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THE IMPACT OF QUEUE LENGTH ROUNDING AND DELAYED APP INFORMATION ON DISNEY WORLD QUEUES

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ABSTRACT

Many service systems provide queue length information to customers to aid their decisions of what queue to join. One example is at Walt Disney World (WDW), where waiting times are posted to customers via an app. However, it has been observed that the real waiting times are not posted in the app. In fact, WDW rounds the waiting time up to nearest five minute interval. In this paper, we build a simulation model to study the impact of rounding this information and when the information is lagged. We show that rounding or delaying information can result in oscillations in the queue length process. Moreover, increasing the rounding parameter or the delay in information causes oscillations to increase. We also demonstrate that our queueing model can mimic the observed dynamics in the data seen in WDW. Thus, we show the importance of understanding the impact of rounding or delaying information.

1 INTRODUCTION

Smartphone technology has changed the paradigm for communication between customers and service systems. One example of this communication is delay announcements, which have become important tools for service system managers to inform customers of their estimated waiting time. Thus, managers have found tremendous value in understanding the impact of providing waiting time or queue length information to customers as these estimates can affect customers' decisions of whether to remain in the system and receive service. Since these announcements can affect the decisions of customers, they clearly affect the underlying queue length dynamics of the system. Thus, the development of methods to support such announcements and interactions with customers has attracted the attention of the operations research community and is growing steadily.

Although many service systems provide waiting time information, the information may not be reliable or may not be provided in real time. One reason is that the information may be delayed from a technological point of view as it might take time to process and distribute the information to customers. Another reason is that there may be a lag from when one must commit to joining a queue and when they actually join. This type of delay occurs in transportation settings where drivers must commit to a road before they actually get on the road itself. On the right of Figure 1 we see an example of this delay at Walt Disney World (WDW) in Orlando, Florida where park goers must commit to going to rides before they get to the line, but their commitment to a ride is based on the information they receive from the Disney app.

On the left of Figure 1, we show a snapshot of the My Disney Experience mobile app. The Disney app lists waiting times and the rider's current distance from each ride in the park and lists the rides in increasing order of the waiting times. Customers have the opportunity to choose which ride they would like to go on, yet this choice depends on the information that they are given through the app. However, the wait times on the app might not be posted in real-time or customers might need travel time to get to their next ride, hence the information upon which they base their decision on is prone to delay. Thus, our queueing analysis is useful for Disney or even other types of service systems to understand how their decision to offer an app that displays waiting time and their decision to round this information will affect the lines for rides in the park.

To this end, this paper introduces a novel stochastic queueing model which describes the dynamics of customer choice with delayed information and queue length rounding. In the queueing model, the customer receives information about the queue length which is delayed by a constant parameter Δ and the queue length is rounded to the nearest multiple of a chosen integer α. Using a modified version of a standard continuous time Markov chain (CTMC) simulation algorithm, we develop a stochastic simulation algorithm to explore the impact of delayed information and rounding. Our results indicate that both rounding and delayed information can cause oscillations in the queues. Finally, we show that we can replicate the important features and dynamics that are observed in real data from WDW. Thus, our stochastic model is relevant and further analysis of this model could yield important insights for these types of service systems. As a result, our paper intends to answer the following questions:

- What is the impact of the rounding on the sample path dynamics of the system?
- How does the scaling of the rounding affect the dynamics of the system?
- How does delaying the information further affect the dynamics of the system?

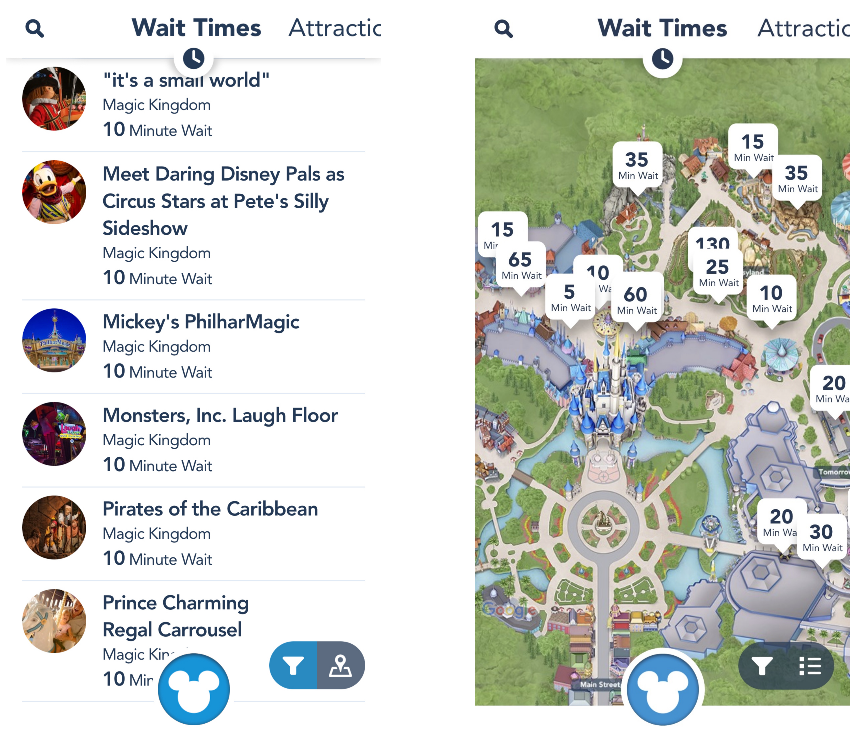


Figure 1: Disney waiting times on smartphone app, displayed in List (left) and Map (right) Forms.

