

## **INVENTORY MANAGEMENT UNDER DISRUPTION RISK**

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### **ABSTRACT**

Inventory management models help determine policies for managing trade-offs between customer satisfaction and service cost. Initiatives like lean manufacturing, pooling, and postponement have been proven to be effective in mitigating the trade-offs by maintaining high levels of service while reducing system inventories. However, such initiatives reduce the buffers, exacerbating supply chain issues in the event of a disruption. We evaluate stocking decisions in the presence of operational disruptions caused by random events such as natural disasters or man-made disturbances. These disruptions represent different risks from those associated with demand uncertainties as they stop production flow and typically persist longer. Thus, operational disruptions can be much more devastating though their likelihood of occurrence may be low. Using stochastic simulation, we combine the newsvendor model capturing demand uncertainty costs with catastrophe models capturing disruption/recovery costs. We apply data analytics to the simulation outputs to obtain insights to manage inventory under disruption risk.

### **1 INTRODUCTION**

The objective of inventory management models is to determine effective policies for managing the trade-off between customer satisfaction and the cost of service. Over the years, inventory models have become increasingly sophisticated, incorporating many complicating factors that are relevant in practice such as working capital constraints, finite supplier capacity, and yield losses. Guided by these models, innovative operational initiatives such as lean manufacturing, pooling, and postponement have been effective in mitigating this cost-service trade-off by maintaining high levels of service while significantly reducing system inventories. However, such initiatives have also reduced the buffers that a firm could fall back on in the event of a disruption, exacerbating the costly effect of supply chain disruptions. Curiously absent from these models, therefore, is the impact of operational disruptions.

Operational disruptions are fundamentally different from the risks arising from demand uncertainties as they completely stop the production flow and typically persist longer (Kleindorfer and Saad 2005); thus, the impact of supply chain disruptions can be much more devastating while their likelihood of occurrence

is very low. In an empirical study, Hendricks and Singhal (2005) show the negative impact supply chain disruptions inflict on a firm's financial performance and the slow recovery from such shocks. Furthermore, reinsurance companies report that the frequency of natural hazards is on the rise, further increasing the cost associated with supply chain disruptions.

Supply chain disruptions have recently started receiving increasing attention in the literature. A comprehensive review of models is provided by Snyder et al. (2014). An equally comprehensive list of approaches for mitigating supply chain disruptions is discussed by Tang (2006) and Simchi-Levi et al. (2014). These approaches could be tactical, anchored on classical inventory models as in Berk and Arreola-Risa (1994), Parlar (1997), and Arreola-Risa and DeCroix (1998) or they could be strategic, anchored on network design principles as in Lim et al. (2013), Schmitt et al. (2014), and Simchi-Levi et al. (2014). These approaches could further include supplier selection as in Aydin et al. (2011).

A related concept is resilience, defined as the ability of a supply chain and its components to anticipate, absorb, accommodate, or recover from the effects of a shock or stress in a timely and efficient manner (Mitchell and Harris 2012). This definition separates resilience, associated with robustness, resourcefulness, recovery and redundancy, from risk and its common determinants of threat/hazard, vulnerability, and consequences. A number of studies therefore take risk management as an entry point for operationalizing and measuring resilience (e.g., Twigg 2009), advocating effective management of risk to build resilience. Therefore, a system that is effective in managing risk is likely to become more resilient to shocks and stresses, though the exact relationship needs to be tested empirically.

We consider stocking problems in the presence of facility disruptions caused by random events such as natural disasters (e.g., floods) or man-made disturbances (e.g., site contamination). Disruptions may be independent from one facility to another such as a fire in a plant or labor movements or correlated across facilities such as an earthquake or tornado. As suggested by Aydin et al. (2011), *"future research may move beyond generic models of supply risk to look deeper into specific causes of supply disruptions. For example, a plant fire and an impasse in labor negotiations may be similar as far as their end results are concerned – both will cause a temporary halt in production. However, the preparedness and recovery plans for a disruption caused by a plant fire may be significantly different from the plans that are in place for labor negotiations. High-fidelity models, which capture considerations that are relevant to specific causes of supply risks, may be used for prescriptive purposes in supply risk management."* Despite being rare in the Operations Management literature (e.g., Elkins et al. 2007; Lohmann and Yue 2011), these risks have been widely addressed in literature on health and safety (e.g., Henselwood 2009; Jung et al. 2011; and Hadjisophocleus and Fu 2004) and in the literature on insurance (e.g., Gupta et al. 2003). Similar to Deleris et al. (2011), who use a Generalized Semi-Markov Process framework, our models will therefore combine the well-known newsvendor model capturing the cost of demand and supply uncertainty with catastrophe models capturing the cost of disruption and recovery. However, we select stochastic simulation as the technology of choice.

