

## **ELECTRIC VEHICLE ROUTING PROBLEM WITH TIME WINDOWS AND STOCHASTIC WAITING TIMES AT RECHARGING STATIONS**

Merve Keskin

Warwick Business School  
University of Warwick  
Coventry CV4 7AL, UK

Raha Akhavan-Tabatabaei

Faculty of Management  
Sabanci University  
Istanbul 34956, TURKEY

Bülent Çatay

Faculty of Engineering and Natural Sciences  
Smart Mobility and Logistics Lab  
Sabanci University  
Istanbul 34956, TURKEY

### **ABSTRACT**

The Electric Vehicle Routing Problem with Time Windows and Stochastic Waiting Times at Recharging Stations is an extension of the Electric Vehicle Routing Problem with Time Windows where the vehicles may wait in the queue before recharging their battery due to a limited number of available chargers. Long waiting times at the stations may cause delays for both the customers and the depot. In this study, we model the waiting times using  $M/M/1$  queueing system equations. We solve small instances by CPLEX assuming expected waiting times at the stations and calculate the reliability of these solutions by simulating the waiting times. We observe that, while chargers become busier, the reliability of the solutions obtained with average times decreases.

### **1 INTRODUCTION**

Electric vehicles (EVs) have gained increased attention of researchers in recent years because of the growing environmental concerns and the need for decreasing greenhouse gas (GHG) emissions. Hence, route planning of EVs has appeared as a challenging optimization problem in the literature due to the additional complexities it involves. The Electric Vehicle Routing Problem with Time Windows (EVRPTW) was introduced by Schneider et al. (2014) as an extension to the Green Vehicle Routing Problem (GVRP) of Erdoğan and Miller-Hooks (2012). The problem is a variant of the classical Vehicle Routing Problem with Time Windows (VRPTW) where a fleet of EVs is used instead of internal combustion engine vehicles (ICEVs) running with fossil fuel. Different from the ICEVs, an EV is equipped with a battery as the energy source and its driving range is limited. So, it may need to recharge its battery en route in order to complete its tour. The battery may be recharged at any state of charge (SoC); however, its duration depends on the amount of energy transferred and is significantly longer compared to refueling an ICEV.

Most of the papers in the literature assume that EVs start their recharging as soon as they arrive at the recharging stations. However, in practice there may be other EVs which are already being recharged and some others waiting in the queue. Hence, a newly arriving EV may have to wait for some time before it starts recharging its battery. If the recharging stations are privately owned by the fleet operator, then one may schedule the EVs such that they do not arrive at the same stations at the same time. In this way, possible conflicts are eliminated. Bruglieri et al. (2018) studied this problem in the GVRP context. Alternative fuel

vehicles (AFVs) are routed such that they do not overlap in the alternative fuel stations. They minimize the total distance and propose an exact method in which the routes are considered as compositions of paths. Ding et al. (2015) studied this problem considering limited charging capacity in each station and allowing partial charging. They route vehicles such that their recharging times do not conflict at a charger. Their objective minimizes the total distance traveled and they propose a heuristic method which combines variable neighborhood search and tabu search to solve the problem. Froger et al. (2017) solved a similar problem in which the stations have a limited number of chargers and an EV may need to wait before recharging if the chargers are busy recharging other EVs in the fleet. In this problem, the use of chargers depends on the routing and charging decisions. They also used a non-linear charging function for the recharging time. They proposed a mixed integer linear programming formulation as well as a route-first assemble-second matheuristic to tackle the problem. Their objective function minimizes the total time which includes driving, service and recharging times. Kullman and Mendoza (2018) considered the uncertain availability of stations as a Markov decision process using an  $M/M/\psi_c$  queueing system. Their objective is to minimize the total expected time which includes the travel, recharging, and queueing times. The customers do not have time windows and their service times are ignored. The authors proposed four heuristic policies to solve the problem and tested them using a set of instances that vary in the number of customers and stations, the geographic locations of the customers, and the average utilizations of the stations. The mathematical model and the details of the results are not provided in the extended abstract. Recently, Keskin et al. (2019) also addressed queueing at the stations where the waiting times vary depending on the time of the day, i.e., the vehicles may wait longer during rush hours due to high demand of other EVs. Waiting times were estimated using  $M/G/1$  queueing system equations and an adaptive large neighborhood search approach was proposed to solve the problem.