The answers to the above questions will provide novel insights for managers of these systems, aid understanding of the impact of using smartphone technology to inform customers of wait times, and demonstrate how modifying the information can affect the underlying dynamics. The remainder of the paper is structured as follows. Section 2 presents a review of the delay announcement literature for service systems. Section 3 introduces the data that informs the construction of our queueing model. Section 4 constructs a new queueing model with delayed information and queue length rounding and presents an algorithm for simulating our queueing model. In Section 5, we present the findings from simulating our stochastic queueing model and report the observations we learn from our simulation studies. We also show that we can replicate the dynamics seen in the real data from WDW. Finally, we conclude in Section 6 and give ideas for future research.

2 LITERATURE REVIEW AND RESEARCH OBJECTIVES

Most of the current research that analyzes the impact of providing queue length or waiting time information to customers tends to focus on the impact of delay announcements. Delay announcements are useful tools that enable managers of service systems to provide customers with valuable waiting time information. For the most part, the literature only explores how customers respond to the delay announcements or the actual number that is displayed. Previous work by Armony and Maglaras (2004), Guo and Zipkin (2007), Hassin (2007), Armony et al. (2009), Guo and Zipkin (2009), Jouini et al. (2009), Jouini et al. (2011), Allon and Bassamboo (2011), Allon et al. (2011), Ibrahim et al. (2016), Whitt (1999) and references therein focus on this aspect of the announcements. Some of the most relevant work was done by Ibrahim and Whitt (2009), Ibrahim and Whitt (2011), Ibrahim et al. (2016) in which they study several delay announcement predictors in a single station queue. When a new arrival occurs, these predictors return the delay experienced by the last-to-enter-service (LES) customer, head-of-line (HOL) customer, last-to-complete-service (LCS) customer, or most recent arrival to complete service (RCS). LES and HOL predictors have been shown to perform better than their LCS and RCS counterparts since LCS and RCS use less relevant information than LES and HOL.

Despite its relevance, the previous literature does not focus on two important aspects of the reality of WDW queues. The first is the situation where the information given to customers in the form of an announcement is delayed. The second is that the wait time or queue length is rounded in presentation to the customer yet it is still used to provide information about their wait or the actual queue length.

The analysis of this paper is similar to the main thrust of the delay announcement literature in that it is concerned with the impact of the information on the dynamics of the queueing process. However, it differs from the mainstream literature since we focus on when the information itself is delayed and is not given to customers in real-time and the actual information that is given to the customer. Moreover, we include the impact of rounding this information, which separates it from recent work by Pender et al. (2018), Pender et al. (2017a) Burnetas et al. (2017) Ibrahim (2017), Lingenbrink and Iyer (2017). These features of our model are important distinctions from the current literature, which only focuses on delay announcements given in real-time and there is no exploration of rounding effects.

Recently, there also is work that considers how the loss of information can impact queueing systems. Work by Jennings and Pender (2016), Pender (2015a), Pender (2015b) compares ticket queues with standard queues. In a ticket queue, the manager is unaware of when a customer abandons and is only notified of the abandonment when the customer would have entered service. This artificially inflates the queue length process and the previous work compares the difference in queue length between the standard and ticket queue. However, this work does not consider the aspect of customer choice and delays in providing the information to customers, which is the case in many healthcare settings.

The main objective of this work is to not only understand the impact of delayed information, but also understand how rounding the information will also affect the underlying system. However, before we describe our queueing model, we describe some data from WDW theme parks and provide some important observations from our data that inform the construction of our new stochastic queueing model.

3 OBSERVATIONS FROM DISNEY WORLD DATA

In this section, we describe the data that we collected for the wait times for the attractions at WDW. Disney makes the live waiting times publicly available through the My Disney Experience mobile app. These wait times can either be shown in a non-decreasing list or displayed in a map of the park. In addition to reporting wait times, this app provides locations of attractions, dining, and shopping, manage their reservations, view their park photos, order food, as well as many other options. To support our project's investigation, we recorded the public wait times approximately once each minute when the parks were open.

These wait times were collected for each of the attractions across each of the four parks at WDW: Magic Kingdom, Epcot, Hollywood Studios, and Animal Kingdom. Our data collection began on November 2, 2017 and concluded February 3, 2018. Across the seasonal offerings of each park, we recorded times for 50 different attractions. Almost exclusively, the wait times are reported in the app in multiples of 5 minutes. Based on this observation and on how much Disney values the guest experience, we assume that Disney rounds up the wait times to the nearest multiple of 5 except in special cases, which we discuss in the remark below. Compared to other rounding schemes, this policy will not ever present a time shorter than the actual anticipated wait and thus guests will not be disappointed with any posted wait. A potential follow up to this work could study the impact of other types of rounding schemes on customer choice.

Remark. It is interesting to note that our data indicates that Tower of Terror reports a wait time of 13 minutes while 15 minutes never appears in the data. While this seems to contradict the 5 minute rounding policy, we can note that this is instead a special case that demonstrates Disney's operating guidelines. As detailed on company webpages, the Walt Disney Company prioritizes "Show," the environmental aspects conveying the park's stories to its guests, over efficiency Walt Disney Company (2018). Because the Tower of Terror attraction is a *Twilight Zone*-inspired, supernatural hotel with 13 floors, the 13 minute display extends the attraction's theming to its posted wait time.

In Figures 2 and 3, we plot sample data for wait times across a week (Monday through Sunday) at the attractions Toy Story Midway Mania! and "it's a small world," respectively. Toy Story Midway Mania! is an interactive ride mimicking carnival-style target games themed after the Toy Story franchise, and "it's a small world" is a water-based dark ride showcasing numerous international cultures. Toy Story Midway Mania! is in the Hollywood Studios park whereas "it's a small world" is in Magic Kingdom. In each of these plots, the horizontal axis tick marks correspond to noon on the specified date. It is interesting to note that there appears to be a dampening effect of the daily 3:00 parade on the wait times for "it's a small world."

