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HOW TO SUPPLEMENT THE SAFETY REQUIREMENTS

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ABSTRACT

Safety requirements based on Probability of Exceedance (POE) take into account only probabilities, but do not control magnitude of outcomes in the tail of the probabilistic distribution. The requirements do not constrain very large outcomes leading to industrial catastrophes. The paper shows how to upgrade safety requirements with the Buffered Probability of Exceedance (bPOE). The bPOE risk function equals a tail probability with known mean of the tail (i.e., it is a probability of the tail such that the mean of the tail equals to the specified value). The paper considers two application areas: 1) Credit ratings of financial synthetic instruments (e.g., AAA, AA, ... credit ratings); 2) Materials strength certification (A-basis, B-basis).

1 INTRODUCTION

The safety requirements in many engineering fields are optimistically designed and do not control magnitude of very large outcomes. Some thresholds (lower bounds) are specified that should be exceeded with low probability. For instance, in nuclear safety, a level of release of radiation in the environment is specified, which may be exceeded, say, only once in 10,000 years. Nevertheless, the magnitude of exceedance is not controlled, even for extremely large outcomes. Safety regulations do not distinguish magnitude of outcomes exceeding a high threshold corresponding to a major accident. For instance, to our knowledge, safety requirements were not violated in the Fukushima nuclear accident. We can verify calculations of safety models for the Fukushima nuclear power plant and come to a conclusion that the plant worked as designed (no violation because the probability of radiation release is low, in spite of a very large magnitude of the release, which is not controlled). Similar concerns arise in defining financial ratings of companies, certification of materials (A/B basis), designing of dams, etc. This paper suggests to upgrade risk constraints in various engineering areas to take into account both chances and magnitudes of outcomes. This paper only formulates the idea. A huge diverse effort is needed to make changes in actual practices: 1) development of statistical and mathematical theory of risk measures; 2) computer modeling to demonstrate that the upgraded requirements/regulations prevent past catastrophes; 3) collection of appropriate data; 4) setting new risk requirements (legal issues); etc.

Majority of the current safety requirements are formulated with low bounds on values of undesirable outcomes. There are two equivalent variants of the “optimistic lower-bound risk management approach”:

1) Fix a threshold, which is the lower bound of outcomes in a tail of the distribution, and constrain POE, also known as the Survival, Survivor, or Reliability function;

2) Fix the probability of the tail, and constrain the lower bound of the tail outcomes, called the quantile or Value-at-Risk (VaR) in finance.

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Majority of risk constraints in various engineering areas are set with POE.

It has been recognized in finance that the second approach based on VaR needs to be improved. VaR has been supplemented with the Conditional Value-at-Risk (CVaR), also called Expected Shortfall, Average VaR, Tail VaR, and Superquantile. By definition, CVaR is the average value of outcomes in the tail with some specified probability. For instance, if probability of the tail is 10%, than CVaR is the average of the worst-case 10% outcomes. So, by construction, CVaR takes into account both probability and magnitude of the tail. CVaR is a so-called "coherent risk measure" and it has exceptional mathematical properties.

It was recently discovered that there is a conservative analog of POE, called bPOE, which takes into account both the chances and magnitudes of losses. By definition, bPOE is a probability of the distribution tail with the known average value of this tail. So, bPOE is an inverse function of CVaR. The safety constraints with CVaR and bPOE are equivalent. Therefore, POE requirements can be upgraded with bPOE, similar to how VaR requirements were upgraded with CVaR. Recently, a general formula for bPOE was derived, and it was shown that, similar to CVaR, the bPOE risk management/optimization can be reduced to convex and linear programming.

2 MOTIVATION FOR IMPROVING THE REGULATIONS

Let us list some recent major technological catastrophes: 1) Fukushima nuclear accident in Japan; 2) Financial crisis in 2008; 3) Space shuttle Columbia accident; 4) Hurricane Katrina flood because of levee failures. These catastrophes resulted in deaths of people and astronomical financial losses. However, to our knowledge, in all of these cases safety requirements were not violated (at least from a legal point of view) and nobody was held accountable for the faulty design of engineering systems.