In this study, we extend the EVRPTW by considering stochastic waiting times at the recharging stations. We also extend the objective function and minimize the total cost associated with the energy consumption, driver wages and acquisition of EVs. We use an  $M/M/1$  queueing system to model the waiting times at the recharging stations and we perform a simulation to measure the reliability of routes.

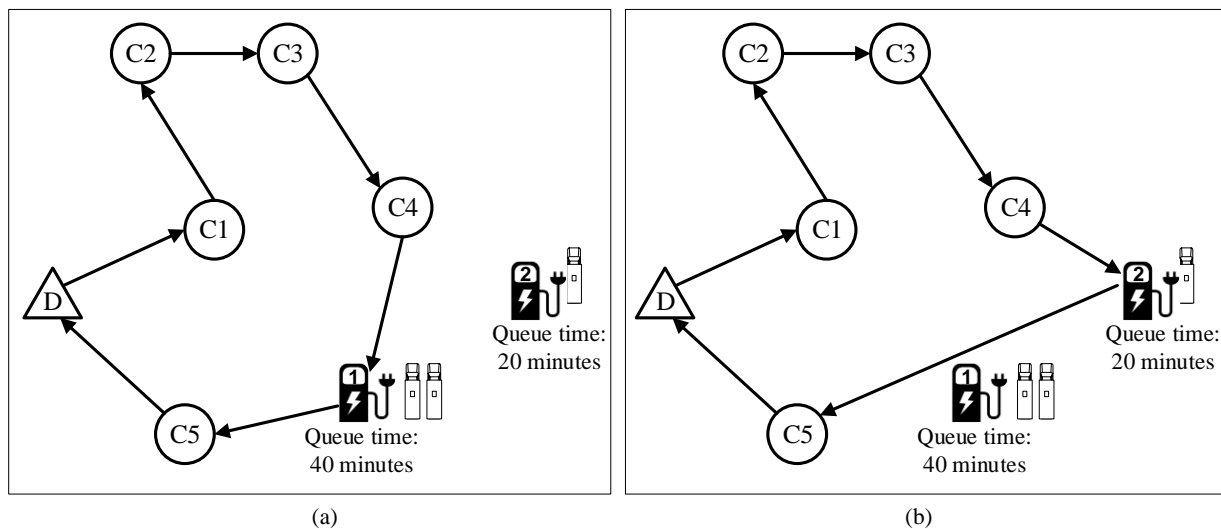


Figure 1: Impact of the queueing time on routing decisions: Solutions obtained (a) without considering, (b) with considering queueing at stations.

## 2 PROBLEM DESCRIPTION AND FORMULATION

The problem establishes a set of routes which are operated by a homogeneous fleet of EVs. Routes should cover all customers which have known demands and time windows. The customers should be visited within their time windows. If an EV arrives before the early service time, it waits until that time. On the other hand, arriving later than the late service time is not allowed. All EVs depart from the depot and should return to the depot before its due date. Furthermore, the SoC of the EVs should be nonnegative throughout the journey. The EVs may visit recharging stations to recharge their batteries and continue their routes. Since most of the charging stations are public (e.g., [Esarj](#) and [Sharz](#) in Istanbul), the EVs may need to queue up for recharging service in urban areas. Then, the waiting times at the stations become very crucial for effective route planning. In this paper, we address the waiting times within the context of EVRPTW where the objective is to minimize the total cost of energy, driver wages, and vehicle costs subject to time, demand and SoC constraints.

The most important part of this problem is the waiting times at the recharging stations which affects the routing decisions significantly. Figure 1 shows the impact of waiting times on a 5-customer instance. The nodes with letters “C” and “S” represent the customers and recharging stations, respectively whereas “D” stands for the depot. The truck figures next to the stations represent the EVs waiting in the queues at the stations. Figure 1(a) illustrates the case where the waiting times are ignored. Obviously, visiting S1 to recharge the battery is a better decision than visiting S2 in terms of the total distance. However, S1 has a longer waiting time than of S2. If C5 has tight time windows, then the vehicle may not be able to visit C5 within its time windows because of long waiting at S1. In this case, the vehicle may recharge its battery at S2, as shown in Figure 1(b). Although the total distance slightly increases, C5 can be visited feasibly within its time window.