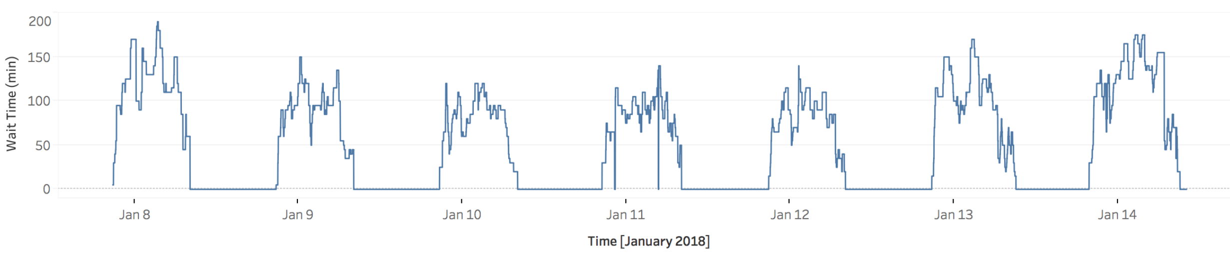


Figure 2: Wait times for Toy Story Midway Mania! during the week of January 8-14, 2018.

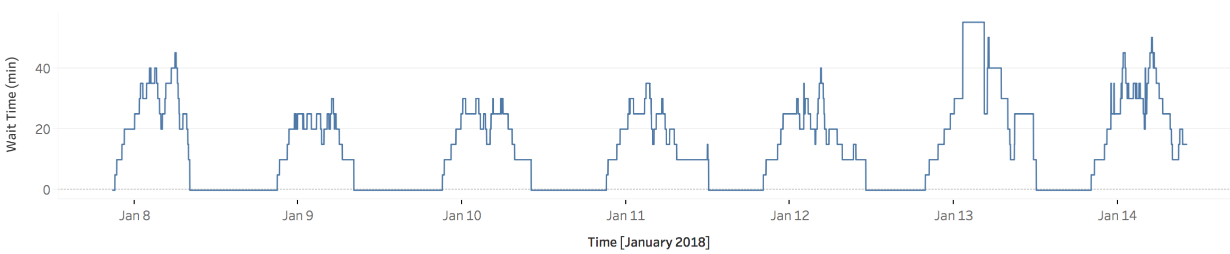
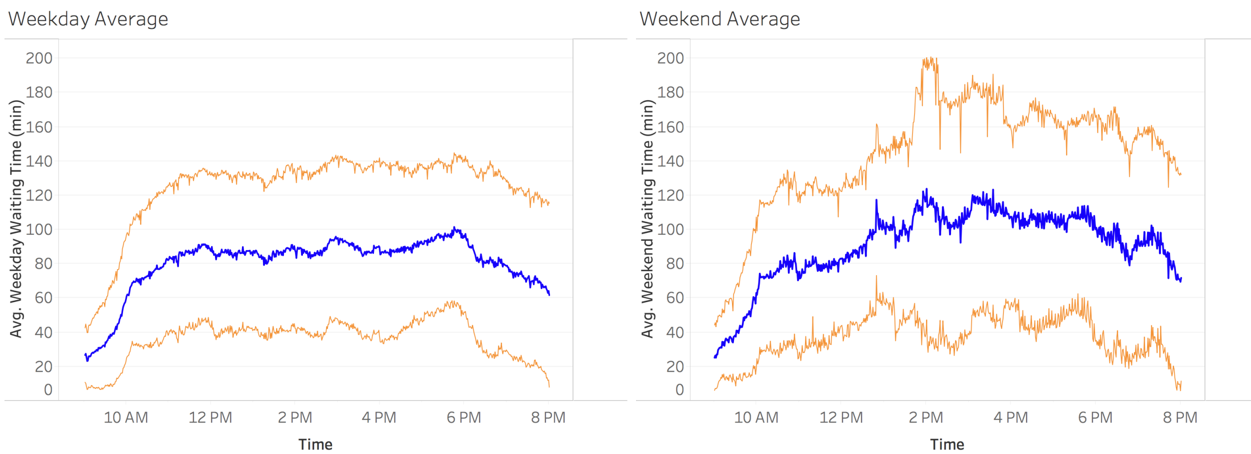


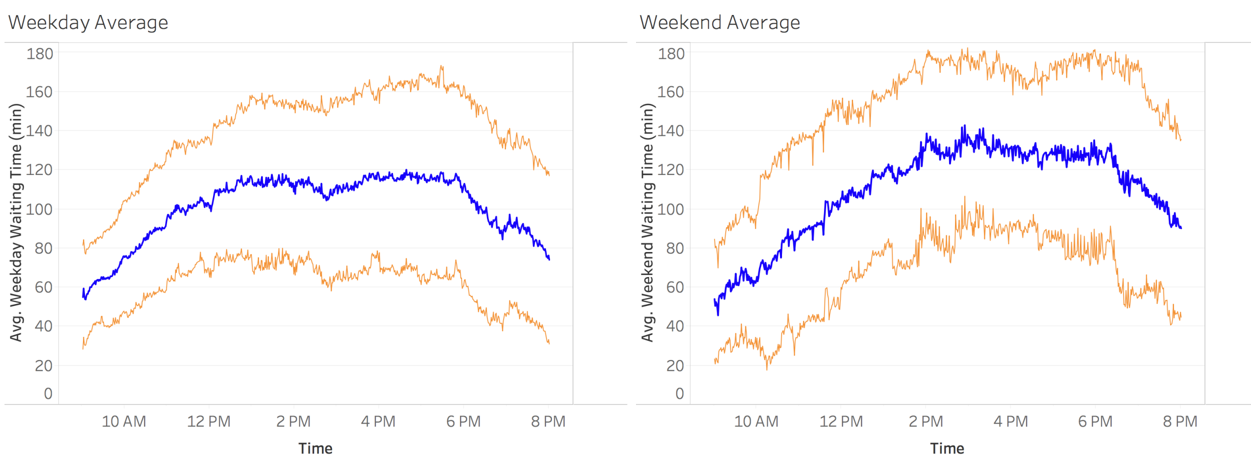
Figure 3: Wait times for "it's a small world" during the week of January 8-14, 2018.