We anticipate that a conservative upgrade of safety requirements, which takes into account both magnitude as well as chances of losses, will benefit virtually all engineering fields. For instance, nuclear safety regulations require extremely low "core melt frequency". However, they do not distinguish between a large and an extremely large radiation release (which may be contaminating the whole globe and leading to a "nuclear winter"). We think that this inconsistency can be fixed by using the bPOE risk function.

Another major example is the 2008 Financial Crisis. At least partially, it can be attributed to defaults of financial companies overloaded with exposure to derivative instruments. Derivatives may lead to very high losses with very low probability. This was the case for the AIG, a giant insurance company, which overloaded the trading book with Collateralized Debt Obligation (CDO) upper tranches. The chance of default was low and AIG kept the AAA rating in spite of an extremely high exposure. The U.S. Government had to bailout AIG for $85 Billion.

This paper discusses the so called Buffered Credit Ratings based on bPOE, which take into account the exposure as well as chances of default. Buffered Credit Ratings were quickly discussed in (Pertaia and Uryasev 2019). The topic is of paramount importance because the 2008 Crisis cost tops $22 trillion, according to some estimates. We are planning to write an extensive paper covering this topic.

One more example is a non-conservative Material Certification process (A and B basis). B basis is a 95% lower confidence bound on the tenth percentile and the A basis is a 95% lower confidence bound of the first percentile. Let us consider that 1000 coupons of a ceramic material were tested for certification, and all coupons passed the test (i.e., the material was certified with A and B basis). Now suppose that we replace 9 coupons with "fake" coupons having zero strength. This may happen because of a flaw in the manufacturing process, so that a small portion of coupons have zero strength. The chance of getting a defective coupon is 0.9%. The A-Basis and B-Basis requirements would still be valid because they do not take into account the magnitude of outcomes below the threshold. Engineers design strong materials in spite of non-conservative strength requirements. However, requirements need to be upgraded to be more conservative. The ceramic plates covering the Columbia Shuttle passed A-Basis and B-Basis certification. In this case, even if a small number of plates fail, it leads to catastrophic consequence.

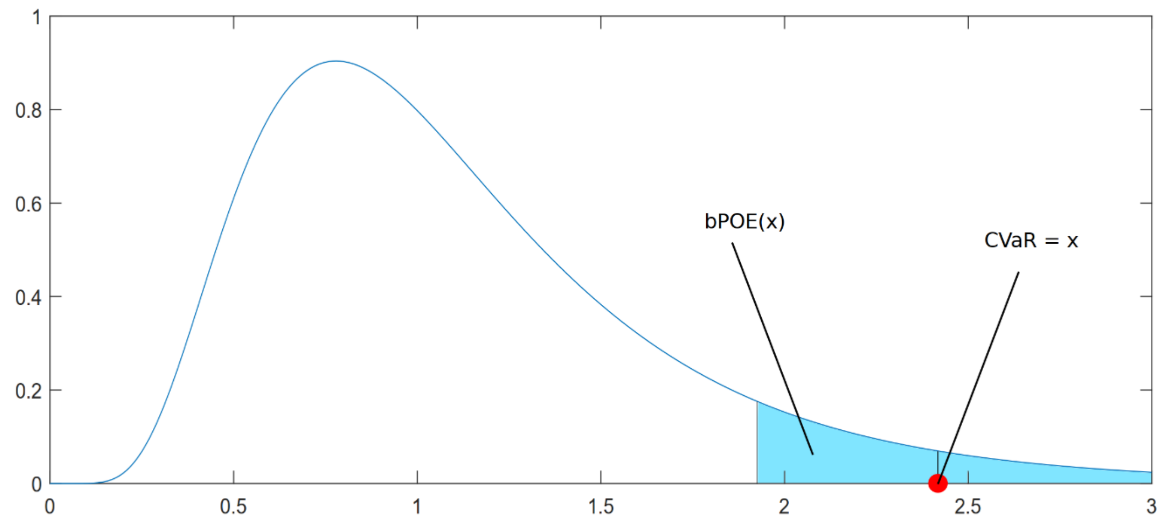

Figure 1: Relationship between bPOE and CVaR.