### 2.1 Mathematical Formulation

The mathematical formulation of the problem is based on the formulation proposed in Keskin and Çatay (2016). Let  $V = \{1, \dots, N\}$  and  $F$  denote the set of customers and recharging stations. Since multiple visits to the recharging stations are allowed, we create a new set,  $F'$ , including the stations and their copies to permit several visits to each vertex in the set  $F$ .  $V^d$  and  $V^a$  stand for the departure and arrival depot vertices. Although there is only one physical depot where EVs are based, we create dummy copies of it in order to keep track of the return times of different EVs. Each vehicle departs from one of the vertices in  $V^d$  and ends its route at one of the vertices in  $V^a$ . Let  $V' = V \cup F'$ ,  $V'_d = V \cup V^d$ ,  $V'_a = V' \cup V^a$ . Now the problem can be defined on a complete directed graph  $G = (V'_{d,a}, A)$  where  $V'_{d,a} = V^d \cup V'_a$  and  $A = \{(i, j) | i, j \in V'_{d,a}, i \neq j\}$ . Each arc  $(i, j)$  has a distance  $d_{ij}$  and a travel time  $t_{ij}$ . The SoC is consumed at the rate of  $h$  and each traveled arc  $(i, j)$  consumes  $hd_{ij}$  of the remaining battery. Battery is recharged at the rate of  $g$ , which means one unit of recharge takes  $g$  amount of time. Each customer  $i \in V$  has a positive demand  $q_i$ , a service time  $s_i$  and a time window  $[e_i, l_i]$ . Load and battery capacity of the EVs are  $C$  and  $Q$ , respectively. If an EV visits recharging station  $i$ , it waits in the queue for  $\mathbb{W}_i$  time units, which is a random variable, before being recharged. The objective function includes three components: cost of the energy used, drivers' cost and EVs' operating cost. The unit energy cost is  $c_e$  while the drivers are paid  $c_d$  on a unit time basis. Furthermore, each EV has a fixed operating cost  $c_f$ . The decision variables,  $\tau_i$ ,  $u_i$ , and  $y_i$  keep track of the service starting time, the remaining cargo and charge levels at vertex  $i \in V'_{d,a}$ , respectively whereas the SoC level at the departure from a station  $i \in F'$  is tracked by variables  $Y_i$ . Finally, binary decision variable  $x_{ij}$  takes value 1 if arc  $(i, j)$  is traversed and 0 otherwise. The mathematical notation is summarized in Table 1.

Table 1: Mathematical notation.

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<b>Sets</b>	
$V$	Set of customers
$F$	Set of recharging stations
$F'$	Set of recharging stations with their copies
$V'$	Set of customers and recharging stations with their copies ( $V \cup F'$ )
$F'_d$	Set of departure depots and recharging stations with their copies ( $V_d \cup F'$ )
$V_d$	Set of departure depots and customers ( $V_d \cup V$ )
$V_a$	Set of customers and arrival depots ( $V \cup V_a$ )
$V'_d$	Set of departure depots, customers, and recharging stations with their copies ( $V_d \cup V'$ )
$V'_a$	Set of customers, recharging stations with their copies, and arrival depots ( $V' \cup V_a$ )
$V'_{d,a}$	Set of all vertices ( $V_d \cup V'_a$ )
<b>Parameters</b>	
$d_{ij}$	Distance from vertex $i$ to vertex $j$
$t_{ij}$	Travel time from vertex $i$ to vertex $j$
$q_i$	Demand of customer $i$
$s_i$	Service time of customer $i$
$e_i$	Early service time of customer $i$
$l_i$	Late service time of customer $i$
$C$	Cargo capacity of the vehicles
$Q$	Battery capacity of the vehicles
$g$	Battery recharging rate
$W_i$	Average waiting time at station $i$
$h$	Fuel consumption rate
$c_e$	Unit energy cost
$c_d$	Driver wage per unit time
$c_f$	Fixed vehicle cost
$M$	A sufficiently large number
<b>Decision variables</b>	
$x_{ij}$	1 if EV departs from vertex $i$ and arrives at vertex $j$ , 0 otherwise
$u_i$	Remaining cargo capacity upon arrival at vertex $i$
$y_i$	Battery SoC at vertex $i$
$Y_i$	Battery SoC when departing from station $i$

