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STOCHASTIC MIXED-INTEGER NONLINEAR PROGRAMMING MODEL FOR DRONE-BASED HEALTHCARE DELIVERY

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ABSTRACT

This paper proposes a mixed integer set covering model with congestion and chance constraints to design a network of drones to deliver defibrillators in response to out-of-hospital cardiac arrest (OHCA) occurrences. The incident rate of OHCAs is assumed to be random and congestion in the system is modeled as a network of M/G/1 queues. Both the arrival and service rates are determined endogenously in the model, which results is a stochastic nonconvex mixed integer nonlinear location-allocation problem with decision-dependent and exogenous uncertainty. We derive a Boolean-based (Lejeune and Margot 2016) deterministic reformulation and use a simulation-optimization solution approach to solve the stochastic model.

1 INTRODUCTION

Out-of-hospital cardiac arrests (OHCA) are a time-critical healthcare emergency and require a fast response time (Holmberg et al. 2000). This motivates the present research, in which we propose a stochastic optimization model to design a network of drones to deliver automated external defibrillators to out-of-hospital cardiac arrests (OHCAs). The model is a stochastic discrete location-allocation problem with exogenous and endogenous uncertainty, and random technology matrix. The rate of OHCA occurrences is assumed to be a random variable with finite support. We model congestion in the system by assuming that each drone launching station can be modeled as an M/G/1 queuing system, with a generally distributed service time and Poisson arrivals. The probability that the response time does not exceed a certain value for a prescribed probability level is modeled as a chance constraint with random technology matrix.

The problem described has some inherent challenges: multiple stochastic inequalities that must be satisfied jointly, the stochastic inequalities involve a nonconvex function of the decision variables and the random variable, and random technology matrix. We address these challenges by deriving convex reformulations of the deterministic problem that are used in a simulation-optimization solution approach based on the Boolean optimization framework (Lejeune and Margot 2016). We implement the model using real-life data for a large US city.

2 PROPOSED MODEL

The optimization model minimizes the total number of drone bases to be opened while satisfying a requirement on the expected response time at each OHCA location. The drone launching stations locations are selected from a predetermined set, making this a discrete facility location problem. Let I and J be, respectively, the set of OHCA locations and the set of candidate drone bases, where $i \in I$ and $j \in J$. The random OHCA incidence rate is given as ξ_i. A_i and A_j are vectors of coordinates of the OHCA incidents and the candidate drone bases, respectively. The maximum allowed response time is given as u, which must be satisfied with tolerance $1-\delta$. The decision binary variables x_j and y_{ij} represent, respectively, whether a drone launching station was opened at location j, and the allocation of drone launching stations to OHCA incident locations. The speed of the drone is given as v_j, β is a constant that allows for different travel speeds to and from the scene of the call, W_{ij} is the non-travel portion of the service time, $d_{ij} = \|A_i - A_j\|$ is the euclidean distance between a candidate base and an OHCA location. Assuming that each drone launching station is an M/G/1 queue, and letting $s_{ij} = W_{ij} + \frac{\beta}{v} d_{ij}$, the chance-constrained queuing set covering problem is given below:

$$\begin{aligned}
\textbf{SCP}: \quad & \min \sum_{j \in J} x_j && && (1)\\
& \text{s.t.} \quad \textstyle\sum_{j \in J} y_{ij} = 1 && i \in I_j && (2)\\
& y_{ij} \leq x_j && i \in I, j \in J && (3)\\
& \mathbb{P}\left(\begin{array}{lc} \sum_{i \in I} \xi_i y_{ij} s_{ij} \leq 1 & j \in J \\ \sum_{j \in J} \left[\frac{y_{lj} \sum_{i \in I} \xi_i y_{ij} s_{ij}^2}{2\left[1 - \sum_{i \in I} \xi_i y_{ij} s_{ij}\right]} + \frac{d_{lj} y_{lj}}{v} \right] \leq u & l \in I \end{array} \right) \geq 1 - \delta && && (4)\\
& x_j, y_{ij} \in \{0,1\} && i \in I, j \in J && (5)
\end{aligned}$$

The objective function (1) minimizes the number of drone bases to be opened. Constraint (2) establishes that each OHCA location can only be served by one drone launching station, a requirement of an M/G/1 queuing model. Equation (3) requires that a drone launching station must be open if it is assigned to cover an OHCA location. The joint chance constraint (4) requires that the queuing system operates in the steady-state condition, which is represented by the first equation in the chance constraint. Simultaneously, it also requires that the total response time at any OHCA location does not exceed the prescribed limit. Constraint (5) enforces the binary conditions on the decision variables.

3 SOLUTION APPROACH

The resulting optimization model is highly nonconvex. In addition to having binary decision variables and chance constraints, the technology matrix is random and it involves nonconvex inequalities. The solution approach will be based on the Boolean programming framework introduced by Lejeune and Margot (2016), which can adequately deal with quadratic inequalities and random technology matrices, as well as convex optimization techniques. The first step is to derive an equivalent convex reformulation for the inequalities involved in the chance constraint. The second step will consist of deriving a reformulation based on the Boolean programming framework. In the Boolean approach, a Boolean function is used to characterize the probability distribution of OHCA occurrences. One major advantage of this approach is that the number of constraints in the reformulation does not increase with the number of simulated scenarios. This is extremely relevant, considering that the model will be implemented using real-life data for a large US city.

4 CONCLUSION

This paper introduced a stochastic location allocation set covering model with exogenous and endogenous uncertainty. We introduced the main characteristics of the model and described the solution approach. Future work includes implementing the proposed model with historical cardiac arrest data collected for a large US city and deriving data-driven insights that can improve decision-making.

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