One final example is the 2005 levee failure in New Orleans following the passage of Hurricane Katrina. Levee designers have not taken into account the magnitude of possible losses of the failure. Safety margins should take into account the magnitude of possible losses.

3 DEFINITION OF BPOE

The concept of bPOE has been studied by (Mafusalov and Uryasev. 2018) as an extension of the buffered failure probability introduced by (Rockafellar 2009) and explored by (Rockafellar and Royset 2010). Optimization representation of bPOE was suggested in PhD dissertation of M. Norton and was further discussed in the paper (Norton and Uryasev. 2018)

Let X be some random variable and let $v \in \mathbb{R}$ be a threshold. bPOE of X with threshold v equals

$$bPOE_X(v) = \min_{a \geq 0} \mathbb{E}\left[a(X - v) + 1\right]^+ , \tag{1}$$

where $[*]^+ = \max\{*, 0\}$. (Mafusalov and Uryasev. 2018) show that bPOE is one minus inverse of CVaR (see figure 1). CVaR was introduced and studied by (Rockafellar and Uryasev. 2000), (Rockafellar and Uryasev. 2002), it is an expectation of the right α tail of the distribution of X. CVaR is called Expected Shortfall in financial applications. Given some confidence level $\alpha \in [0,1)$, CVaR of X is defined as

$$CVaR_X(\alpha) = \min_C \left\{ C + \frac{1}{1-\alpha} \mathbb{E}\left[X - C\right]^+ \right\}. \tag{2}$$

Both, CVaR and bPOE are monotonic functions of parameters. Constraint on CVaR can be transformed into an equivalent bPOE constraint. Specifically, $CVaR_X(\alpha) \leq v$ is equivalent to $bPOE_X(v) \leq \alpha$.

CVaR is a convex function with respect to the random variable X and bPOE is the quasi-convex function of X. Convexity and quasi-convexity makes CVaR and bPOE desirable functions in optimization setting. In many cases, inequality constraints on CVaR and bPOE can be re-formulated as linear inequality constraints. For example, bPOE was used by (Shang and Uryasev. 2018) to solve a cash-flow matching problem for a bond portfolio.

4 SUPPLEMENTING POE WITH BPOE IN SAFETY REQUIREMENTS

The VaR based regulations were supplemented by the CVaR based regulation in the finance. VaR was widely criticized due to its undesirable properties. VaR is not a coherent measure of risk, as defined by (Artzner and Heath. 1999). Namely, VaR does not satisfy the *subadditivity* property, which can lead to counter intuitive results. For example, it is possible to construct a portfolio of two assets, with random returns X and Y, that has $VaR(X+Y) > VaR(X) + VaR(Y)$, which contradicts the principle of diversification (a portfolio can not have more risk than total risk of individual assets). CVaR on the other hand is a coherent measure of risk. Also, by definition, VaR is an optimistic measure of risk, since it shows the smallest quantile (lower bound of the tail) for the given confidence level (see figure 2). Therefore, in financial regulations, CVaR has supplemented VaR as the measure of risk.

Similar, POE based requirements can be supplemented by requirements on bPOE. (Pertaia and Uryasev 2019) proposed a bPOE conversion factor to replace the current POE based requirements.

4.1 UPGRADING POE BASED FINANCIAL RATINGS

(Pertaia and Uryasev 2019) suggested to upgrade the credit ratings for synthetic financial instruments, with bPOE based credit ratings, by using the existing rating assignment tables. For example, if Standard and Poor's (S&P) gives "AA" rating to a company, then historically it has less than 0.02% chance of default in one year (see table 1). To assign rating to a synthetic instrument, first the default probability of the instrument is calculated and then a corresponding credit rating is determined based on credit rating table (similar to 1). Default probability is frequently estimated using the cumulative loss distribution of the instrument and some threshold value. For instance, in the Merton model, a company defaults if the cumulative loss exceeds the company's equity, so the default probability for the company is $\mathbb{P}(loss > equity)$. Therefore, the default probability is POE of the loss distribution with the equity threshold.