The remainder of the paper is organized as follows. In Section 2, we review principal approaches to catastrophe modeling. After introducing the notation in Section 3, we study a newsvendor model exposed to multiple sources of uncertainty, first with a mathematical analysis in Section 4.1 and then with stochastic simulation in Section 4.2. We primarily rely on simulation's ability to model uncertainty for any input process and for any number of input processes. We conclude the paper with a summary and suggestions for future research in Section 5.

## 2 CATASTROPHE MODELING

Catastrophic risk typically refers to natural or human losses with major consequences and unacceptable lasting effects, usually involving significant harm to humans, substantial damage to the environment along with possible loss of the firm's ability to operate (Lohmann and Yue 2011). Manufacturing firms are uniquely susceptible to these categories of loss due to the diverse range of interconnected supply chains. For example, as reflected in *100 Largest Losses* compiled by Marsh, the accumulated value of the hundred

largest losses in the energy sector is more than \$34 billion and the single largest loss is approximately \$1.8 billion due to an explosion at Piper Alpha in the North Sea. Although progress has been made by firms and regulatory agencies, further research is needed in the area of modeling catastrophic losses to advance our understanding of the potential for loss and ultimately mitigate the risk to humans, the environment, and the operability of the firm.

The potential for future loss events can be informed through frequency modeling by considering events external to the firm or natural disasters (e.g., earthquakes, hurricanes, floods) and those internal to the firm, which arise from human or asset failures. Many modeling techniques have been developed and utilized in various industries; however, only a representative set will be summarized here.

Natural disasters have multiple impacts. In addition to their direct impact on human lives and assets, they also disrupt the functioning of supply chains, leading to additional losses — often referred to as indirect loss of an economic nature. Hallegatte (2013) introduces an adaptive regional input-output model, which explicitly incorporates inventories, to assess the direct and indirect losses triggered by natural disasters. A comprehensive treatment of modeling extreme events is offered in Embrechts et al. (2012).

When considering human or asset failures, there are two types of errors to address: (1) errors whose primary causal factors are individual human characteristics unrelated to the work situation and (2) errors whose primary causal factors are related to the design of the work situation (Lorenzo 1990). However, human factors specialists estimate that only 15-20% of workplace errors are primarily caused by internal characteristics and the vast majority (80-85%) primarily result from environments that management directly controls (Swain 1993). Imperfect management systems such as process design, workload, training, and individual stress have significant impact on human error probability (Reason and Hobb 2003). The frequency at which an error can be expected is a function of both the predicted error rate per task and how many times the task is performed within a given period of time. Well-defined approaches for computing human error rates are available in the literature (Swain and Guttman 1983 and Gertman and Blackman 1994). As shown in Table 1, likelihood of human error can be quite high.

Table 1: Tabulation of probability due to human error (Atallah 1980).

Activity or Task	Probability of Error/Task
Critical Routine Task	$10^{-3}$
Non-Critical Routine Task	$3 \times 10^{-3}$
General Error Rate in High-Stress that Occurs Rapidly	$2.5 \times 10^{-1}$
Non-Routine Operations (Start-Up)	$10^{-2}$
Checklist Inspection	$10^{-1}$
Walk-Around Inspection	$5 \times 10^{-1}$
High Stress Operations and Responding after a Major Accident in:	
First Minute	1.0
After 5 Minutes	$9 \times 10^{-1}$
After 30 Minutes	$10^{-1}$
After Several Hours	$10^{-2}$
General Human Errors of Observance	$5 \times 10^{-2}$
Operator Actuation of a Switch	$10^{-3}$
Operator Actuation of a Key-Operated Switch	$10^{-4}$
General Human Errors of Omission	$10^{-2}$

In the absence of generic data sources, a number of techniques are available to predict human reliability, such as *Standardized Plant Analysis Risk-Human Reliability Analysis* method (SPAR-H) (Gertman et al. 2005), *Human Event Repository and Analysis* (HERA) (Hallbert et al. 2007), *Technique for Human Error*

*Rate Prediction* (THERP) (Swain and Guttman 1983), and *Human Error Assessment and Reduction Technique* (HEART) (Kirwan 1994).