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The mathematical programming model of the problem is formulated as follows:

$$\text{minimize } c_e \sum_{i \in V'_d} \sum_{j \in V'_a} d_{ij} x_{ij} + c_d \sum_{i \in V^a} \tau_i + c_f \sum_{i \in V'_0} \sum_{j \in V^a} x_{ij} \quad (1)$$

subject to

$$\sum_{j \in V'_a} x_{ij} = 1 \quad i \in V \quad (2)$$

$$\sum_{j \in V'_a} x_{ij} \leq 1 \quad i \in F' \quad (3)$$

$$\sum_{i \in V'_d} x_{ij} = \sum_{i \in V'_a} x_{ji} \quad j \in V' \quad (4)$$

$$\sum_{j \in V'} x_{ij} \leq 1 \quad i \in V^d \quad (5)$$

$$\sum_{i \in V'} x_{ij} \leq 1 \quad i \in V^a \quad (6)$$

$$\sum_{i \in V^d} \sum_{j \in V'} x_{ij} = \sum_{i \in V^a} \sum_{j \in V'} x_{ji} \quad (7)$$

$$0 \leq \tau_i + (t_{ij} + s_i)x_{ij} - l_0(1 - x_{ij}) \leq \tau_j \quad i \in V_d, j \in V'_a \quad (8)$$

$$0 \leq \tau_i + t_{ij}x_{ij} + g(Y_i - y_i) + \mathbb{W}_i - M(1 - x_{ij}) \leq \tau_j \quad i \in F', j \in V'_a \quad (9)$$

$$e_j \leq \tau_j \leq l_j \quad j \in V'_{d,a} \quad (10)$$

$$0 \leq u_j \leq u_i - q_i x_{ij} + C(1 - x_{ij}) \quad i \in V'_d, j \in V'_a \quad (11)$$

$$u_0 \leq C \quad (12)$$

$$0 \leq y_j \leq y_i - (hd_{ij})x_{ij} + Q(1 - x_{ij}) \quad i \in V, j \in V'_a \quad (13)$$

$$0 \leq y_j \leq Y_i - (hd_{ij})x_{ij} + Q(1 - x_{ij}) \quad i \in F'_d, j \in V'_a \quad (14)$$

$$y_i \leq Y_i \leq Q \quad i \in F'_d \quad (15)$$

$$x_{ij} \in \{0, 1\} \quad i \in V'_d, j \in V'_a \quad (16)$$

Objective function (1) minimizes the total cost of energy used, drivers and EVs. Constraints (2) and (3) establish the connectivity of customers and the visits to recharging stations, respectively. Constraints (4) are the flow conservation constraints which ensure that number of incoming arcs should be equal to the number of outgoing arcs for each vertex. Constraints (5) and (6) keep track of the departures from the depots and arrivals at the depots. Constraint (7) ensures that number of departure and arrival depots used in the solution coincide. Constraints (8) keep track of the time if the EV departs from a customer, whereas Constraints (9) keep track of the time when departing from the depot or from a station. Constraints (10) ensure that all the vertices are visited within their time windows. Constraints (11) and (12) guarantee that the demand of each customer is satisfied and the cargo load is always non-negative. Finally, Constraints (13)–(15) keep track of the SoC level and ensure that it is always non-negative and bounded above by the battery capacity while Constraint (16) defines the domain of the flow variables.

## 2.2 Waiting Times

A Poisson process is very useful for modelling purposes in many applications and it is widely used to model this type of demand processes (Rezgui et al. 2012). Also, although the amount of service for each customer may differ, all services are in the same general domain. Hence, exponential distribution is a reasonable assumption for the service times. We restricted the number of chargers with one to keep the model simple, and better observe and analyse the effects of queueing. Besides, if a station is equipped with multiple chargers, it is more likely for the EVs to queue in front of each charger instead of one long queue at the entrance of the station. In this study, we assume that the recharging stations operate based on an  $M/M/1$  queueing system. The arrivals of EVs at stations follow a Poisson process with rate  $\lambda$  and the recharging time is exponentially distributed with rate  $\mu$ . Here service rate is the recharging rate and is approximated by assuming that recharge amount for each EV is uniformly distributed between 10% and 100% of the capacity. Hence, average recharging time is the time required to recharge 55% of the capacity. It is calculated by  $\mathbb{E}[\text{Recharging time}] = 0.55 \times Q \times g$ . Hence  $\mu = 1/\mathbb{E}[\text{Recharging time}]$ . In

$M/M/1$  systems, with probability  $(1 - \rho)$  the waiting time in the queue is 0, whereas with probability  $\rho$  the waiting time is an exponential random variable with parameter  $\mu(1 - \rho)$ , where  $\rho = \lambda/\mu$  is the utilization level of the servers. Hence, upon arrival at a station, expected waiting time in the queue can be calculated by the formula  $(1 - \rho)0 + \rho(1/(\mu(1 - \rho))) = \rho/\mu(1 - \rho) = \lambda/\mu(\mu - \lambda)$ . In the experimental study, we used approximation of the waiting times. The random variable  $\mathbb{W}_i$  in the mathematical formulation is replaced with its expectation  $\mathbb{E}[\mathbb{W}_i] = \lambda/\mu(\mu - \lambda)$ .

