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BUDGET ALLOCATION PROBLEM IN SIMULATION ANALYTICS

Xiao Jin

Department of Industrial Systems Engineering and Management
National University of Singapore
1 Engineering Drive 2, Blk E1A #06-25
Singapore, 117576, SINGAPORE

ABSTRACT

Simulation as a decision supporting tool, in principle is time-consuming and lacks responsiveness. "Time consuming" means that a simulation model often costs a long time to build and validate. Beyond establishment, for each simulation trial, it also takes long to run. On the other hand, the "lacking responsiveness" can be considered in a way that simulation is often used to measure long-term average performance unconditional on any particular system status. In this sense, when an instant decision depending on the current situation is needed, a classical simulation-based optimization tool could do very little. In this research, I shall propose a possible way of exploiting the potential of applying simulation-based optimization in solving the real-time decision problem. Under this new problem setting, a corresponding budget allocation shall be properly derived. A numerical study shall be attached at the end.

1. INTRODUCTION

We started by understanding the problem in the classical setting. In the original simulation-based optimization problem, the classical objective is to find the optimal from an alternative pool with respect to one performance measure. Due to the observation noise, the actual performance of each candidate is estimated by averaging multiple simulation results. By taking the rank of the sample average as an estimation of real mean, we are able to gradually approximate the real rank based on the law of large numbers. Though this usually works in an offline setting where all the future behavior of the real system is properly encoded in the simulation model, it may be of a challenge when part of the information is uncertain yet has a definitive impact on the behavior of all alternatives. A reference to such information is that it can be regarded as "scenario". The main issues in this new setting are that alternatives may have totally different rankings in different scenarios and the number of scenarios is often too large or even continuous. If one could afford to apply simulation to differentiate the best from others in one scenario, it is practically impossible to exhaust all scenarios. Therefore a new simulation scheme is needed if a simulation is still compulsory in the setting. Through this new scheme, these following two questions should be systematically answered:

- If for a scenario occurred currently is unobserved, how could we measure the performance of each alternative in this scenario with reference of the simulated results from other scenarios?
- If a decision is to be made in the near future, providing the chance for new observations, in which scenarios and which alternatives should we observe to increase our knowledge in the future?

Consider the following definition:

$$\min_{x \in \chi} f(x,\theta) = E_{w|x,\theta}[J(x,\theta,w)] \quad \theta \in \Theta$$

We would like to find the optimal decision x for each scenario θ with respect to f, which is the corresponding expected performance of J under the noise of $w|x,\theta$. The real value of f for each x and θ is unknown, we therefore execute multiple simulations and estimate such value using sample average (I denote such an estimation as $\hat{f}(x,\theta)$). When θ is given with $|\chi|$ being finite, there has been a lot of literature stressing this problem. However, when we want to know the optimal knowledge of x across all θ and $|\Theta|$ being extraordinarily large, new scheme is needed to tell under what θ to be simulated.

This problem is evidently not possible to solve fully in a real circumstance. However, for practical use, it can be properly stressed in two subproblems. The first one is when a decision maker need an instant reaction to a problem where the scenario θ is already given. With a pile of historically simulated data of $\{J(x_i,\theta_i,w_i)\}_{i\in I} := \mathbf{D}$ provided, how could we construct a proper estimation function $\hat{f}(x,\theta)$ for each f of x in this particular θ so that we can minimize the potential loss of selection x in this scenario θ. One reasonable way is to introduce fitting technique across the θ dimension for each x separately to construct $\hat{f}(x,\theta)$. I also want to include the confidence level for such a fitting so that the cost can be regarded in a systematic way. Therefore, $\hat{f}(x,\theta)$ may follow some Bayesian posterior distribution inferred from data $\mathbf{D}$. Then the objective of the first subproblem could be illustrated as:

$$\min_{x\in\chi} \int_{\hat{f}(x,\theta)} \int_{\hat{f}(x',\theta)} [\hat{f}(x,\theta) - \hat{f}(x',\theta)]^p dF_{\hat{f}(x',\theta)|\mathbf{D}} dF_{\hat{f}(x,\theta)|\mathbf{D}} \quad \text{given } \theta$$

When $p=0$, it is equivalent to the classical Probability of Correct Selection *PCS* criteria while when $p=1$, it is the Expected Opportunity cost (*EOC*). As we can see, as long as $\hat{f}$ is properly structured, the solution of this problem yield a selection policy which is cheap and can provide an instant response to a random θ.

The second subproblem is more challenging. Suppose we are not required to decide instantly, rather a period of time or equivalently several times of observations is provided. In this situation, which f of (x,θ) is suggested observed based on the current $\mathbf{D}$ we know? Intuitively, we would expect such a scheme to allocate observations to those (x,θ) which can minimize the expected loss of the first subproblem. Since this loss is defined on an uncertain θ, an assigned distribution is needed for θ. One could either construct it by historical data or simply by rational believe of the decision maker. Also, the data $\mathbf{D}$ should also be substituted by the anticipated data after new observations. Suppose the newly added data is denoted by $\mathbf{d}$ and $\mathbf{d}$ is decided by our allocation of $N_{x,\theta}$ ($\mathbf{N} := \{N_{x,\theta}\}$ for matrix representation) for each x,θ (e.g. $\mathbf{d} = \mathbf{d}(\mathbf{N})$). Further we let $\tilde{\mathbf{D}} := \{\mathbf{d},\mathbf{D}\}$. Then a proper objective can be defined:

$$\min_{\mathbf{N}} \min_{x\in\chi} \int_{\theta} \int_{\mathbf{d}} \int_{\hat{f}(x,\theta)} \int_{\hat{f}(x',\theta)} [\hat{f}(x,\theta) - \hat{f}(x',\theta)]^p dF_{\hat{f}(x',\theta)|\tilde{\mathbf{D}}} dF_{\hat{f}(x,\theta)|\tilde{\mathbf{D}}} dF_{\mathbf{d}|\mathbf{D}} dF_{\theta}$$

Intuitively speaking, this objective described an allocation scheme of $\mathbf{N}$ where the anticipated cost of selecting a certain alternative x under scenario θ according to the previously mentioned selection policy is minimized across all θ.

I shall briefly illustrate the numerical results and a case study in the colloquium.