The methods available for estimating the frequency of equipment failure rates associated with manufacturing equipment are numerous and subtle. Therefore, the reader is referred to appropriate expert information sources as a detailed review is beyond the scope of this paper. Publicly available, generic data are accessible through a variety of sources (e.g., Center for Chemical Process Safety 1989 and Offshore and Onshore Reliability Data 2009) to assist in modeling effort. When generic data is not available, it is often necessary to estimate failure rates. In these situations, predicted rates for total failures of equipment are developed using techniques such as the parts count or parts stress method (see U.S. Department of Defence 1990 for an example study). In certain situations limited data is available because no failures have been observed. When failure is physically possible and no failure event has occurred, statistical approximation methods such as the rule of  $1/(3n)$ , where  $n$  refers to the time period in years during which no events have occurred, are typically used to estimate the mean failure rate (Welker and Lipov 1974). More sophisticated methods are available using advanced quantitative risk analysis (e.g., Bailey 1997 and Freeman 2011).

In this paper, we consider a newsvendor inventory system and define catastrophe as a disruption that causes a certain fraction of the on-site inventory to be lost. We govern the underlying random event via disruptive event probability and introduce its notation in the following section.

### 3 NOTATION

Our focus is on inventory stocking problems in the presence of facility disruptions caused by random events such as natural disasters or man-made disturbances. Although we assume disruptions to be independent from one facility to another (e.g., facility contamination or a fire in the plant), our simulation design and analysis introduced in the following section can be readily extended to the case of correlated disruptions across facilities (e.g., an earthquake or tornado). Our models will therefore combine the well-known newsvendor model capturing the cost of demand and supply uncertainty with catastrophe models capturing the cost of disruption and recovery. To this end, we will use the following notation in the remainder of the paper:

#### Random Variables:

- $X$  demand with cumulative distribution function  $F_X(\cdot; \Psi)$  and probability density function  $f_X(\cdot; \Psi)$
- $\Psi$  vector denoting the (deterministic) parameters of the demand distribution function
- $p$  probability of a disruptive event
- $\ell$  fraction of inventory lost due to the disruptive event

#### Cost Parameters:

- $h$  unit inventory holding cost
- $s$  unit shortage cost
- $c(k)$  cost of recovering  $k$  units of inventory lost following the disruptive event

#### Capacity Parameter:

- $K$  quantity of on-site inventory

In the following section, we choose the demand distribution function as lognormal with a location parameter  $\xi$  of zero and shape parameters of  $\gamma$  and  $\delta$ . Therefore, the numerical experiments of Section 4.2 are designed for  $\Psi = (\xi, \gamma, \delta)'$  and the cumulative demand distribution function  $F_X(x; \Psi) = \Phi(\gamma + \delta \log(x))$  where  $x > \xi = 0$  and  $\Phi$  is the cumulative distribution function of the standard normal random variable.

Without loss of generality, we set  $h = 1$  throughout Section 4 and identify the shortage cost  $s$  as equivalent to  $\beta h / (1 - \beta)$  where  $\beta$  is the Type-1 service level. Therefore, unit holding and shortage costs are chosen such that the non-stockout probability is  $\beta$  in the absence of any disruptive event.

#### 4 INVENTORY MODEL

We consider a case where inventory is held at a location that may experience a disruptive event with low probability and high impact. Disruptive events could be triggered by severe weather (e.g., lightning strikes causing a factory fire, hail destroying crops on a field) or man-made disturbances (e.g., contamination or explosion). As a result, a portion,  $\ell$ , of the available inventory is lost. Under the assumption of complete inventory loss following a disruptive event (i.e.,  $\ell = 1$ ), Section 4.1 presents a mathematical analysis demonstrating the impact of disruption probability on the optimal on-site inventory which minimizes the expected cost. We extend this analysis beyond the consideration of expected cost as the performance measure by designing a stochastic simulation in Section 4.2. We build the risk profiles for the inventory model performance measures, in particular, total cost and unit fill rate, and investigate the impact of operational disruptions on these measures of performance.