### 3 EXPERIMENTAL STUDY AND DISCUSSION

We conduct experiments on the 10-customer instances created by Schneider et al. (2014). These instances differ by the time windows and geographical locations of the customers. In C-type instances, customers' locations are clustered, whereas in R-type instances, customers are located randomly. In RC-type instances, half of the customers are randomly distributed and the remaining half are clustered. If the first number in the instance name is 1, then the customers have narrow time windows whereas in those with number 2, time windows are wide. The number of charging stations in each instance is indicated in the instance name in Table 2. For instance, C101-S5 involves five stations. The average distance between the nodes is 30 and the speed is assumed as one distance per unit time. The energy consumption of the vehicle is also one unit per unit distance (time). The mathematical model is implemented using IBM CPLEX 12.6.3 within a Java environment on a workstation with 16 GB RAM and Intel Xeon E5 2.10 GHz processor. CPLEX is a commercial solver widely used for solving various optimization problems. It finds the optimal solution to the problem, if sufficient time is given; otherwise, it reports the best feasible solution found within the given time limit. In our experiments, we set the time limit to 7200 seconds. We keep the service rate fixed as explained above and calculate different arrival rates to allow the chargers have different utilization levels. So, we consider different expected waiting time values for each utilization level.

Next, we solve the mathematical model considering these average waiting times at the stations in a deterministic setting and conduct a simulation study to estimate the reliability of the solution obtained. For each instance, we perform 10,000 replications. We use the same number of replications used by Gutierrez et al. (2018) who estimated the statistics of the arrival times at the nodes through simulation. So, we generate 10,000 random waiting time values for every visit to a station in the solution using the corresponding distribution function. Then, we update the values of all time-related decision variables according to these waiting times. Since the deterministic solutions are obtained using expected waiting times, in some experiments, a solution may become infeasible because of time-window violation(s) due to longer (random) waiting times. Then, the reliability is calculated by dividing the number of feasible solutions by 10,000.

#### 3.1 Setting

We adopt the objective function components of Taş et al. (2013) who considered fuel, driver and vehicle costs. Since their fleet consists of diesel vehicles, we adopt the fuel and vehicle cost coefficients to an EV. In Feng and Figliozzi (2013), a diesel and an electric truck are compared for their fuel consumptions and purchase prices. The EV is three times more expensive than the diesel vehicle. In addition, using the consumption values reported in Feng and Figliozzi (2013) and the current electricity and fuel prices reported by the US Energy Administration ([www.eia.gov](http://www.eia.gov)), we can argue that the cost of the energy consumed by a diesel truck is 2.5 times higher compared to an electric truck. Hence, the unit cost of energy and the vehicle operating cost are determined as  $c_e = 0.4$  and  $c_f = 1200$ , respectively.

#### 3.2 Results

We performed experiments with different charger utilization levels, i.e.,  $\rho = 90\%$ ,  $80\%$ ,  $50\%$ ,  $40\%$  and  $10\%$ . Since the utilization is calculated as  $\rho = \lambda/\mu$ , the arrival rate  $\lambda$  can be determined for any given  $\rho$  and  $\mu$

values. We assume a constant  $\mu$  in each scenario. Hence, the values of  $\lambda$  vary according to the utilization levels.

Table 2: Results for different utilization levels.

Instances	$\rho = 90\%$		$\rho = 80\%$		$\rho = 50\%$		$\rho = 40\%$		$\rho = 10\%$		No Waiting
	TC	Rel	TC	Rel	TC	Rel	TC	Rel	TC	Rel	TC
C101-S5		Inf.		Inf.	8692	0.32	6961*	0.28	6843*	0.94	6825
C104-S4		Inf.	7029	0.34	4657	0.90	4469	0.76	4144	0.99	4100
C202-S5	11230	0.78	7469	0.46	4352	0.39	4352	0.93	4352	0.93	4352
C205-S3	11015	0.55	7998	0.42	5714	0.84	5578	0.88	5513	0.80	5374
R102-S4		Inf.	7127	0.52	5622	0.66	5620	0.75	4279	0.98	4263
R103-S3	6590	1.00	4172	0.37	2897*	0.53	2892	0.86	2880*	0.96	2850
R201-S4	3733	0.49	3686	0.77	2011*	0.74	1962	0.68	1953*	0.98	1951
R203-S5	3432	0.63	2003	0.58	1981	0.97	1981	0.98	1981	1.00	1981
RC102-4		Inf.	8268	0.51	6908	0.64	5652	0.71	5630	0.95	5626
RC108-S4		Inf.	5765	0.60	4408	0.80	4332	0.91	4305	0.98	4299
RC201-4	5341	0.72	3939*	0.26	3469*	0.50	3468*	0.91	2174	0.96	2171
RC205-4	5350	0.51	3772	0.41	3690	0.95	3678*	0.90	3636	0.95	3634
Average	6670	0.67	5565	0.47	4533	0.69	4245	0.80	3974	0.95	3952