In addition to the sample week figures, we give week-day and weekend-day average wait times across the three months of data in Figures 4, 5, and 6 for Space Mountain, Seven Dwarfs Mine Train, and Avatar Flight of Passage, respectively. Further, each of these plots features curves for the mean plus and minus one standard deviation. Space Mountain is an indoor, space-themed low-light roller coaster. The Seven Dwarfs Mine Train is an indoor and outdoor roller coaster with swaying ride vehicles themed after Snow White and the Seven Dwarfs. Both of these attractions are in Magic Kingdom. Avatar Flight of Passage is a flying simulator attraction set in Pandora from the movie Avatar; it is one of the latest additions to Animal Kingdom. While all three attractions are popular among park guests, it is interesting to note the immediate high demand at Avatar Flight of Passage, likely an effect of how recently it was opened at the park.



(a) Weekday Average (b) Weekend Average

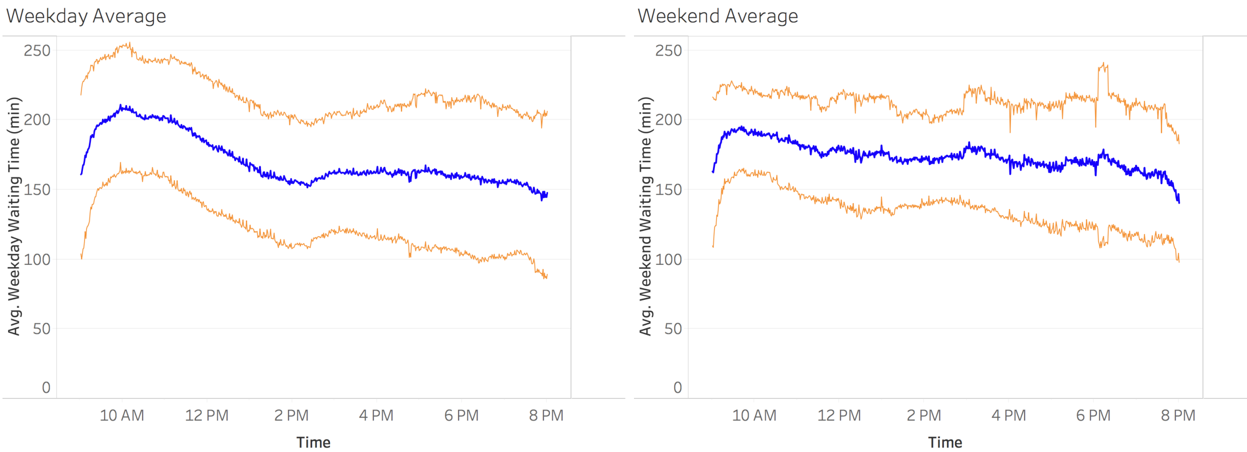
Figure 4: Wait times for Space Mountain.



(a) Weekday Average (b) Weekend Average

Figure 5: Wait times for Seven Dwarfs Mine Train.

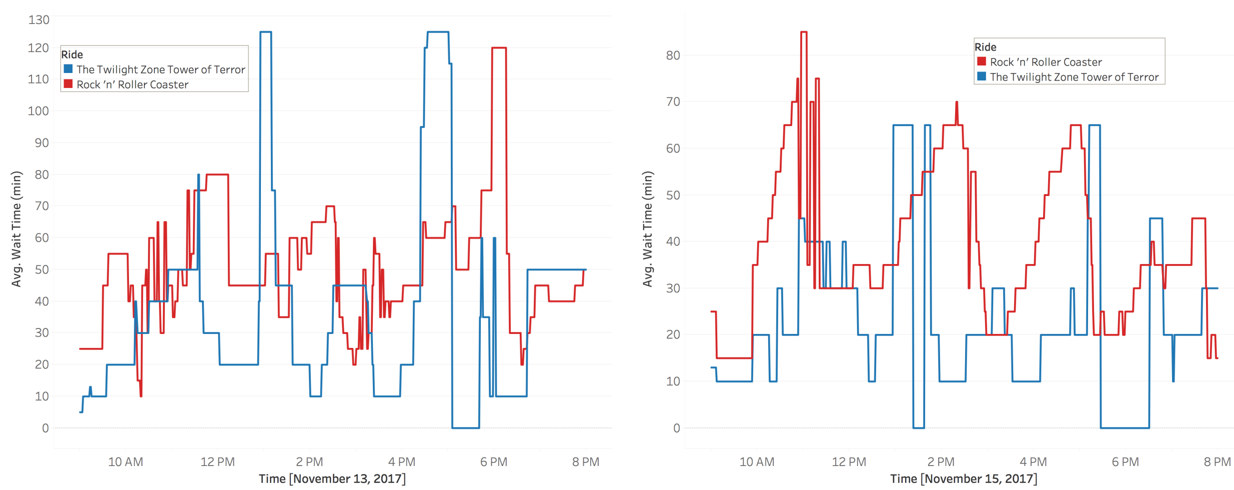
As a final interesting example from data, we plot two separate days of wait time data for the attractions Rock 'n' Roller Coaster and Tower of Terror, an Aerosmith themed roller coaster and Twilight Zone



(a) Weekday Average (b) Weekend Average

Figure 6: Wait times for Avatar Flight of Passage.

inspired drop ride, respectively. Both of these rides are in Hollywood Studios; in fact, they are very near to one another. Moreover, the two rides are both quite popular and are often considered the two most thrilling rides in that park. This combination of proximity and similarity create an interesting dynamic seen in Figure 7 when the wait time for one attraction increases park guests start to prefer its neighbor. In the following sections, we will explore this phenomenon further through simulation models that feature delayed announcements of rounded wait times.



(a) November 13th (b) November 15th

Figure 7: Observed oscillations in neighboring attractions Rock 'n' Roller Coaster and Tower of Terror.

