Proceedings of the 2019 Winter Simulation Conference
N. Mustafee, K.-H.G. Bae, S. Lazarova-Molnar, M. Rabe, C. Szabo, P. Haas, and Y.-J. Son, eds.

ROLLING HORIZON MODELS FOR INTER-DEPOT EMPTY CONTAINER REPOSITIONING

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ABSTRACT

Export customers requesting empty containers in the hinterland areas are serviced by maintaining sufficient inventory at each regional depot. The supply-demand imbalance at the regional level is stabilized by repositioning empty containers between inland depots. We propose an inter-depot empty container repositioning problem and a heuristic real-time decision algorithm to solve it. Initially, single-period travel time is considered and three models: Allocation Problem (AP), Value Approximation Model (SPL-VA), and Node Decomposition Heuristic (NDH-SP) are presented. The system is simulated over a certain time horizon by generating real-time supply and demand values, and the system's evolution is studied under each of the proposed models. The VA models perform better than the AP with modest computational effort. The NDH-SP is further generalized to accommodate multi-period travel times. By simulating this algorithm with a demand rejection policy, we observe that maximum demand satisfaction is obtained by allowing medium-sized demand queues at the depots.

1 INTRODUCTION

Packing and transporting goods in standardized containers has increased the efficiency of multimodal transport by reducing the time and costs associated with cargo handling at multiple terminals. These containers can be directly loaded on the ships, barges, trains, or trucks and moved from the shipper to the receiver over large distances. Once the goods are unloaded, the empty container is free to be assigned to another customer. However, the regions with a high level of imports generate a supply of empty containers with few export commodities to be loaded in them. On the other hand, regions with higher exports have a high demand for empty containers, which may not be locally available. Thus, there is an inevitable need to transport empty containers from supply to demand locations. The empty container repositioning is done at both global as well as regional levels.

The repositioning of empty containers at different levels is subject to distinct constraints. At the global level, containers are stacked on the ships carrying loaded containers and no excess costs are incurred. However, at the regional level, where the containers have to be moved individually by trucks or pooled together to be relocated by trains, the costs are significant. The time required to procure international containers is much larger than that involved in getting them from some hinterland location. For these reasons, the two problems are modelled and studied separately.

Most of the research concerning empty container repositioning focuses on the global imbalance, whereas the work related to inland repositioning is limited (Sterzik and Kopfer 2013). The inland repositioning papers mainly focus on the movement of containers between ports\depots and customer locations. The containers available at the depot are allocated to the requests arising in the real-time during the operational stage. Repositioning empty containers between multiple depots leads to the availability of the containers to the customers in the shortest possible time and thus dictates the service level. This aspect has not been explored extensively in the existing research. As per our knowledge, this is the first work that explicitly models and solves the problem of inter-depot repositioning of empty containers.

This work defines the Inter-depot Empty Container Repositioning Problem (IDCRP) and presents a heuristic real-time decision model to address it. Initially, single-period travel time is assumed and three models are proposed that use predicted demand and supply values for a time window of two time periods. Problems of this type are traditionally modelled using bipartite graphs separating the demand and supply locations. Hence, first, the problem is modelled as an Allocation Problem that assigns excess supply during time period t to the excess demands during $t+1$. However, as this model does not consider the effects of repositioning decisions at t, a Value Approximation Model based on Dynamic Programming is described. To reduce the complexity of the model, a heuristic based on Node-wise Decomposition of the Value Approximation model is proposed. To study the effectiveness of the model, the system is simulated over a certain time horizon and the evolution of the system under each of the three models is studied. Statistical significance of the results is tested using Wilcoxon signed rank test, and it is concluded that the heuristic is suitable for practical applications. The single time period heuristic is further generalized to allow realistic multi-period travel times. A simulation study is conducted on the modified heuristic and the demand rejection policy is tested.

The paper is organized as follows. A review of the inland repositioning literature is presented in Section 2. The detailed description of the IDCRP investigated in this paper is given in Section 3. Various models to solve the described problem are detailed in Section 4 with a simple allocation model in Section 4.1, Value Approximation Model in Section 4.2, and a Node Decomposition Heuristic in Section 4.3. All these models assume a single-period travel time between depots. Section 5, provides a comparative study of these models. A generalized model for multi-period travel time is presented and analyzed in Section 6.

2 LITERATURE REVIEW

The maritime Empty Container Repositioning Problem has been addressed by numerous researchers. However, only the literature pertaining to inland repositioning is reported in this section. A detailed review of the empty container management literature is given by Braekers et al. (2011). They differentiate the papers based on the level of planning (Strategical, Tactical or Operational) as well as the type of model proposed (Deterministic, Stochastic or Simulation). In our view, Crainic et al. (1993) have published the first work concerning the inland repositioning of empty containers. They formulate dynamic deterministic models for both single and multi-commodity empty container allocation problems and a two-stage recourse model for the dynamic stochastic single commodity problem.