Table 2 compares the results obtained with these utilization levels as well as those achieved without any waiting at the stations. The column heading “TC” stands for the total cost of the solution, whereas “Rel” stands for the reliability which shows the probability of feasibility of the solutions. The results with an asterisk (\*) are the best found bounds achieved in 7,200 seconds, whereas the remaining results are optimal. In the highest utilization level ( $\rho = 90\%$ ), five instances are infeasible since the waiting times are very long due to the high arrival rate of EVs. In the case of 80% utilization, only one instance remains infeasible. In the other lower utilization levels, a feasible solution is always found, which is the optimal solution in most of the cases. As expected, higher arrival rates to the stations usually increase the total cost due to the fact that the EVs need to make longer detours to catch the customer time windows and/or additional vehicles are needed to serve all the customers. The increase in total cost can be significantly large when the utilization rate is high. On the other hand, in some instances we observe that waiting times have no or less effect on total costs, in particular when  $\rho \leq 50\%$ . These instances are 2XX-type instances where the customers have wide time windows and routing decisions are not significantly affected by the recharging decisions.

When we analyze the reliability of the solutions given by CPLEX, we see that the solutions become less reliable as  $\rho$  increases, as expected. Similar to the total costs results, we observe that the effect of longer queues is more significant in type 1XX instances where the time windows are narrow. Notice that in some cases, the reliability increases even though  $\rho$  increases. This is due to the fact that the solution requires more vehicles at high utilization levels. For example, consider the instances C202, C205, R103, R203, RC201, and RC205 where the solutions become more reliable when  $\rho = 90\%$  compared to when  $\rho = 80\%$ . These results are not surprising because when  $\rho = 90\%$  the total costs increase dramatically since a larger fleet is needed to serve the customers and EVs make fewer stops for recharging en-route. In some cases, even increasing the fleet size cannot remedy the solution, e.g. in C101, C104, R102, RC102, RC108, which are infeasible for  $\rho = 90\%$ . We surprisingly observe that the reliability of the solution in R103 is 1.00 when  $\rho = 90\%$ . This is due to the fact that the solution involves a larger fleet where all EVs complete their tours without running out of energy and they do not need to visit stations for recharging. As expected, we observe that the number of EVs monotonically increases as  $\rho$  increases. The increase can be as high as 300% in C202 where one EV can serve all customers if the charger is immediately available when it arrives at a station whereas three or four EVs are needed if longer queues are observed at stations.

Table 3: Influence of utilization level on fleet size.

Instances	$\rho = 90\%$		$\rho = 80\%$		$\rho = 50\%$		$\rho = 40\%$		$\rho = 10\%$		No Waiting	
	#Veh	#Rech	#Veh	#Rech	#Veh	#Rech	#Veh	#Rech	#Veh	#Rech	#Veh	#Rech
C101	Inf.	Inf.	Inf.	Inf.	4	4	3	5	3	5	3	5
C104	Inf.	Inf.	3	3	2	3	2	3	2	3	2	3
C202	4	1	2	3	1	5	1	5	1	5	1	5
C205	4	2	3	3	2	4	2	3	2	4	2	5
R102	Inf.	Inf.	5	3	4	2	4	2	3	4	3	3
R103	5	0	3	2	2	3	2	3	2	4	2	4
R201	2	3	2	3	1	6	1	5	1	4	1	4
R203	2	3	1	4	1	4	1	4	1	4	1	4
RC102	Inf.	Inf.	6	2	5	3	4	4	4	4	4	4
RC108	Inf.	Inf.	4	3	3	3	3	4	3	4	3	4
RC201	3	3	2	5	2	6	2	5	1	6	1	6
RC205	3	3	2	4	2	6	2	7	2	6	2	6
Average	3.3	2.1	3	3.2	2.4	4.1	2.3	4.2	2.1	4.4	2.1	4.4

In Table 3 we compare the number of vehicles and recharging stations in the solutions for different utilization levels to investigate the influence of the waiting times on the fleet size as well as on the frequency of stops for recharging. #Veh and #Rech show the number of vehicles used and the total number of recharges in each solution, respectively. When  $\rho$  decreases, although in some cases the number of stops decreases, on average vehicles tend to visit more recharging stations since the waiting times are getting shorter.