##### 4.1 A Mathematical Analysis

The objective is to characterize the optimal on-site inventory minimizing the mean of the total cost function. Specifically, the total cost is the sum of on-hand inventory holding cost, demand shortage cost, and the cost of recovering the inventory lost due to the disruptive event. Therefore, expected (total) cost  $I$  as a function of the on-site inventory  $K$  can be written as

$$I(K) = (1 - p) \left\{ \int_0^K h(K - x) f_X(x; \Psi) dx + \int_K^\infty s(x - K) f_X(x; \Psi) dx \right\} + p \left\{ \int_0^{(1-\ell)K} h((1-\ell)K - x) f_X(x; \Psi) dx + \int_{(1-\ell)K}^\infty s(x - (1-\ell)K) f_X(x; \Psi) dx + c(\ell K) \right\}. \quad (1)$$

As defined earlier,  $f_X(x; \Psi)$  is the probability density function of the demand, which is assumed to be independent of the random disruptive event. Note that the first part of the equation in (1) is the usual newsvendor (demand-supply mismatch) cost in the absence of any disruptions whereas the second part of the equation reflects the shortage and recovery costs due to the loss of inventory triggered by the disruption.

The optimal inventory level  $K^*$  satisfies

$$\left. \frac{\partial I(K)}{\partial K} \right|_{K=K^*} = 0 \quad \text{and} \quad \left. \frac{\partial^2 I(K)}{\partial K^2} \right|_{K=K^*} > 0.$$

Therefore, we first derive  $\partial I(K)/\partial K$  using the first-order optimality condition:

$$\begin{aligned} \frac{\partial I(K)}{\partial K} &= (1 - p)h \int_0^K f_X(x; \Psi) dx - (1 - p)s \int_K^\infty f_X(x; \Psi) dx \\ &\quad + ph(1 - \ell) \int_0^{(1-\ell)K} f_X(x; \Psi) dx - ps(1 - \ell) \int_{(1-\ell)K}^\infty f_X(x; \Psi) dx + p \frac{\partial c(\ell K)}{\partial K}. \end{aligned}$$

Since

$$\int_0^K f_X(x; \Psi) dx = F_X(K; \Psi) \quad \text{and} \quad \int_K^\infty f_X(x; \Psi) dx = 1 - F_X(K; \Psi),$$

$$\frac{\partial I(K)}{\partial K} = (1 - p)(h + s)F_X(K; \Psi) + p(h + s)(1 - \ell)F_X((1 - \ell)K; \Psi) + p \frac{\partial c(\ell K)}{\partial K} - s(1 - p\ell).$$

If  $\ell = 1$ , then all the inventory is lost due to the catastrophic event (e.g., flooding or contamination). In that case,

$$\frac{\partial I(K)}{\partial K} = (1 - p)(h + s)F_X(K; \Psi) + p \frac{\partial c(K)}{\partial K} - s(1 - p).$$

Similarly,

$$\frac{\partial^2 I(K)}{\partial K^2} = (1-p)(h+s)f_X(K; \Psi) + p \frac{\partial^2 c(K)}{\partial K^2}.$$

$$\left. \frac{\partial I(K)}{\partial K} \right|_{K=K^*} = 0 \Rightarrow (1-p)(h+s)F_X(K^*; \Psi) - (1-p)s + p \left. \frac{\partial c(K)}{\partial K} \right|_{K=K^*} = 0.$$

Thus, it holds that

$$F_X(K^*; \Psi) = \frac{s}{h+s} - \frac{p}{(1-p)(h+s)} \cdot \left. \frac{\partial c(K)}{\partial K} \right|_{K=K^*} \quad (2)$$

and

$$f_X(K^*; \Psi) = -\frac{p}{(1-p)(h+s)} \cdot \left. \frac{\partial^2 c(K)}{\partial K^2} \right|_{K=K^*}.$$

Because

$$\left. \frac{\partial^2 I(K)}{\partial K^2} \right|_{K=K^*} \geq 0,$$

we obtain

$$(1-p)(h+s)f_X(K; \Psi) + p \frac{\partial^2 c(K)}{\partial K^2} \geq 0.$$

That is,

$$\frac{\partial^2 c(K)}{\partial K^2} \geq -\frac{(1-p)}{p} \cdot (h+s) \cdot f_X(K; \Psi).$$

We are now ready to make several observations about the impact of operational disruptions on the optimal inventory level which would minimize the expected cost under disruption risk. The first term in Equation (2) represents the usual newsvendor fractile or Type-1 service level (i.e., probability of not stocking out) in the absence of any catastrophic event. We denote it by

$$\beta = \frac{s}{h+s}.$$

Then, we can rewrite Equation (2) as

$$F_X(K^*; \Psi) = \beta - \frac{p}{(1-p)} \cdot (1-\beta) \cdot \left. \frac{\partial c(K)}{\partial K} \right|_{K=K^*}.$$

It is natural to expect  $\left. \frac{\partial c(K)}{\partial K} \right|_{K=K^*} \geq 0$ ; i.e., the higher the inventory loss, the more costly it will be to recover. Hence, when a catastrophic event is likely to occur with positive probability,  $p > 0$ , it is optimal to reduce the inventory level by an amount which decreases the probability of not stocking out by

$$\frac{p}{(1-p)} \cdot (1-\beta) \cdot \left. \frac{\partial c(K)}{\partial K} \right|_{K=K^*}.$$

This is because, when a catastrophic event is likely to occur with positive probability, the risk lies in overstocking. Therefore, the optimal inventory level is set to hedge against the risk of overstocking, and the optimal quantity of inventory to keep is reduced.