The repositioning decision is classified as global, inter-regional or regional by Boile et al. (2008). They focus on the regional repositioning of containers between ports, empty container depots and import or export customers. The depot positioning and container allocation problem is formulated as an ILP and solved by Branch and Bound technique. Further, the model is used to analyze the port of the New York-New Jersey region. Jula et al. (2006) assess the empty container movements in the Los Angeles and Long Beach port areas. A Transportation Problem is solved over a bipartite network of demand and supply locations to determine the number of containers to be moved. To study the effect of reuse cost on depot direct and street turn methodologies, simulations are performed over multiple demand-supply scenarios. Finke and Kotzab (2017) conduct a case study on the empty container problem in the German hinterland region. They demonstrate the effectiveness of the inland depots in reducing the total distance travelled by trucks and overall operating costs by modelling the problem as an ILP and solving it by branch and bound method. None of these models (Boile et al. 2008; Jula et al. 2006; Finke and Kotzab 2017) assume direct movement between depots or import and export customers.

Another practical application concerning a transportation company in Germany with consideration of future demands is presented by Jansen et al. (2004). Various sub-problems of operation planning like modality choice, repositioning, combination of orders, order planning and planning improvement are solved separately. The repositioning problem is modelled as a minimum cost flow problem including both rail and road movements. Reinhardt et al. (2016) study the cost reductions obtained by allowing triangulation with predetermined as well as free destination cases. They enumerate all feasible paths and select the most profitable ones by an order covering model and solve the model using column enumeration technique. The imbalance at each depot is avoided by solving a container yard balancing model. A Decision Support System based on a detailed model including terminals, depots and customer locations for optimization of empty container movements in Valencia is studied by Furio et al. (2013). They indicate similar insights of cost saving in hinterland operations by allowing street-turn operations.

The repositioning and leasing decision at a single port is studied by Yun et al. (2011) using an inventory control model. They determine the near-optimal inventory levels at the port under different conditions by varying the parameters like (s,S) policy, simulation period, leasing, ordering and holding costs and the supply-demand gap. Dang et al. (2012) use the inventory model on a multi-depot region with a port. The optimal policy determines the number of containers to be acquired from other ports, depots or leased from other companies and is obtained by a genetic algorithm. However, they do not attempt to optimize the repositioning decisions but allocate containers from other depots based on fixed rules.

The container repositioning problem is similar to the empty rail car assignment problem where the available empty cars have to be allocated to the demands in the network. This problem has been explored by Narisetty et al. (2008) who use the transportation problem to determine the number of cars to be assigned during each time period. Spieckermann and Voss (1995) model the problem as a scheduling problem where the demands have to be assigned to the rail-cars and are solved using a heuristic approach. The models based on Transportation Problem and minimum cost flow formulation are described in Gorman et al. (2011). These models are similar to the allocation model considered in Section 4.1 of this paper.

3 PROBLEM DESCRIPTION

Consider a network of hinterland depots distributed over a region. We assume that there is a single depot in each region and a set of neighbouring locations are allocated to each depot. After unloading the import containers, they are returned to this depot and the container requests in the neighbourhood for loading export goods are met by empty containers available here. Thus the demand or supply at the depot indicates aggregate values of the neighbouring customers. The allocation of empty containers to a demand location is an operational decision and depends on the availability of the containers at the depot. For this, sufficient inventory of empty containers should be maintained at each depot. As supply and demand values in each region may vary, there is a need to reposition the empty containers from depots with excessive supply to the container deficit regions. This decision has to be optimized such that maximum demands can be served with minimum empty movement.

The containers need to be repositioned at each time period over the specified time horizon. However, at a given time period, the demand and supply at each depot are known over a certain time window. For this, we assume that the demands are known at least $'k'$ days before they are required such that k is greater than the time required to reposition the containers from some other depot. Since the container requests are placed ahead of time, this is a realistic assumption.

Similarly, as the import containers enroute their destination are known, the supply for each time period in the given window can be determined. Though the demand and supply values are deterministic in the observed window, the containers are relocated to other depots only when they are available at the depot. Thus, the repositioning decision has to be taken in real time.

The parameters used in the model are as following:

- N- Set of nodes in the network indicating depots, $n \in N$
- E- Set of edges indicating paths between two depots in the network $e_{ij} \in E$ for $i, j \in N$
- T- Set of time periods, $t \in T$
- t_{ij}- Travel time between nodes i and j
- c_{ij}- Travel cost between nodes i and j

- r_n^t- Number of containers available at depot n at start of time period t.
- s_n^t- Estimated supply at depot n during the time period t
- d_n^t- Estimated demand at depot n during the time period t
- p_n^t- Pending demands at depot n during at time period t from previous periods
- Q_n- Revenue earned by serving demand at n

A two node representation of the inter-depot container repositioning is shown in Figure 1. Note that the decision to move containers x_{ij}^t containers from i to j is taken at the start of time period t. For the node i, the demands and supply of containers by neighbourhood customers are realized during the time period t. The demands d_i^t can be served by the containers available at i i.e., $r_i^t - x_{ij}^t$ as well as the incoming supply during the same time period i.e., s_i^t. The containers moved from i are available at j at the start of time period $t + t_{ij}$. However, at t we do not know the number of containers at j at the start of $t + t_{ij}$ as it depends on decisions at j between time t and $t + t_{ij}$.