Table 4: Share of the waiting times in maximum tour time.

Instances	$l_0$	$\rho = 90\%$		$\rho = 80\%$		$\rho = 50\%$		$\rho = 40\%$		$\rho = 10\%$	
		Avg.WT	%	Avg.WT	%	Avg.WT	%	Avg.WT	%	Avg.WT	%
C101	1236	Inf.		Inf.		148	12	165	13.3	27	2.2
C104	1236	Inf.		594	48.1	222	18	149	12.1	24	1.9
C202	3390	334	9.9	891	26.3	740	21.8	495	14.6	80	2.4
C205	3390	667.5	19.7	594	17.5	296	8.7	148.5	4.4	32	0.9
R102	230	Inf.		39	17	8	3.5	6	2.6	3	1.3
R103	230	0	0	43	18.7	24	10.4	17	7.4	4	1.7
R201	1000	221	22.1	98	9.8	96	9.6	55	5.5	8	0.8
R203	1000	221	22.1	260	26	64	6.4	44	4.4	8	0.8
RC102	240	Inf.		22	9.2	10	4.2	11	4.6	2	0.8
RC108	240	Inf.		50	20.8	17	7.1	15	6.3	3	1.3
RC201	960	150	15.6	168	17.5	51	5.3	28	2.9	12	1.3
RC205	960	150	15.6	134	14	51	5.3	39	4.1	6	0.6

To further analyze how much time the EVs spend waiting in the queue compared to the planning horizon (maximum tour length), Table 4 presents the average waiting times per vehicle and the percentages of total time spent for queueing. The column headings  $l_0$ , Avg WT, and % stand for the planning horizon, the average waiting time spent at the stations per vehicle, and share of the waiting times in the allowed total tour time, respectively. As expected, while utilization level increases, the percentage of the time spent in the queues increases, as well. We observe a few exceptions such as R203, where the percentage drops from 26% at  $\rho = 80\%$  to 22.1% at  $\rho = 90\%$ . The reason of this decrease is the increased fleet size. Since there are more vehicles in the fleet, vehicles cover fewer customers and hence, they visit fewer stations.



As we discussed above, the reliability in R103 is 1.00 when  $\rho = 90\%$  due to the utilization of a large fleet such that no vehicles need recharging en-route, hence, the average and percentage waiting times are 0.

Table 5: Percentage increases in cost and reliability when one additional vehicle is used.

	$\rho = 90\%$	$\rho = 80\%$	$\rho = 50\%$	$\rho = 10\%$
$\Delta$ Cost	20%	23%	31%	31%
$\Delta$ Reliability	3%	26%	10%	3%

To investigate the influence of the fleet size on reliability, we resolved all the instances by adding one more vehicle to the optimal fleet size and fixing it. The percentage increases in reliability and cost compared to those of the best found solutions are reported in Table 5. The figures are the averages corresponding to the utilization levels 10%, 50%, 80%, and 90% for the whole data set. As the utilization level goes from 10% to 80%, the reliability increases gradually, as well. Since the fleet size increases, vehicles visit fewer customers and hence, they need less energy to complete their tour. This decreases the number of visits to the recharging stations, and vehicles face less uncertainty in terms of waiting times. In addition, because the reliabilities are already higher in low utilization levels, increase in reliability level in these cases is less compared to the higher levels. On the other hand, we do not observe the same behavior when the utilization level is 90% because many instances are infeasible in that case and the reliabilities are already high in the feasible instances due to the larger fleet size in the original optimal solutions (see Tables 2 and 3). To analyse the impact of additional vehicles on reliability and cost, we performed an additional experiment by increasing the fleet size by 3 vehicles. This yielded an average improvement of 15% in reliability but 62% increase in cost on average. So, we observe that higher reliabilities may be achieved at the expense of significant costs.