Second, the reduction in the Type-1 service level is highly sensitive to the likelihood of the catastrophic event. In particular,

$$\frac{d}{dp} \left\{ \frac{p}{1-p} \cdot (1-\beta) \cdot \left. \frac{\partial c(K)}{\partial K} \right|_{K=K^*} \right\} = -\frac{1}{(1-p)^2} \cdot (1-\beta) \cdot \left. \frac{\partial c(K)}{\partial K} \right|_{K=K^*}.$$

As  $\beta$  increases ( i.e.,  $(1 - \beta)$  decreases), the deterioration in the Type-1 service level becomes smaller. However, the impact of  $\beta$  is overwhelmed by the effect of the disruptive event probability  $p$ .

Third, the shape of the recovery cost function plays an important role in the determination of the optimal inventory level. While the impact may be negligible for linear cost (e.g., when  $c(k) = \alpha k$  with  $\alpha > 0$ ), it could be quite significant for polynomial cost functions (e.g., when  $c(k) = \alpha k^2$  with  $\alpha > 0$ ).

Finally, within the context of risk management, focusing on expected cost as the objective function may mask some important trade-offs. We will, therefore, design a stochastic simulation in the next section and perform a numerical investigation into the impact of inventory model parameters not only on the expected cost but also on the risk profile of the service level given an on-site inventory level, which would minimize the total cost in the absence of any disruptive event. In light of the insights obtained in this section, we will focus on the impact of disruptive event probability  $p$ , fraction of inventory loss  $\ell$  in the presence of disruption and the form of the recovery cost function  $c(k)$  on the distributional properties of the total cost and the unit fill rate for given demand distribution function  $F_X(x; \Psi)$  and the Type-1 service level  $\beta$  which is expected to hold in the absence of disruption.

### 4.2 Stochastic Simulation: Design and Analysis

In this section, we use SAS Simulation Studio to represent the process flow of the inventory system under consideration. SAS Simulation Studio is a Java-based application for building and working with both Monte Carlo and discrete-event simulation models. SAS Simulation Studio models dynamic system operations as a discrete sequence of events, each of which occurs at a specific point in time and triggers a change in system state as objects move within the stochastic simulation as entities. In the inventory system simulation of interest, an object refers to demand, and the demand quantities flow through the stochastic simulation logic as it generates a variety of attributes and system key performance indicators (KPIs). The inventory simulation designed to support our discussion reports values for two KPIs, namely, total cost and unit fill