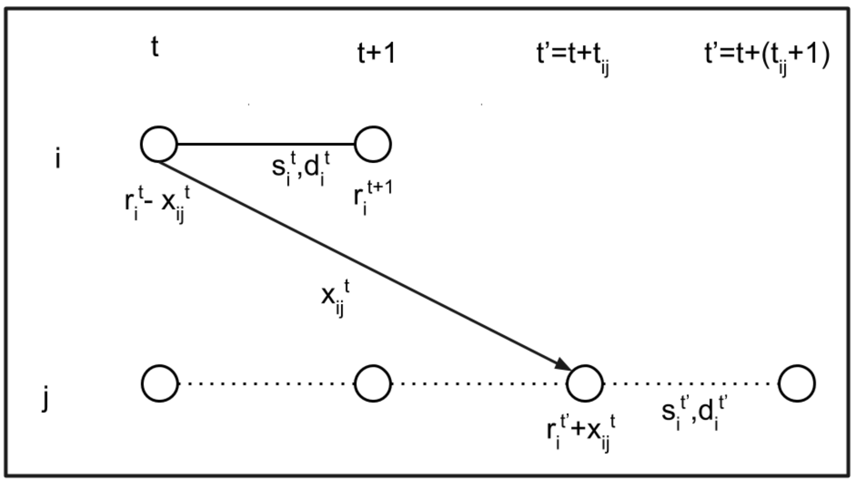

Figure 1: Representation of container repositioning problem.

This problem can be solved by modelling it as a dynamic transportation problem. However, for large data sets, this takes a considerable amount of time to solve as compared to single-period models. The demand and supply are subject to uncertainty in real life applications. In this paper, the problem is modelled as a real-time repositioning problem where the decision is taken at the start of every time period t and only the containers available at the depot at this time period can be relocated.

4 MODELS FOR INTER-DEPOT REPOSITIONING PROBLEM

This paper aims to develop a heuristic to solve the inter-depot empty container repositioning problem with multi-period travel time. Initially, the problem is simplified by assuming single-period travel time between any two depots. Hence, only a time window of two time periods, t and $t+1$ is considered. First, the problem is formulated as a Container Allocation Problem in Section 4.1. This maximizes the revenue earned by the excess containers at t repositioned during the time period t. The total revenue obtained by supplying empty containers to requests includes the demands satisfied at current as well as future time periods. However, the AP does not consider the effect of the decision at t on the demand satisfaction during t and only the revenue generated by repositioned containers is considered. A Value Approximation based inclusive model considering demand and supply over the entire time window is presented in Section 4.2. A heuristic method developed by decomposing this model over individual nodes is given in in Section 4.3.

4.1 Container Allocation Problem

In the model presented here, a rolling horizon of two time periods is considered, where the number of containers to be repositioned between depots is determined in the first time period and the results of this decision are observed in the second time period. At every depot, the demands during t are prioritized and only excess available containers are considered for repositioning. The net supply and demand at each $n \in N$ are calculated using known supply and demand values during the given time window.

Apart from the variables described in Section 3, in the preprocessing step, the following parameters are utilized to calculate the net supply and demand of containers at depot n at the start of time period t:

- p_n^t- Unserved demands at n from time $t' < t$, $\forall t' \in T$
- $d_n^{t^+}$- Excess demands at n, during time the period t ($d_n^{t^+} = |d_n^t + p_n^t - s_n^t|^+$)
- $\bar{r}_n^t$- Containers available for assignment at n at the start of time period t ($\bar{r}_n^t = |r_n^t - d_n^{t^+}|^+$)
- I_n^{t+1}- Imbalance of supply at n during time period $t+1$ ($I_n^{t+1} = \bar{r}_n^t + s_n^{t+1} - d_n^{t+1}$)
- $\bar{S}_n^t$- The net supply at n indicates the number of excess containers available at start of t apart from the containers reserved for demands during time t and $t+1$ at n. $\bar{S}_n^t = min(\bar{r}_n^t,\ |I_n^{t+1}|^+)$
- $\bar{D}_n^t$ The net demand at n is the number of requests that cannot be served by the available containers at n over the given time window. $\bar{D}_n^t = \left| |I_n^{t+1}|^- - |s_n^t + d_n^t - p_n^t|^+ \right|^+$

The decision variable used in the models is x_{ij}^t which indicates the number of containers to be repositioned from depot i to depot j at time t. The final allocation at the current time period can be determined by solving the following model at a given time period t

$$\text{Maximize} \sum_{ij \in E} (Q_j - c_{ij})\ x_{ij}^t \tag{1}$$

such that

$$\sum_{i:(ij) \in E} x_{ij}^t \le \bar{S}_i^t \qquad \forall i \in N \tag{2}$$

$$\sum_{j:(ij) \in E} x_{ij}^t \le \bar{D}_j^t \qquad \forall i \in N \tag{3}$$

$$x_{ij}^t \in \mathbb{Z}^+ \qquad \forall ij \in E \tag{4}$$

Objective (1) maximizes the net revenue earned by repositioned containers. The number of containers supplied from each depot is restricted by net supply available at that depot by Equation (2). As the repositioning costs have to be minimized, Equation (3) restricts the number of containers moved to every depot beyond its net demand.