4 THE STOCHASTIC QUEUEING MODEL

In this section, we present a stochastic queueing model with customer choice based on the queue length with a constant delay that was studied by Pender et al. (2017b) and add a rounding component. Thus, we begin with N infinite-server queues operating in parallel, where customers choose according to a choice model which queue to join by using the size of the queue as information. However, we make the slight and realistic modification that the queue length information that the customer receives is delayed by a constant Δ for all of the queues. Therefore, the queue length that the customer receives is actually the queue length Δ time units in the past. Moreover, we also round this queue length to nearest multiple of the parameter α i.e. $\alpha\lceil x/\alpha \rceil$.

The choice model that we use to model the customer choice dynamics is a Generalized Multinomial Logit Model (GMNL). This is one of the most frequently used choice models, especially in transportation situations. Although the GMNL model has a utility maximization interpretation, it also has a probabilistic interpretation. In the context of probabilities, the probability of going to the i^{th} queue is given by

$$p_i(Q(t),\Delta) = \frac{g\left(\alpha\left\lceil \frac{Q_i(t-\Delta)}{\alpha}\right\rceil\right)}{\sum_{j=1}^{N} g\left(\alpha\left\lceil \frac{Q_j(t-\Delta)}{\alpha}\right\rceil\right)},$$

where $Q(t) = (Q_1(t), Q_2(t), ..., Q_N(t))$ and $g(\cdot)$ is a non-negative, decreasing, and continuously differentiable function.

It is evident from the above expression that if the queue length in station i is larger than the other queue lengths, then the i^{th} station has a smaller likelihood of receiving the next arrival. This decrease in likelihood as the queue length increases represents the disdain customers have for waiting in longer lines. Using these probabilities for joining each queue, we are able to construct the following stochastic model for the queue length process of our N dimensional queueing system for $t \geq 0$:

$$Q_i(t) = Q_i(0) + \Pi_i^a\left(\int_0^t \frac{\lambda \cdot g\left(\alpha\left\lceil \frac{Q_i(s-\Delta)}{\alpha}\right\rceil\right)}{\sum_{j=1}^{N} g\left(\alpha\left\lceil \frac{Q_j(s-\Delta)}{\alpha}\right\rceil\right)}\,\mathrm{d}s\right) - \Pi_i^d\left(\int_0^t \mu Q_i(s)\,\mathrm{d}s\right),$$

where each $\Pi(\cdot)$ is a unit rate Poisson process and $Q_i(s) = \varphi_i(s)$ for all $s \in [-\Delta, 0]$. In this model, for the i^{th} queue, we have that

$$\Pi_i^a\left(\int_0^t \frac{\lambda \cdot g\left(\alpha\left\lceil \frac{Q_i(s-\Delta)}{\alpha}\right\rceil\right)}{\sum_{j=1}^{N} g\left(\alpha\left\lceil \frac{Q_j(s-\Delta)}{\alpha}\right\rceil\right)}\,\mathrm{d}s\right)$$

counts the number of customers that decide to join the i^{th} queue in the time interval $(0,t]$. Note that the rate depends on the queue length at time $t-\Delta$ and not time t, hence representing the lag in information. Similarly,

$$\Pi_i^d\left(\int_0^t \mu Q_i(s)\,\mathrm{d}s\right)$$

counts the number of customers that depart the i^{th} queue having received service from an agent or server in the time interval $(0,t]$. However, in contrast to the arrival process, the service process depends on the current queue length and not the past queue length.

Initially, one might be interested in showing fluid and diffusion limits for this queueing model. However, the nature of the rounding precludes using standard methodology like strong approximations or continuous mapping arguments. Since the rounding function is discontinuous, new methodologies would need to be created in order to derive new limit theorems for this type of queueing system. Thus, in this paper we use simulation to gain insight on the model behavior and explore the impact of delayed information and queue length rounding.

4.1 Simulation Algorithm

In this section, we provide an algorithm for the stochastic simulation of the stochastic queueing model. We need an algorithm for two reasons. One reason is that the stochastic queueing model has delays in information, therefore a standard CTMC algorithm does not work. Thus, our simulation algorithm is based on a modification of an implementation of continuous time Markov chains (CTMC) for a number of N parallel infinite server queues. This simulation approach is much more convenient and neat compared to other event-based approaches.

1. Initialize each of the N queues with the initial number of customers on the interval $[-\Delta, 0]$.
2. Generate three independent Uniform(0,1) random variables u_1, u_2, and u_3.
3. Calculate the sum of all rates of all the possible events that can occur next i.e.

$$\mu^* = \sum_{i=1}^{N} \lambda \cdot \frac{g\left(\alpha \left\lceil \frac{Q_i(t-\Delta)}{\alpha} \right\rceil\right)}{\sum_{j=1}^{N} g\left(\alpha \left\lceil \frac{Q_j(t-\Delta)}{\alpha} \right\rceil\right)} + \sum_{i=1}^{N} \mu Q_i(t) = \lambda + \sum_{i=1}^{N} \mu Q_i(t).$$

   Note that these rates depend on $Q_i(t)$ and $Q_i(t-\Delta)$ for each $i \in \{1, 2, ..., N\}$.
4. Lets $T^* = -\frac{\log(u_1)}{\mu^*}$ be the time of the next event.
5. If $u_2 < \frac{\lambda}{\lambda + \sum_{i=1}^{N} \mu Q_i(t)}$, then the next event is an arrival otherwise it is a departure. If, there is an arrival, then we find the integer k such that

$$\sum_{i=1}^{k-1} \lambda \cdot \frac{g\left(\alpha \left\lceil \frac{Q_i(t-\Delta)}{\alpha} \right\rceil\right)}{\sum_{j=1}^{N} g\left(\alpha \left\lceil \frac{Q_j(t-\Delta)}{\alpha} \right\rceil\right)} < u_3 \leq \sum_{i=1}^{k} \lambda \cdot \frac{g\left(\alpha \left\lceil \frac{Q_i(t-\Delta)}{\alpha} \right\rceil\right)}{\sum_{j=1}^{N} g\left(\alpha \left\lceil \frac{Q_j(t-\Delta)}{\alpha} \right\rceil\right)}.$$

