information and decision-making risk caused by preference uncertainty and preference conflict of the decision makers into consideration) still uses the same clustering method in situation 1 (considers these factors into consideration), so situation 2 can directly use the preference matrix of the clusters in situation 1 to make decision.

First of all, we transform the IVIF preference of the clusters into real numbers by the score function in Definition 4. Then combined with prospect theory, the prospect value of alternatives under each attribute of the clusters can be obtained. Next, combined with the cluster weight, the prospect value of alternatives under each attribute can also be obtained. The results are shown in Table 9.

Table 9: The prospect values of alternatives under each attribute.

<table>
<thead>
<tr>
<th></th>
<th>( f_1 )</th>
<th>( f_2 )</th>
<th>( f_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>0.12</td>
<td>0.09</td>
<td>-0.01</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>-0.03</td>
<td>-0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>-0.07</td>
<td>-0.08</td>
<td>-0.09</td>
</tr>
<tr>
<td>( x_4 )</td>
<td>-0.04</td>
<td>0.04</td>
<td>0.05</td>
</tr>
</tbody>
</table>

And then, we regard the maximization of the comprehensive prospect value as the target and the attribute weight partially known as constraint condition. By constructing following model, we can get the weight of every attribute:

\[
\max Y = -0.02 w_1 + 0.01 w_2 - 0.01 w_3
\]

s.t. \( 0.55 \leq w_1 \leq 0.65, 0.05 \leq w_2 \leq 0.15, 0.35 \leq w_3 \leq 0.45, \sum_{j=1}^{N} w_j = 1, 0 \leq w_j \leq 1; \ j = 1, 2, 3 \). We can get the weight \( w = (0.55, 0.10, 0.35)^T \). Combining the prospect values of alternatives, we can obtain the comprehensive prospect values are \( q_1 = 0.07, q_2 = -0.08, q_3 = -0.080, q_4 = -0.002 \). Then the ranking of the alternatives is \( x_1 > x_4 > x_2 > x_3 \), \( x_1 \) is selected as a quick response to the coal mine accident.

It is clear that the two methods have different ranking with consider the loss of preference information and the decision-making risk or not.
5 CONCLUSION

In this paper, a risk minimizing method of large group decision is proposed, which provides a reference for emergency decision under fuzzy environment in the background of engineering. On the one hand, the IVIF distance in the form of IF number is proposed in order to reduce the loss of preference information. On the other hand, both the ambiguity of the emergency decision-making information and preference conflicts for large group may bring decision-making risk, so large group fuzzy-conflict entropy emergency decision-making model is constructed of which the goal is based on minimizing the decision-making risk in the process of emergency decision-making.

Although this paper is about the decision problem which the preference information is in the form of IVIF numbers, the proposed method is not limited to this, it can also be applied to other preferences in fuzzy environments. The limitation of this study is embodied in the research background of single-stage static emergency decision making for large group, so we can further explore and study the multi-stage risky dynamic emergency decision making for large group in the future study. In addition, the proposed method also provides a kind of feasible solution for processing big data, such as the big data in social networks. Thus, the proposed method should be useful and effective both in theoretical and practical aspects.

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REFERENCES

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