The post decision container availability at n is given by, $r_n^{xt} = r_n^t - \sum_{i \in N} x_{ni}^t, \forall n \in N$. This represents the number of available containers at the start of the time period t after the repositioning decision is made. When actual supply, $\hat{s}_n^t$ and demand $\hat{d}_n^t$ are realized during the time period t, the number of containers at time $t+1$ is the sum of available containers after decision at t, net supply generated during time period t and the containers arriving as per the decision taken at start of period t. This is calculated as $r_n^{t+1} = \left| r_n^{xt} + |\hat{s}_n^t + \hat{d}_n^t - p_n^t|^+ \right|^+ + \sum_{i \in N}\ x_{in}^t, \forall n \in N$. The number of pending requests at the start of time $t+1$ is also updated based on the observed supply and demand, $p_n^{t+1} = |\hat{s}_n^t + \hat{d}_n^t - p_n^t|^-,\ \forall n \in N$.

The parameters are updated and the above model is solved for the next time period to make the repositioning decisions at $t+1$. The revenue obtained by serving demands at time $t+1$ by repositioning decision taken at time t is maximized. To ensure that the demands in the current time period t at depot n are not affected, the excess supply of containers, $\bar{S}_n^t$ that can be relocated is determined. This model does not implicitly maximize the current revenue, which is an important factor during repositioning.

4.2 Single-Period Lookahead Value Approximation (SPL-VA)

In this section, a single-period look-ahead model incorporating the effect of decision x^t_{ij} on the demands during time period t and $t+1$ is presented. In this model, v^t_n represents the number of demands served at depot n during the time period t and $\hat{V}^{t+1}(x^t)$ the approximate revenue earned during time period $t+1$ given the decision x^t at time t. Under the current assumption of single-period travel time, the decisions at t affect only the states at t and $t+1$, and thus only revenue over these two periods is considered while allocating containers at time t. The net revenue at time period t by the Dynamic Programming Model can be determined as follows

$$\text{Max}_{x^t}\Big(\sum_{i\in N} Q_i v^t_i - \sum_{ij\in E} c_{ij}x^t_{ij} + \hat{V}^{t+1}(x^t)\Big). \tag{5}$$

The number of demands served during time period t for every $n \in N$ is given by

$$v^t_n = \min(r^t_n - \sum_{k\in N} x^t_{nk} + s^t_n, d^t_n). \tag{6}$$

The revenue at $t+1$ also depends on decision at $t+1$. Demands at the current node are prioritized by the model due to the negative repositioning costs in the objective. Thus, the containers at n will not be repositioned during $t+1$ if there is a demand at n at the start of $t+1$. As the decisions at $t+1$ which determine the supply and thus the revenue at $t+1$ are not considered, the approximate revenue at $t+1$ is given by $\hat{V}^{t+1}(x^t) = \sum_{i\in N} Q_i v^{t+1}_i$. The number of demands served at n at $t+1$ can be approximated as,

$$v^{t+1}_n = \min(r^{t+1}_n + s^{t+1}_n + \sum_{k\in N} x^t_{kn}, d^{t+1}_n + (d^t_n - v^t_n)). \tag{7}$$

where $(d^t_n - v^t_n)$ is the pending demand from time period t and the pending demands prior to $t-1$ are included in d^t. Let g^t_n represent the number of containers available at t after assignment. The dynamic program described above, is modelled and solved as an ILP. Equation 5 is the objective which can be directly written as

$$\text{Max} \sum_{i\in N} Q_i v^t_i - \sum_{ij\in E} c_{ij}x^t_{ij} + \sum_{i\in N} Q_i v^{t+1}_i \tag{8}$$

such that,

$$\begin{aligned}
\sum_{k\in N} x^t_{nk} &\le r^t_n && \forall n\in N && (9)\\
v^t_n &\le g^t_n + s^t_n && \forall n\in N && (10)\\
v^t_n &\le d^t_n && \forall n\in N && (11)\\
v^{t+1}_n &\le r^{t+1}_n + s^{t+1}_n + \textstyle\sum_{k\in N} x^t_{kn} && \forall n\in N && (12)\\
v^{t+1}_n &\le d^{t+1}_n + (d^t_n - v^t_n) && \forall n\in N && (13)\\
g^t_n &= r^t_n - \textstyle\sum_{k\in N} x_{nk} && \forall n\in N && (14)\\
r^{t+1}_n &= g^t_n + s^t_n - v^t_n && \forall n\in N && (15)\\
v^t_n, v^{t+1}_n &\in \mathbb{Z}^+ && \forall n\in N && (16)
\end{aligned}$$

In this formulation, the objective (8) maximizes the revenue earned during the time periods t and $t+1$ while minimizing the repositioning costs. Equation (9) restricts the number of assignments from every n to be less than the number of available containers. Further, Equations (10) to (13) linearize the value functions at t and $t+1$ given in Equations (6) and (7). The number of containers available at the start of t after assignment decision is made i.e., g^t is calculated by Equation (14). The number of containers available at start of $t+1$ is determined by Equation (15). Lastly, Equation (16) constraints decision variables to be positive integers.