(Pertaia and Uryasev 2019) proposed to scale the rating table by the factor of $e = 2.71...$ and replace POE with bPOE, calculated for the same threshold. This scaling factor comes from the fact that the exponential distribution, with PDF, $p(x) = \lambda e^{-\lambda x}$, and corresponding $POE_X(v) = \mathbb{P}(X > v) = e^{-\lambda v}$, has a fixed ratio of $bPOE_X(v)/POE_X(v) = e$, for any threshold v and any parameter λ. The exponential distribution can be used to distinguish between heavy tailed and light tailed distributions. A distribution is heavy tailed if

$$\lim_{v \to \infty} e^{zv}\, \mathbb{P}(X > v) = \infty \quad \forall z > 0, \tag{3}$$

or, stated differently, if the distribution has heavier tails than an exponential distribution with arbitrary parameter $\lambda = z$. Note that, for an exponential distribution with parameter z, the left-hand side of (3) is equal to $\lim_{v \to \infty} e^{zv} e^{-zv} = 1$. Therefore, if the empirical cumulative loss distribution of the synthetic instrument is heavy tailed, then the bPOE based rating will be lower (worse) than corresponding POE ratings for that instrument.

Table 1: Probabilities of default (in %) over time and corresponding credit ratings given by S&P.

Rating \Time	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
AAA	0	0.03	0.13	0.24	0.35	0.46	0.52	0.61	0.66	0.72
AA	0.02	0.06	0.13	0.23	0.34	0.45	0.55	0.63	0.71	0.79
A	0.06	0.15	0.26	0.4	0.55	0.72	0.92	1.1	1.28	1.48
BBB	0.19	0.53	0.91	1.37	1.84	2.3	2.71	3.11	3.5	3.89
BB	0.73	2.25	4.07	5.86	7.51	9.03	10.34	11.49	12.53	13.45
B	3.77	8.56	12.66	15.82	18.27	20.26	21.89	23.19	24.32	25.37
CCC/C	26.36	35.54	40.83	44.05	46.43	47.28	48.24	49.05	49.95	50.6
Investment grade	0.1	0.28	0.48	0.73	0.98	1.24	1.49	1.72	1.94	2.17
Speculative grade	3.8	7.44	10.6	13.15	15.24	16.94	18.38	19.58	20.65	21.61
All rated	1.49	2.94	4.21	5.27	6.17	6.92	7.57	8.12	8.62	9.09