Table 6: Comparison of reliabilities for no-waiting and average waiting-time considerations.

	$\rho = 90\%$	$\rho = 80\%$	$\rho = 50\%$	$\rho = 40\%$	$\rho = 10\%$
No-waiting	45%	35%	55%	60%	85%
Average waiting time	61%	47%	69%	80%	95%

Finally, we investigate the reliability of the solutions obtained by ignoring the waiting as compared to considering average waiting times when planning the routes. The average reliability values for different utilization levels are given in Table 6. As expected, ignoring waiting times leads higher share of infeasible solutions compared to the case that average waiting times are considered in determining the routes.

In sum, these results reveal that waiting times at stations should be taken into account in routing decisions when the fleet consists of EVs; otherwise, the solutions may lead to disruptions in delivery operations.

#### 4 CONCLUSION AND FUTURE RESEARCH DIRECTIONS

In this study, we considered limited availability of recharging stations in EVRPTW and the resulting queue times before the recharging starts. We assumed that each recharging station is equipped with a single charger and the recharging time is exponentially distributed with parameter  $\mu$ . EVs arrive at stations according to a Poisson process with rate  $\lambda$ . Hence, each station has an  $M/M/1$  queueing system. To investigate the effect of waiting times in routing decisions and costs, we first solved the mathematical model with CPLEX using the expected waiting times. We used several utilization levels of chargers and obtained the results for each level. Then, using these optimal solutions, we performed a set of simulations for different  $\mu$  and  $\lambda$  values for each recharging station visited along the routes and calculated the reliability of the solutions. While the chargers are utilized at higher levels, the fleet size tends to increase since catching the time windows of the customers becomes more difficult by fewer EVs. On the other hand, waiting times have less influence

on the solution when the customers have wide time windows. The experiments also reveal that increasing fleet size has positive effect on reliability level.

In this study, we showed the effect of waiting times at stations for recharging the EVs and the importance of considering queuing in route planning. Future research on this topic may address the development of exact and heuristic methodologies to solve the problem by taking into account the characteristics of waiting times in order to achieve reliable solutions. Among various heuristic methods, simheuristics, which combine optimization with discrete event simulation, have been successfully applied for solving similar VRPs with stochastic nature (Juan et al. 2015). The authors are planning to implement a similar approach for effectively solving EVRPTW with stochastic waiting times at recharging stations.

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## AUTHOR BIOGRAPHIES

**MERVE KESKIN** is a postdoctoral researcher at Warwick Business School. She received her B.Sc. degree in Industrial Engineering from Istanbul Technical University in 2010 and her M.Sc. and Ph.D. degrees from Sabanci University in 2014 and 2018. She is currently working in a project to improve the logistics operations of a waste oil collection company. Her research interests include transportation systems, green logistics, routing of electric vehicles, and developing heuristic solution methods. Her email address is [merve.keskin@wbs.ac.uk](mailto:merve.keskin@wbs.ac.uk).

**RAHA AKHAVAN-TABATABAEI** is an associate professor of Operations Management and director of Masters in Business Analytics at Sabanci School of Management. Prior to this position, she was associate professor of Industrial Engineering and the founding director of Masters in Analytics at Universidad de los Andes in Bogota, Colombia. Before that, she worked as senior industrial engineer at Intel Corporation in Arizona, USA. She has received her Ph.D. and M.Sc. degrees in Industrial Engineering and Operations Research from North Carolina State University, and her B.Sc. from Sharif University of Technology. Her

research is focused on stochastic modeling and data-driven decision making under uncertainty with applications in healthcare, logistics, revenue management and reliability. Her email address is [akhavan@sabanciuniv.edu](mailto:akhavan@sabanciuniv.edu).

**BÜLENT ÇATAY** is a Professor of Industrial Engineering in the Faculty of Engineering and Natural Sciences and founding director of the Smart Mobility and Logistics Laboratory at Sabanci University. He received his B.Sc. degree in Industrial Engineering from Istanbul Technical University in 1992 and his Ph.D. degree from the University of Florida in 1999. He worked as a consultant at IBM Microelectronics in Burlington, Vermont in 1997 and as a visiting lecturer at the University of Florida during 1999-2000. He joined Sabanci University in 2000. His research interests include transportation and logistics systems, sustainable transport planning, the utilization of electric vehicles in logistics operations, applied optimization, heuristic methods to solve large-scale combinatorial optimization problems. His email address is [catay@sabanciuniv.edu](mailto:catay@sabanciuniv.edu).