Based on the observed demand and supply values, the number of available containers at the start of $t+1$ are, $r_n^{t+1} = |g_n^t + \hat{s}_n^t - \hat{d}_n^t|^+ + \sum_{k\in N} x_{kn}^t, \quad \forall n \in N$. The pending demands $p_d^{t+1} = \hat{d}_n^t - min(\hat{d}_n^t, v_n^t)$ are, added to the demands in next time period, $d_n^{t+1} = d_n^{t+1} + p_d^{t+1}$, $\forall n \in N$.

The above model determines the optimal repositioning decision for all the depots in the network for the time period t. For large-sized real networks, this model involves n^2 variables and $7 \times n$ constraints. The complexity of the problem can be reduced by determining optimal decision at each depot instead of solving for all the depots simultaneously. The Constraint (12) makes the model inseparable in n as it includes repositioning decisions from all other nodes $k \in N$ to n at time t. The remaining constraints are independent of the decisions made in t from nodes other than n. To simplify the model, an approximate heuristic by node wise decomposition of the above model given in Section 4.3 is developed by separating the initial problem at each n and approximating V^{t+1} without the knowledge of x. This reduced formulation has $n-1$ variables and $(5+2*n)$ constraints and thus it is easier to solve as compared to the SPL-VA model.

4.3 Node Decomposition Heuristic Based on SPL-VA (NDH-SP)

A heuristic presented in this section is devised by, decomposing and solving SPL-VA individually at each node, $n \in N$. The final repositioning is then determined from the cumulative solutions. Thus for the individual depots $n \in N$, the objective 8 is redefined as

$$\text{Max } Q_n v_n^t - \sum_{i\in N} c_{ni} x_{ni}^t + \sum_{i\in N} Q_i v_i^{t+1}. \tag{17}$$

This objective function maximizes the revenue earned in time t at n and demands served in $t+1$ at all nodes $n \in N$, while minimizing the total cost of repositioning empty containers. This is maximized under the constraints, 9-15 described in section 4.2 for a single node n. The value of v_k^{t+1}, $\forall k \in N \backslash n$ has to be approximated as the decision x_{kj}^t, $\forall j \in N$ is not determined by this model. The number of demands served at depot k at time t and $t+1$ is approximated as

$$v_k^t = min(r_k^t + s_k^t, d_k^t)$$
$$v_k^{t+1} = min(r_k^{t+1} + s_k^{t+1} + \sum_{j\in N} x_{jk}^t, d_k^{t+1} + p_k^{t+1}).$$

By the above definition, $\sum_{j\in N} x_{kj}^t$ is not included in v_k^t since it is not known at this stage, however at the node at which repositioning decision is made, i.e., n the dependency of v_n^t on x_{nk}^t exists. Also note that, r_k^{t+1} is also dependent on x_{kj}^t, $\forall j \in N$. If the containers are repositioned from n to k at time period t, this satisfies deficit of demand during $t+1$. Such k should not have any containers repositioned during t, as it doesn't have sufficient supply during t to satisfy deficit in the next time period. Thus we approximate $g_k^t = r_k^t$ and $r_k^{t+1} = r_k^t + s_k^t - v_k^t$. In order to determine the demands served at $k \in N \backslash n$ at $t+1$ we consider the demands served by relocated containers x_{nk}^t and available containers r_n^t from previous time period rather than overall demands. The revenue earned by serving the demands d_k^{t+1} by incoming supply s_k^{t+1} during same time period remains fixed for all n. Thus, v_n^{t+1} represents the demands satisfied by available or repositioned containers and can be given by

$$v_k^{t+1} = min(r_k^t + x_{nk}^t, |d_k^{t+1} - s_k^{t+1}|^+ + d_k^t - v_k^t).$$

Thus, following constraints are added to the model

$$v_k^{t+1} \le r_k^t + x_{nk}^t \quad \forall k \in N \backslash n \tag{18}$$
$$v_k^{t+1} \le \left|d_k^{t+1} - s_k^{t+1}\right|^+ + d_k^t - v_k^t \quad \forall k \in N \backslash n \tag{19}$$

The solution of the above model for each $n \in N$ determines the number of containers allocated from n to serve demands arising over the entire network. This could lead to multiple solutions from different depots $n \in N$ to a depot k. To ensure that excess containers are not assigned the following post-processing steps are implemented:

- For every $k \in N$, if $\sum_{n \in N} x_{nk}^t > \left| d_k^{t+1} - s_k^{t+1} \right|^+$, assign containers from the nearest locations till the demands at k are satisfied.
- All the remaining assignments are cancelled i.e., $x_{nk}^t = 0$ and the containers are assumed to be available at this location, considered to be a temporary supply at n, s_n^{temp}
- If the excess demand at some k is not satisfied, the containers from nearest s_n^{temp} $\forall n \in N$ are assigned.
- Thus, the final assignments at t are intended to satisfy all excess demands at k at $t+1$.