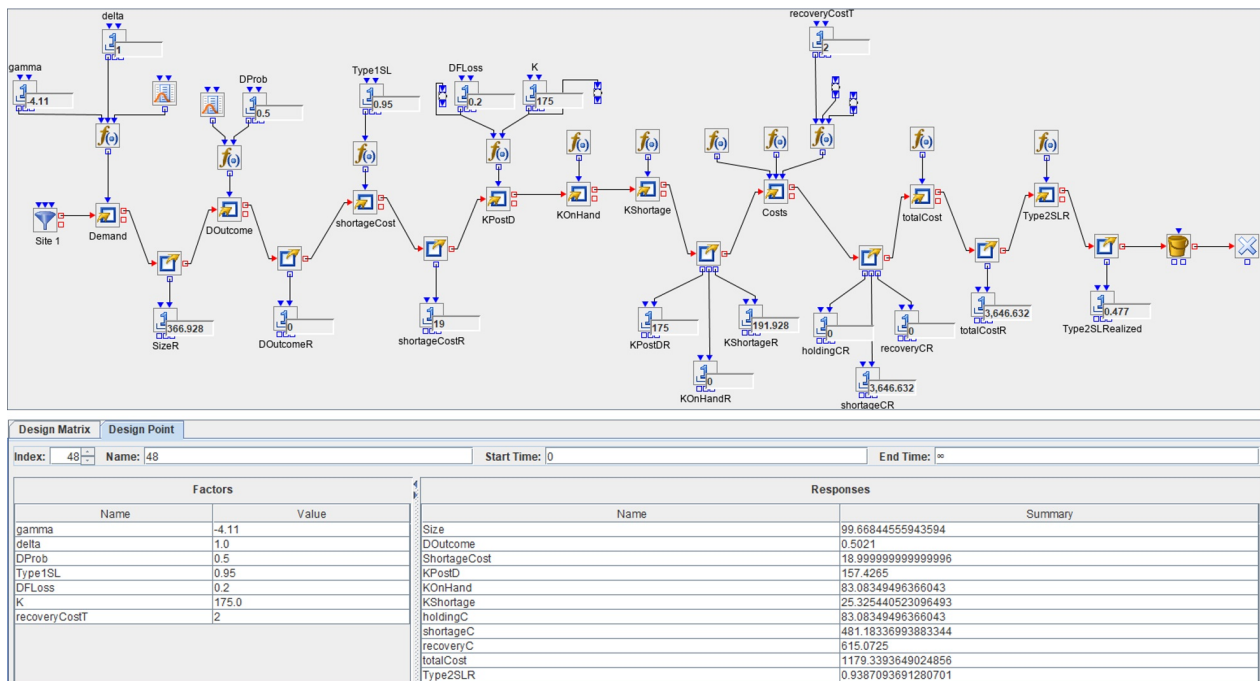


Figure 1: SAS Simulation Studio: Inventory system simulation model and experimental design.

rate. The examples of the attributes include (i) the demand size; (ii) whether a disruptive event occurs or not; (iii) amounts of inventory lost due to disruption and left on-site; (iv) costs of inventory holding, demand shortage, and loss recovery; and (v) total cost and fill rate. Figure 1 illustrates a simplified view of how the blocks available in SAS Simulation Studio for drag-and-drop construction lead to an inventory system simulation model and experimental design view. We refer the reader to Biller et al. (2019) for a discussion of scalability, data-driven nature, and flexibility of SAS Simulation Studio models.

#### 4.2.1 Stochastic Inventory Simulation Design

We design the inventory system simulation for an independent demand process with a mean of 100 units. Its coefficient of variation is chosen as 1.31, implying a highly variable demand process. More specifically, we assume a lognormal demand process with location parameter  $\xi = 0$  and shape parameters  $\gamma = -4.11$  and  $\delta = 1.0$ . Therefore, the square root of the coefficient of skewness of the demand random variable is 6.18 and the coefficient of kurtosis is 113.94 with 157 as the 0.95-quantile. This is in comparison to a normal random variable with 0 as the coefficient of skewness and 3 as the coefficient of kurtosis.

We calculate a unit shortage cost associated with a non-stockout probability of 0.95 in the absence of any disruptive event (i.e.,  $\beta = 0.95$ ). Furthermore, we assume  $p \in \{0\%, 1\%, 2\%, 5\%, 10\%, 20\%, 30\%, 50\%\}$  (motivated by Table 1),  $\ell \in \{20\%, 50\%, 100\%\}$ , and  $c(k) = \alpha k$  or  $c(k) = \alpha k^2$  with  $\alpha = 1$ . This corresponds to

Table 2: Mean cost (C) and mean fill rate (FR) for each of the 48 experimental designs.

Linear Recovery Cost Function				Polynomial Recovery Cost Function			
$p$	$\ell$	C	FR	$p$	$\ell$	C	FR
0%	100%	\$523.89	95.05%	0%	100%	\$523.89	95.05%
1%	100%	\$539.99	94.04%	1%	100%	\$862.76	94.04%
2%	100%	\$558.67	92.97 %	2%	100%	\$1,225.52	92.97 %
5%	100%	\$605.59	90.11 %	5%	100%	\$2,198.12	90.11%
10%	100%	\$689.09	84.89 %	10%	100%	\$3,941.15	84.89%
20%	100%	\$833.37	75.67%	20%	100%	\$7,011.67	75.67%
30%	100%	\$986.24	66.34%	30%	100%	\$10,160.82	66.34%
50%	100%	\$1,299.74	47.33%	50%	100%	\$16,588.69	47.33%
0%	50%	\$523.89	95.05%	0%	50%	\$523.89	95.05%
1%	50%	\$528.01	94.95%	1%	50%	\$608.24	94.95%
2%	50%	\$533..11	94.82 %	2%	50%	\$698.86	94.82 %
5%	50%	\$545.37	94.52 %	5%	50%	\$941.21	94.52%
10%	50%	\$567.32	93.94 %	10%	50%	\$1,375.66	93.94%
20%	50%	\$601.86	93.04%	20%	50%	\$2,137.56	93.04%
30%	50%	\$639.74	92.09%	30%	50%	\$2,920.20	92.09%
50%	50%	\$722.11	90.01%	50%	50%	\$4,522.38	90.01%
0%	20%	\$523.89	95.05%	0%	20%	\$523.89	95.05%
1%	20%	\$525.07	95.03%	1%	20%	\$537.68	95.03%
2%	20%	\$526.71	94.99 %	2%	20%	\$552.77	94.99 %
5%	20%	\$530.33	94.92 %	5%	20%	\$592.57	94.92%
10%	20%	\$536.79	94.78 %	10%	20%	\$663.88	94.78%
20%	20%	\$546.74	94.57%	20%	20%	\$788.19	94.57%
30%	20%	\$557.66	94.36%	30%	20%	\$916.20	94.36%
50%	20%	\$581.84	93.87%	50%	20%	\$1,179.34	93.87%



