

## **DISCRETE EVENT OPTIMIZATION: A SIMULATION BASED BENDERS CUT GENERATION APPROACH**

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### **ABSTRACT**

Large solution space is one of the main features of simulation–optimization problems. Reducing the cardinality of the set of alternatives is a key point for increasing the efficiency of simulation–optimization methods. In this work, a new cutting approach is proposed for this purpose. The approach exploits the Benders Decomposition framework that can be effectively applied when the simulation–optimization problems are represented using Discrete Event Optimization models. Benders Decomposition subproblems represent the simulation components, hence, cuts can be easily generated observing the values of the variables while a system alternative is simulated, without solving any subproblem. Using this cut generation procedure, the solution space of tandem queueing system optimization problems can be effectively decreased.

### **1 LITERATURE REVIEW**

Discrete Event Simulation (DES) has established itself as the main tool for the evaluation of the performance of complex stochastic systems in a plethora of applications. As a result of this success, optimization and control have increasingly included the use of DES as part of their procedures.

Several families of techniques, such as random search approaches, surrogate model based algorithms, and partitioning driven procedures, have been adapted from non–linear optimization field in order to include Monte Carlo based techniques that account for the noise of simulation replications. All of the aforementioned approaches share a common characteristic: they consider simulation as a pure black box and very little is used of the simulation model structure. Most of the methods consider the relationship between input and output and use it to inform the search procedure, indeed. Instead, perturbation analysis approaches use events from simulation to calculate first order derivatives of performance measures. Perturbation analysis has been successfully applied in pair with gradient based methods, but it is limited by the structural assumptions that the discrete event system must satisfy. Another family of approaches considering the information about the simulation events, called Discrete Event Optimization (DEO), has been recently proposed by the authors (Pedrielli et al. 2018). The basic idea behind DEO is to fully integrate the simulation and the optimization in a unique model, using mathematical programming.

## 2 SIMULATION BASED BENDERS CUT GENERATION

The DEO approach shows how DES can be used not only for performance evaluation of the modeled stochastic system but also to define the feasible region of the optimization problem. However, several challenges need to be solved in order to make DEO a usable tool, especially when solving large scale optimization problems: (1) one major challenge is the improvement of the computational efficiency (DEO models are large and complex due to the fact that events appear as decision variables, and that the variables are sometimes binary); (2) DEO approaches are single run; as a result, efficient methods to adaptively set the simulation run length are necessary, and such an extension is not trivial.

While both challenges are particularly important, this paper focuses on the first problem and explores the use of Benders Decomposition (BD) to effectively cut the solution space thus reducing the size of the original DEO model. This decomposition approach is not new in simulation–optimization. In fact, the buffer allocation problem of open flow lines was solved in Weiss and Stolletz (2015) using a BD approach. This approach is also used to solve the bottleneck detection problem (Zhang and Matta 2018).

Instead of focusing on a specific problem, in this work, we develop a general solution procedure. In any DEO model, the simulation aspect constructs the subproblem, and cuts will be generated from simulation. Both exact and approximate cut can be generated, depending on the type of subproblems. The investigation of the general solution approach will include the convergence of cuts, effectiveness of approximation, efficiency of cuts.

## 3 ORIGINALITY

The original aspect is that the cuts are generated exploiting the information included in the simulation sample path instead of solving mathematical programming problems as standard BD approaches require. The fact that cuts can be easily generated observing the values of the variables while a system alternative is simulated is an important advantage. Another advantage is that, in the cases where the simulation components include binary variables, simulation sample path enables to generate approximate cuts without much loss in the effectiveness. Further, the cut is independent from the method used to select the alternative to simulate, and this makes possible to combine the proposed cut generation approach to other simulation–optimization approaches as random searches or partitioning procedures.

## REFERENCES

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