Further, as described in Section 4.2, we update the values of r_n^{t+1} and d_n^{t+1} for the period $t+1$. Note that not all possible supply-demand pairs are balanced in this case. This heuristic is compared with the standard AP and the VA model in Section 5 to determine its usability in real life applications.

5 MODEL COMPARISON

In Section 4, three single-period models to solve the IDCRP are presented. The performance of these models are compared across the 30 instances over six network sizes $|N| = \{5, 10, 15..., 30\}$ over a 50×50 grid. At each node n, the average supply (s_n) and demand (d_n) values are obtained from a uniform distribution $\sim U(2,4)$. For each instance, the decisions are simulated over a time horizon of $T = 50$ and the number of demands served and the total empty repositioning distance is observed over each simulation. The supply and demand values at each node n over the time horizon T are drawn from normal distributions $\sim N(s_n, 2)$ and $\sim N(d_n, 2)$ and form dynamic inputs to the models. To avoid the end effects, availability of a certain number of containers is assumed at the start of time period 0 at each depot and the demand and supply at each node during the last 5 time periods is set to zero. This ensures that the results do not vary by partial realization of decisions. The simulations for the models are performed by using Python. The ILPs are modelled in PuLP in Python 2.7.13 and solved by COIN-OR solver. All the computations are performed on a machine equipped with 4 Intel Xeon 2.13 GHz cores and 64 GB RAM.

The maximum number of demands that are satisfied at each t is is bounded by the $min(\sum_{t,n} s_n^t, \sum_{t,n} d_n^t)$ as the import or leasing decision is not considered in this work.

It is observed that for all the instances, maximum possible demands are served using any of the models. Since the value of satisfying each demand is considered to be the same and the maximum possible demands are served, the revenue earned by satisfying demands remains constant. Therefore, to compare the performance of these models, only the repositioning costs are studied. Partial results of this study for three grid sizes and ten instances are reported in Table 1.

Table 1: Empty travel costs for different models.

	N=10			N=20			N=30		
Instance	AP	SPL-VA	NDH-SP	AP	SPL-VA	NDH-SP	AP	SPL-VA	NDH-SP
1	3431	2795	**2713**	6015	5248	**4989**	12147	**11076**	11115
2	6993	**6167**	6191	10566	**8787**	9148	10679	9967	**9878**
3	6103	**5897**	5978	8708	**6798**	7074	6118	**4817**	4907
4	404	**288**	293	5516	4923	**4895**	10079	9161	**8120**
5	2187	**2185**	**2185**	9882	**8646**	9061	5242	**4536**	4753
6	4594	**4183**	4380	4408	**4219**	4497	4845	4690	**4572**
7	4312	4033	**3773**	6890	**5713**	6251	10086	**7848**	8323
8	7726	**7265**	7320	7977	8058	**7947**	10490	**9703**	9706
9	6883	**5631**	5674	5021	**4709**	4710	12278	**11027**	11135
10	**3748**	3811	3814	9316	**8166**	8294	11398	**9758**	10399

The minimum repositioning costs are obtained by the SPL-VA model in 72 percent of the instances and NDH-SP model in 24 percent. In 35.5 percent of the total cases, NDH-SP performs equal to or better than SPL-VA whereas in 85 percent of the remaining cases the NDH-SP cost is within 5 percent of the SPL-VA solution. The maximum deviation seen is 10 percent.

The non-parametric Wilcoxon signed-rank test is applied to test the statistical significance of the results. Pairwise tests are performed on (AP, SPL-VA),(AP, NDH-SP) and (SPL-VA, NDH-SP). We test the hypothesis that the means are equal against the alternative hypothesis, $\mu_{AP} > \mu_{SPL-VA}$, $\mu_{AP} > \mu_{NDH-SP}$ in the first two cases and $\mu_{SPL-VA} \neq \mu_{NDH-SP}$ in the third with significance level 0.05. The results state that the null hypothesis is rejected in the first and the second case, concluding that there is a significant improvement in the performance of Value Approximation models when compared to the traditionally used Allocation Model for the empty container repositioning. The test suggests that there is not enough evidence to reject the null hypothesis in the (SPL-VA, NDH-SP) pair when $|N| > 10$. Thus, from the tests, we conclude that NDH-SP heuristic gives comparable solutions to SPL-VA on larger networks.

Due to lower repositioning costs than AP and lesser complexity than SPL-VA, the NDH-SP heuristic can be considered to be more suitable for practical applications. Further, the heuristic is generalized with some approximations in order to be applied to realistic container data, with multi-period travel time.

6 MULTI-PERIOD HEURISTIC

In this section, the NDH-SP is generalized to accommodate the realistic multi-period travel times to formulate the Node Decomposition Heuristic with Multi-period Travel Time (NDH-MP). A demand rejection criteria to provide higher service level to the customers is described in Section 6.1 and the simulations under different rejection parameters is provided in Section 6.2.