   However, if there was a departure instead, then we find the integer k such that

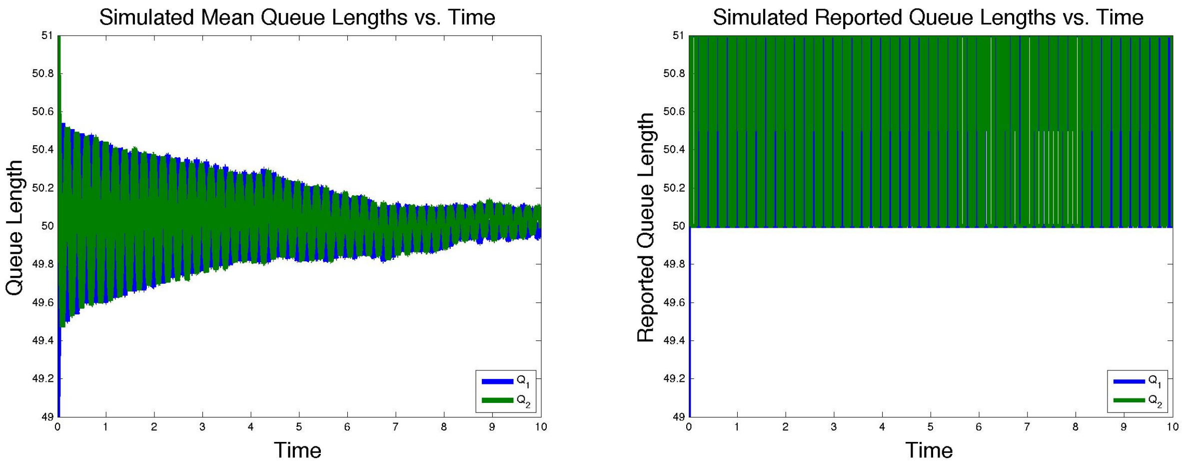
$$\sum_{i=1}^{k-1} \frac{Q_i(t)}{\sum_{j=1}^{N} Q_j(t)} < u_3 \leq \sum_{i=1}^{k} \frac{Q_i(t)}{\sum_{j=1}^{N} Q_j(t)}.$$

6. Set $t = t + T^*$ and update the number of each queue.
7. Return to step 2 or finish.

5 RESULTS

Now that we have described our simulation algorithm, we can use the algorithm to simulate the queueing processes of our system. For each simulation, we computed 10,000 sample paths and we plot the average of the sample paths over a time interval [0,10] as many of the rides are open for about 10 hours.

In Figures 8-12, we plot a sample two dimensional queueing process where we vary the delay in information and the amount of rounding we do. In Figure 8, we simulate a queueing process with rounding where $\alpha = 1$ and minimal amount of delayed information $\Delta = .05$. We see that the queue length oscillates for a while with the rounding even though it is quite small and integer valued. In Figure 9, we simulate the queue length process with a larger delay $\Delta = .5$ we see that the queues start to oscillate and the oscillations are larger. In Figure 10, we increase the value of the rounding to the exact value that Disney uses i.e. $\alpha = 5$. We observe that the queue length oscillates even more and the reported values are also larger as well when the delay is close to negligible. We also find in Figure 11 that the oscillations are larger when the information delay is increased. Thus, we observe that the rounding can induce oscillations in the system and the increase in delay only increases those oscillations. Finally in Figure 12, we simulate a non-stationary arrival rate and we observe that our queueing model can mimic some of the dynamics seen in Figure 7. Thus, if we were to obtain the actual arrival data for the two rides, we could replicate the waiting time dynamics on the app.



(a) Mean Queue Length

(b) Reported Queue Length in App

Figure 8: Simulation experiments features $\lambda = 100$, $\mu = 1$, $\Delta = .05$, and $\alpha = 1$.

6 CONCLUSION AND FUTURE WORK

In this paper, we have investigated the impact of queue length rounding and delayed information in the context of amusement park queues at Disney theme parks. We develop a simulation algorithm that is used to simulate the dynamics of Disney queues and how the information is disseminated to park goers. We find that both delayed information and queue length rounding can have adverse effects on the dynamics of the rides since they both induce oscillations in the queue lengths or wait times for rides. These oscillations were observed in several rides in the DisneyWorld theme park and, we believe that they are caused by the rounding and delayed information provided in the smartphone app. Thus, we believe that the oscillations can be reduced by giving more real-time and accurate information and by not rounding the wait times for customers. We find that there are several directions for future research. For one, we hope to prove limit theorems about the convergence of our queueing model to fluid and diffusion limits, however, the rounding introduces discontinuities that make the current theory inapplicable. Second, we would like to understand further the impact of non-stationary arrival rates, which is very relevant for WDW queues. It is well known that non-stationary arrival rates are quite difficult to analyze, see for example Engblom and Pender (2014). Finally, we would like to explore the possibility of using other types of ways to disseminate the wait time information to customers. We have seen that rounding the information can be problematic, but perhaps other modifications might work well for achieving balance among the queues.