48 different simulation experiments (i.e., scenarios) each of which is simulated for 10,000 replications to control stochastic simulation variance. Each experiment is conducted assuming an on-site inventory level  $K$  of 175 units, which would deliver a unit fill rate of 95% in the absence of any disruptive events.

What is important to recognize here is that the stochastic inventory simulation illustrated in Figure 1 plays the role of a synthetic data program and generates predictions of total cost and unit fill rate in each of the 10,000 replications for 48 different scenarios. This results in a 480,000-row simulation output data set. In the next section, we analyze this data set to obtain risk profiles of total cost and fill rate, and study their relation to  $p$ ,  $\ell$ , and  $c(k)$  as the primary inventory model parameters.

### 4.2.2 Stochastic Inventory Simulation Analysis

Table 2 presents mean cost and mean fill rate for each of the 48 experiments described in Section 4.2.1. As consistent with the mathematical analysis in Section 4.1, we observe the mean fill rate to be a decreasing function of the increasing disruptive event probability. When  $\ell = 1$  and  $c(k) = \alpha k$  (with  $\alpha = 1$ ), disruption probability of 1% results in a mean fill rate of 94.04% while disruption probability of 10% further decreases the mean fill rate to 84.89%. However, the strength of the impact of the disruption probability on the mean fill rate is reduced with the decreasing values of the inventory loss fraction  $\ell$ . The form of the recovery cost function  $c(k)$ , on the other hand, carries significant importance for the resulting expected total cost. The simulation output analysis focusing on the mean performance metrics is beneficial in terms of gauging the role of operational disruptions on inventory management under disruption risk. By focusing on the