The objective function (17) is updated to account for the multi-period travel times as given in equation (20) where the rolling horizon window of two time periods is extended to include the time when the container reaches its destination node.

$$\text{Max } Q_n v_n^t - \sum_{i \in N} c_{ni} x_{ni}^t + \sum_{i \in N} Q_i v^{t+t_{ni}}. \tag{20}$$

The constraints for each individual node n at time t are same as given by Equations (9)–(11) and (14)–(15). As the containers repositioned from n at t will be available to satisfy the demands at location k at time $t + t_{nk}$ where t_{nk} is the travel time between nodes n and k, Equations (18) and (19) where the effect of travel time was observed need to be modified. Let $\bar{x}_{n,k}^{(t,u)}$ be the number of containers repositioned at time t from n and are expected to reach k at u. The total number of containers reaching location k at time u is given by:

$$\bar{x}_k^u = \sum_{t<u} \sum_{n \in N \setminus k} \bar{x}_{n,k}^{(t,u)}.$$

The available containers and pending demands at k at the start during the time period $(t + t_{nk})$ depends on the intermediate time periods and is approximated by using the net imbalance in the corresponding time window, given by:

$$R_k^{(t+t_{nk})} = r_k^t + \sum_{u \in (t, t+t_{nk}-1)} (s_k^u - d_k^u + \bar{x}_k^u).$$

The predicted number of containers available at k at the start of $t + t_{nk}$ is given by $\hat{r}_k^{t+t_{nk}} = |R^{(t+t_{nk})}|^+$. Similarly the demands at k at time $t + t_{nk}$ are predicted to be $\hat{d}_k^{t+t_{nk}} = -|R^{(t+t_{nk})}|^-$. Equations (18) and (19) are modified as given below:

$$v_k^{t+t_{nk}} \leq \hat{r}_k^{t+t_{nk}} + x_{nk}^t$$
$$v_k^{t+t_{nk}} \leq |d_k^{t+t_{nk}} - s_k^{t+t_{nk}}|^+ + \hat{d}_k^{t+t_{nk}}.$$

All the repositioning decisions made at $t < u$ that lead to availability of container at k at the start of time period u, are denoted by A_k^u. Thus, at every time period t, r_k^t is updated as $r_n^{t+1} = |g_n^t + \hat{s}_n^t - \hat{d}_n^t|^+ + A_n^{t+1}, \ \ \forall n \in N$. The pending demands are added to the demands in the next time period as mentioned in Section 4.2.

6.1 Demand Rejection Criteria

The model proposed in Section 5 is simulated and the empty containers are allocated to demand locations at each time period with the aim of maximizing the profits. If the supply is constantly less than demand, pending requests at some locations accumulate with time. In these cases, containers have to be imported or leased from other companies. This also affects the service levels provided to the customer that are remotely located or place demands later in the time. To avoid these situations, a policy to reject incoming demands is evaluated. Consider that each container demand has a time window L and the demands not served within L are cancelled by the customers. The effect of cancelled demands can be seen as a lesser number of served demands at the depots. The containers are assigned to demand locations only if it can be available within a specific time window or the pending requests are not already queued at the given depot.

Service Time: To ensure sufficient service level to the customers, the requests that will not be served within the desired time have to be rejected. Considering this, if the travel time between two locations is greater than $t_{ij} + L$, the containers are not assigned from that location.

Pending Requests: Cancellation of demands that could not be served on time may lead to loss of goodwill. For this, the incoming demands at a depot are also accepted or rejected depending on the pending demands at that depot. Thus, the customers are informed immediately if the containers cannot be supplied so that they can make alternate provisions. If the number of excess demands at a location is more than certain parameter C we immediately reject the new demands at this location.

6.2 Analysis

The final model for realistic container repositioning problem with multi-period travel time and the demand rejection is studied under varying C and L parameters. In this case, all demands are either served or rejected when they are received or cancelled after waiting for a fixed time period.

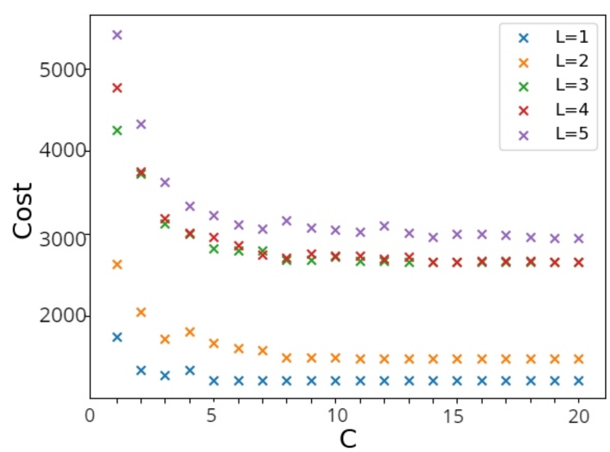

Figure 2: Empty repositioning cost.

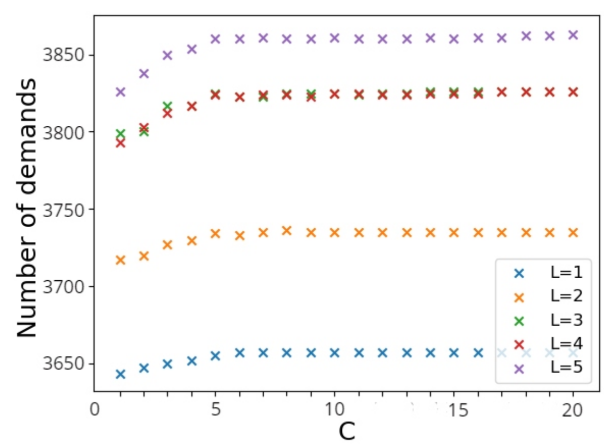

Figure 3: Total demands served.