ACKNOWLEDGMENTS

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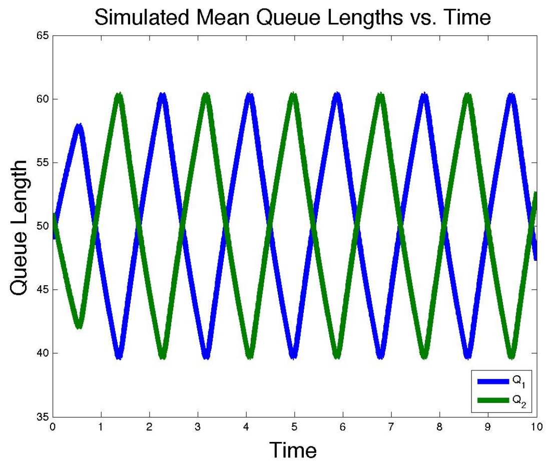
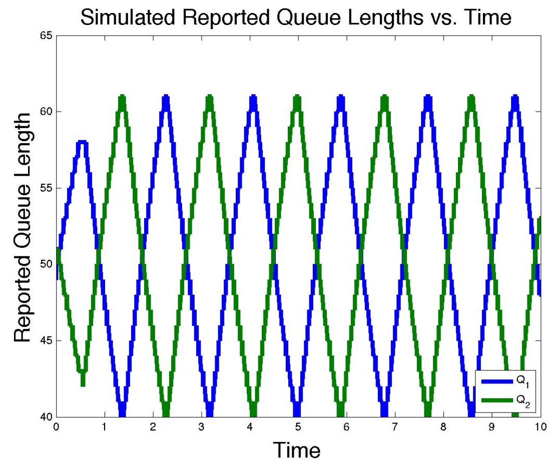
REFERENCES

Allon, G., and A. Bassamboo. 2011. "The Impact of Delaying the Delay Announcements". *Operations Research* 59(5):1198–1210.

Allon, G., A. Bassamboo, and I. Gurvich. 2011. ""We Will Be Right with You": Managing Customer Expectations with Vague Promises and Cheap Talk". *Operations Research* 59(6):1382–1394.



(a) Mean Queue Length



(b) Reported Queue Length in App

Figure 9: Simulations experiments featuring $\lambda = 100$, $\mu = 1$, $\Delta = .45$, and $\alpha = 1$.

Armony, M., and C. Maglaras. 2004. "On Customer Contact Centers with a Call-Back Option: Customer Decisions, Routing Rules, and System Design". *Operations Research* 52(2):271–292.

Armony, M., N. Shimkin, and W. Whitt. 2009. "The Impact of Delay Announcements in Many-Server Queues with Abandonment". *Operations Research* 57(1):66–81.

Burnetas, A., A. Economou, and G. Vasiliadis. 2017. "Strategic Customer Behavior in a Queueing System with Delayed Observations". *Queueing Systems* 86(3-4):389–418.

Engblom, S., and J. Pender. 2014. "Approximations for the moments of nonstationary and state dependent birth-death queues". *arXiv preprint arXiv:1406.6164*.

Guo, P., and P. Zipkin. 2007. "Analysis and Comparison of Queues with Different Levels of Delay Information". *Management Science* 53(6):962–970.

Guo, P., and P. Zipkin. 2009. "The Impacts of Customers' Delay-Risk Sensitivties on a Queue with Balking". *Probability in the Engineering and Informational Sciences* 23(03):409–432.

Hassin, R. 2007. "Information and Uncertainty in a Queuing System". *Probability in the Engineering and Informational Sciences* 21(03):361–380.

Ibrahim, R. 2017. "Sharing Delay Information in Service Systems: A Literature Survey". *Queueing Systems*.

Ibrahim, R., M. Armony, and A. Bassamboo. 2016. "Does the Past Predict the Future? The Case of Delay Announcements in Service Systems". *Management Science* 63(6):1762–1780.

Ibrahim, R., and W. Whitt. 2009. "Real-time Delay Estimation in Overloaded Multiserver Queues with Abandonments". *Management Science* 55(10):1729–1742.

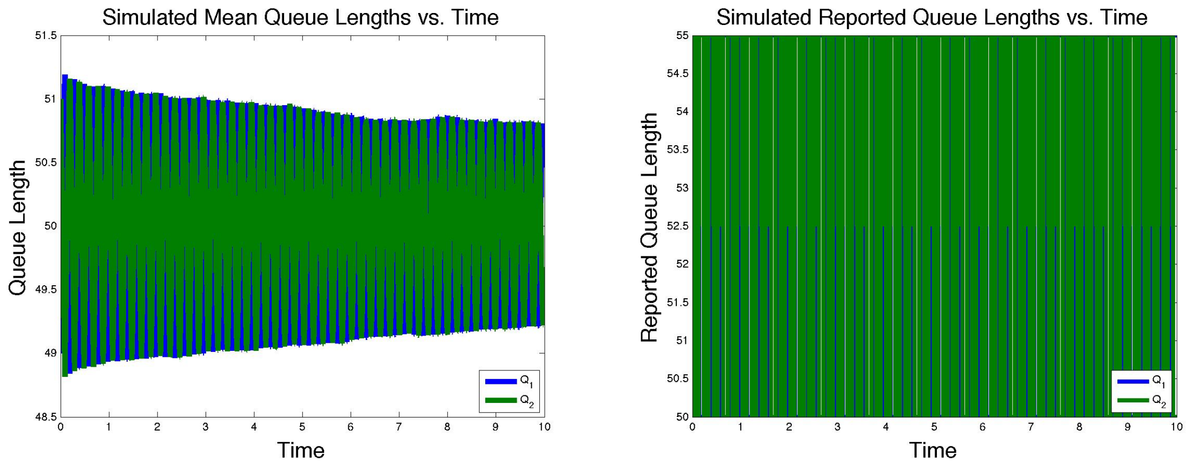
Ibrahim, R., and W. Whitt. 2011. "Real-Time Delay Estimation Based on Delay History in Many-Server Service Systems with Time-Varying Arrivals". *Production and Operations Management* 20(5):654–667.

Jennings, O. B., and J. Pender. 2016. "Comparisons of Ticket and Standard Queues". *Queueing Systems* 84(1-2):145–202.

Jouini, O., Z. Aksin, and Y. Dallery. 2011. "Call Centers with Delay Information: Models and Insights". *Manufacturing & Service Operations Management* 13(4):534–548.

Jouini, O., Y. Dallery, and Z. Akşin. 2009. "Queueing Models for Full-Flexible Multi-Class Call Centers with Real-Time Anticipated Delays". *International Journal of Production Economics* 120(2):389–399.