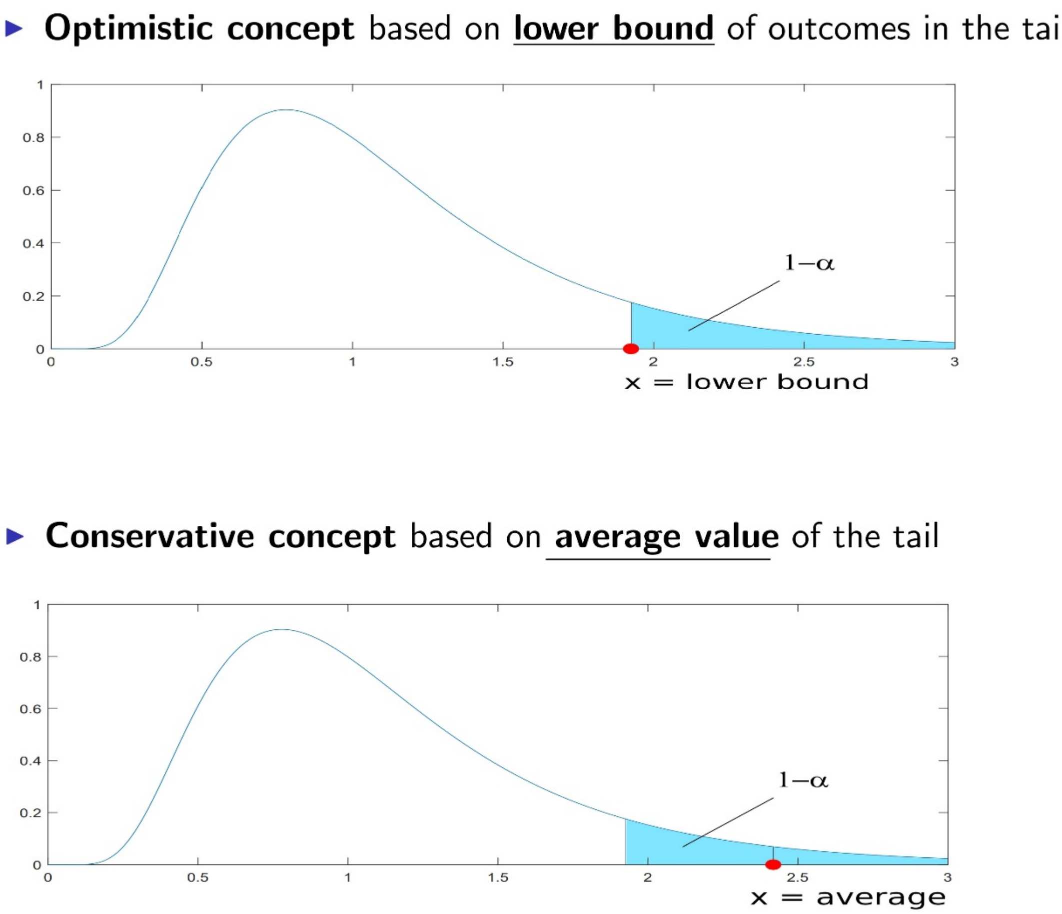

Figure 2: VaR vs CVaR.

4.2 UPGRADING POE BASED CERTIFICATION FOR MATERIALS

The POE $\Rightarrow$ bPOE upgrading procedure considered in the previous section can be applied to various safety constraints outside the financial sector. Let us consider a material certification process. Let us denote by V the force (random variable) which breaks a tested sample (coupon) of material. Suppose that a threshold v is specified and it is required that only some percentage (e.g., 5%) of V outcomes should be below v. Equivalently we can stay this requirements in POE format, $\mathbb{P}(-V > -v) \leq 5\%$. The bPOE variant of this requirement would state that bPOE of $-V$ with threshold $-v$ should be less than or equal to $e \cdot 5\%$, i.e., $bPOE_{-V}(-v) \leq e \cdot 5\% = 13.59\%$. Let us consider a defective material production process, such that a small fraction of the coupons breaks with almost zero force. Obviously, this is unacceptable if the material under question is used for safety sensitive projects, such as bridge construction. Despite this, the material can pass the POE based certification requirement, since it does not take into account the magnitude of breaking force below the threshold v. With bPOE, which takes into account the magnitude of outcomes below the threshold, we set a more conservative requirements for material strength. The bPOE based requirement will account for magnitude of breaking force below the threshold v.

REFERENCES

Artzner, P., F. D. J.-M. E., and D. Heath.. 1999. "Coherent Measures of Risk". *Mathematical Finance* 9:203–228.

Mafusalov, A., and S. Uryasev.. 2018. "Buffered Probability of Exceedance: Mathematical Properties and Optimization". *SIAM Journal on Optimization* 28(2):1077–1103.

Norton, M., and S. Uryasev.. 2018, 07. "Maximization of AUC and Buffered AUC in Binary Classification". *Mathematical Programming*:1–38.

Pertaia, G., and S. Uryasev. 2019. "Buffered Probability of Exceedance (bPOE) Ratings for Synthetic Instruments". In *Advances in Service Science*, edited by H. Yang and R. Qiu, 211–216. Cham: Springer International Publishing.

Rockafellar, R. 2009. "Safeguarding Strategies in Risky Optimization". In *Presentation at the International Workshop on Engineering Risk Control and Optimization, Gainesville, FL.*

Rockafellar, R., and J. Royset. 2010. "On Buffered Failure Probability in Design and Optimization of Structures". *Reliability Engineering and System Safety* 95(5):499–510.

Rockafellar, R., and S. Uryasev.. 2000. "Optimization of Conditional Value-At-Risk". *The Journal of Risk* 2:21–41.

Rockafellar, R., and S. Uryasev.. 2002. "Conditional Value-at-Risk for General Loss Distributions". *Journal of Banking and Finance* 26:1443–1471.

Shang, D., V. K., and S. Uryasev.. 2018, January. "Cash Flow Matching With Risks Controlled by Buffered Probability of Exceedance and Conditional Value-at-risk". *Annals of Operations Research* 260(1):501–514.

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