An inland network with 30 depots is constructed for this study. The demand and supply values are generated as explained in Section 5. The travel time between nodes is proportional to the distance between them. The simulations for the ADH-MP are performed over this data with L varying from 1 to 5 and C from 1 to 20 for 50 time periods. The total empty distance travelled by containers is shown in Figure 2. The smaller distance at low values of L are observed as the overall number of demands that is served

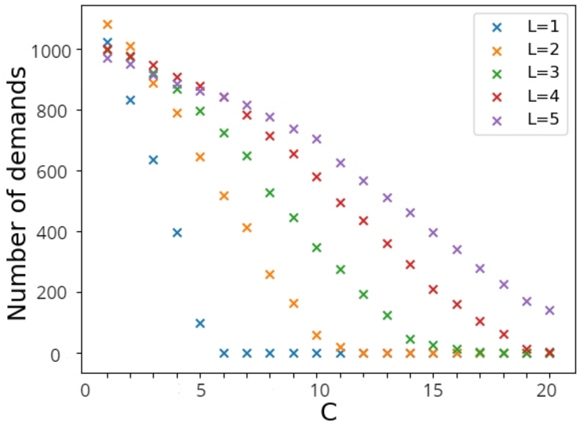

Figure 4: Total demands rejected.

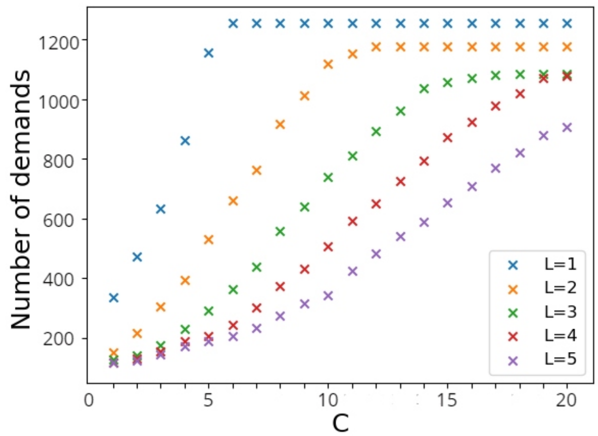

Figure 5: Total demands cancelled.

is small as compared to higher L represented in Figure 3. At smaller C's large number of demands are rejected and the containers have to move larger distances to serve future accepted demands. At values of $C > 6$, the number of demands served is stabilized and thus the distance travelled decreases gradually. The number of rejected demands is very high at low values of L and C (Figure 4) but fewer demands are cancelled (Figure 5). As the value of C is increased, the number of demands rejected is reduced but that of cancelled demands increase. This is due to the pending requests being automatically cancelled after L time periods. Hence at lower L, the number of cancelled demands is high as compared to the higher values. With increasing L, the number of rejected demands increases and the number of cancelled demands decreases.

From the above results, it can be concluded that while applying the model to real networks, demands with larger time windows are preferred so that maximum demands can be catered. Further, C could be set to some medium value to avoid rejection of demands at lower C values and cancellation at higher C's such that while serving maximum demands, majority of demands can be rejected rather than cancelled if service is not provided on time.

7 CONCLUSION

The inland empty repositioning of containers between depots is described in this paper. The inter-depot repositioning is necessary to maintain the inventory levels at each depot in order to ensure empty container availability for demands arising in the neighbourhood regions. However, this problem is not described extensively in the literature.

Initially, Container Allocation Model (AP) is presented which is used traditionally to solve the problems of this type. However, as this model does not consider the effect of decisions on current demands at the supply node a Value Approximation (VA) Model with a single-period lookahead is proposed. To reduce the complexity of the VA model, a heuristic is derived by node wise decomposition of this model (ND-SP).

A comparative study of the models is done by simulating demand and supply over a certain time horizon. By conducting Wilcoxon signed rank test it is observed that the SPL-VA and NDH-SP give better solutions as compared to the AP Model. Also, NDH gives comparable results to the SPL-VA with respect to the repositioning costs over the time horizon by solving ILPs of smaller dimensions. These models are based on a single-period travel time assumption. The NDH-SP model is further generalized by allowing multi-period travel times, by approximating the number of available containers at the future time periods. A demand rejection policy is introduced to govern the demands to be catered such that the service time of customers is not affected based on their location and time of placing the request. This is studied under different time windows and an upper limit on the number of queued requests. The benefit lies in allowing moderate queue size at each depot.

The above models make allocations based on demand and supply predictions. The state at each t can be updated based on the observed values during each time period. However, all the simulations in this paper are performed assuming that the supply and demand information is deterministic. We recommend analysis of the effects of uncertain data on the inter-depot container repositioning decisions. Further, a study over the realistic container data to observe the benefits from inter-depot repositioning and cancellation policy is also proposed.

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