Lingenbrink, D., and K. Iyer. 2017. "Optimal Signaling Mechanisms in Unobservable Queues with Strategic Customers". In *Proceedings of the 2017 ACM Conference on Economics and Computation*, EC '17, 347–347. New York, NY, USA: ACM: ACM.



(a) Mean Queue Length

(b) Reported Queue Length in App

Figure 10: Simulation experiments featuring $\lambda = 100$, $\mu = 1$, $\Delta = .05$, and $\alpha = 5$.

Pender, J. 2015a. "Heavy Traffic Limits for Unobservable Queues with Clearing Times". Technical Report 1, Cornell University, 228 Rhodes Hall Ithaca, NY 14853.

Pender, J. 2015b. "The Impact of Dependence on Unobservable Queues". Technical Report 2, Cornell University, 228 Rhodes Hall Ithaca, NY 14853.

Pender, J., R. H. Rand, and E. Wesson. 2017a. "Queues with Choice via Delay Differential Equations". *International Journal of Bifurcation and Chaos* 27(04):1730016.

Pender, J., R. H. Rand, and E. Wesson. 2017b. "Strong Approximations for Queues with Customer Choice and Constant Delays".

Pender, J., R. H. Rand, and E. Wesson. 2018. "An Analysis of Queues with Delayed Information and Time-Varying Arrival Rates". *Nonlinear Dynamics* 91(4):2411–2427.

Walt Disney Company 2018. "Safety, Courtesy, Show and Efficiency: Our Four Keys". http://careers.disneylandparis.com/en/safety-courtesy-show-and-efficiency-our-four-keys.

Whitt, W. 1999. "Improving Service by Informing Customers about Anticipated Delays". *Management Science* 45(2):192–207.

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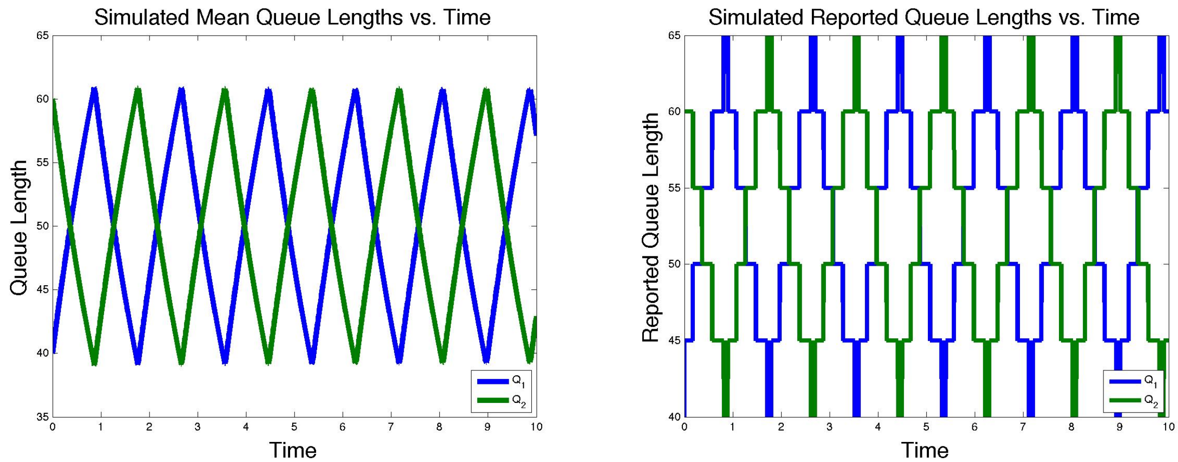
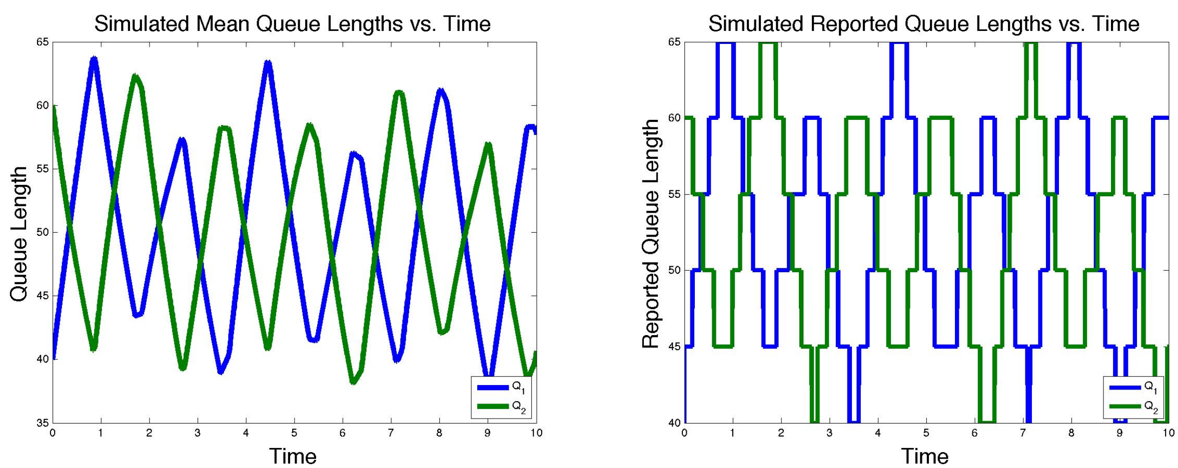


(a) Mean Queue Length (b) Reported Queue Length in App

Figure 11: Simulation experiments featuring $\lambda = 100$, $\mu = 1$, $\Delta = .5$, and $\alpha = 5$.



(a) Mean Queue Length (b) Reported Queue Length in App

Figure 12: Simulation experiments featuring $\lambda = 100 + 20\sin(2t)$, $\mu = 1$, $\Delta = .5$, $\alpha = 5$.