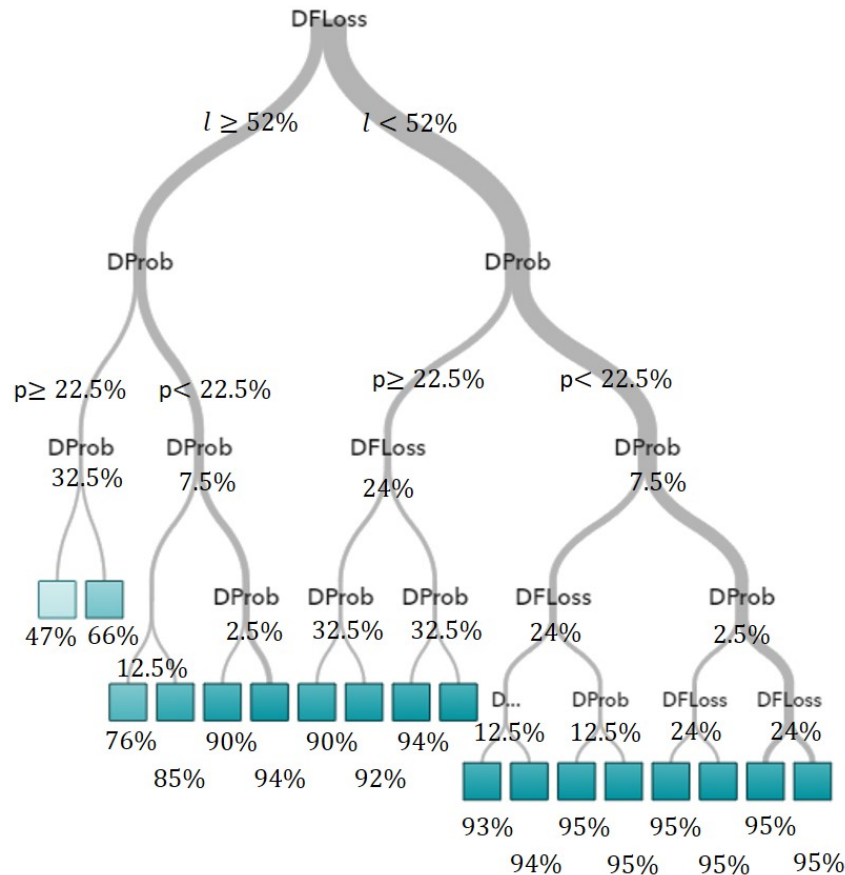


Figure 2: Contents of Table 2 on a decision tree with thresholds on  $p$  and  $\ell$ .

values of the disruption probability  $p$  and the inventory loss  $\ell$  tabulated in Table 2, Figure 2 provides an illustration of the mean fill rate by utilizing the 480,000 different sample paths obtained from the inventory system simulation.

Next, we focus on a different strength of the stochastic inventory system simulation; i.e., the ability to quantify the variability in the performance metrics. Table 3 presents the coefficient of variation associated with each of the mean fill rate tabulated in Table 2.

Table 3: Coefficient of variation (CV) of the fill-rate (FR) measure obtained from the inventory simulation.

$\ell = 100\%$ Loss		$\ell = 50\%$ Loss		$\ell = 20\%$ Loss	
$p$	FR CV	$p$	FR CV	$p$	FR CV
0%	0.158	0%	0.158	0%	0.158
1%	0.189	1%	0.160	1%	0.158
2%	0.219	2%	0.162	2%	0.159
5%	0.285	5%	0.168	5%	0.160
10%	0.384	10%	0.178	10%	0.162
20%	0.536	20%	0.192	20%	0.166
30%	0.684	30%	0.206	30%	0.170
50%	1.028	50%	0.235	50%	0.177

We observe that the sensitivity of the fill rate to the disruption probability decreases as less amount of inventory is lost in the presence of a disruption. The coefficient of variation for the fill rate takes its highest value for 50% disruption probability under the assumption of full loss of inventory in the presence of a disruption. Obviously, the mean fill rate is the most sensitive to the probability of disruption  $p$  followed by the fraction of inventory loss  $\ell$  for any given size of the demand. The relative importance of these two input variables on the mean fill rate is visualized in Figure 3. The numerical values reported in this figure are obtained in two steps: First, SAS GRADBOOST procedure is used to create a gradient boosting model. Second, the change in the residual sum of square errors is calculated for each input variable. For a more detailed description, we refer the reader to the procedures guide on SAS<sup>®</sup> Viya<sup>™</sup> Data Mining and Machine Learning (SAS Institute 2019).

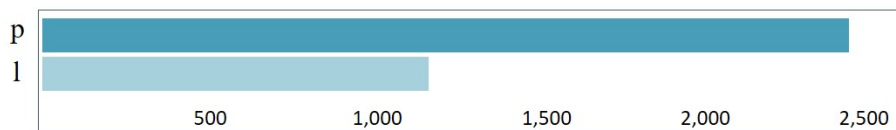


Figure 3: Variable Importance: Impact of  $p$  and  $\ell$  on the mean fill rate.

While it is beyond the scope of this paper, the critical next step is to utilize the risk quantification obtained in this section for devising operational policies to hedge against the risk of disruption (e.g., finding an additional source of supply to meet the product demand when the primary site experiences a disruption).

## 5 CONCLUSION

In this paper, we combine the well-known newsvendor model capturing the cost of demand and supply uncertainty with a catastrophe model capturing the cost of disruption and recovery. Our focus is on stocking decisions in the presence of facility disruptions caused by random events such as natural disasters or man-made disturbances. Disruptions may be independent from one facility to another as assumed in this study (e.g., such as facility contamination or a fire in the plant) . We note that when a catastrophic

event is likely to occur with positive probability, the risk lies in overstocking; the optimal quantity of inventory is therefore reduced where the shape of the recovery cost function plays an important role in the determination of the optimal inventory level. Treating stochastic inventory simulation accounting for disruption as a data-generation program, we provide an example of using data analytics for obtaining insights about the role of disruption probability and inventory loss following a disruptive event on cost and service level. Natural extensions include scenarios with multiple products and multiple locations across multiple periods where the duration of the disruption (i.e., time to recovery) would also be a performance measure of interest. It would also be important to consider disruptions that are correlated across facilities, e.g., as it is the case for earthquakes or tornados.

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