Proceedings of the 2018 Winter Simulation Conference
M. Rabe, A. A. Juan, N. Mustafee, A. Skoogh, S. Jain, and B. Johansson, eds.

INTEGRATING BIASED-RANDOMIZED GRASP WITH MONTE CARLO SIMULATION FOR SOLVING THE VEHICLE ROUTING PROBLEM WITH STOCHASTIC DEMANDS

Paola Festa
Tommaso Pastore

Dept. of Mathematics and Applications
University of Napoli FEDERICO II
Via Cintia
80126 Napoli, ITALY

Daniele Ferone
Angel A. Juan
Christopher Bayliss

IN3 – Computer Science Dept.
Universitat Oberta de Catalunya
Av. Carl Friedrich Gauss 5
08860 Castelldefels, SPAIN

ABSTRACT

Few problems in Operations Research are regarded as highly as the Vehicle Routing Problem (VRP). Its relevance within management and industrial settings has led to the variants of this problem being widely studied by the scientific community. With the aim of solving the VRP with stochastic demands we analyze an extension of the classical GRASP metaheuristic. This work hybridizes a biased-randomized GRASP (BR-GRASP) with a two-stage Monte Carlo simulation which has the ability to attain robust and competitive solutions. In the first stage, a promising set of local optimum solutions is identified based on a short simulation evaluation. In the second stage, the promising solutions are tested for reliability using a larger number of simulation runs. The most reliable solution is the final solution. Experiment results are provided that demonstrate that the proposed integrated algorithm leads to higher quality solutions than the equivalent approach without such an integration.

1 INTRODUCTION

During the last two decades, we have experienced several methodological improvements in solving large-scale combinatorial optimization problems. These methodological improvements have generated a noticeable rise in the industrial demand for optimization tools. These new algorithms, which hybridize established solution strategies with problem-specific implementation features, coupled with the availability of powerful yet inexpensive computing technology, have led scientific research towards new horizons and challenges. One of the recent challenges has been in the intensive research on stochastic optimization problems. Given the non-deterministic nature of real-world scenarios, properly addressing stochasticity is essential for embedding optimization techniques into each of the supply-chain systems that determine production, scheduling, distribution, location-allocation, etc. With this aim in mind, we study one of the most iconic problems in the landscape of stochastic combinatorial optimization, i.e., the Vehicle Routing Problem with stochastic demands (VRPSD).

The vast majority of combinatorial problems are computationally intractable by nature, and the Vehicle Routing Problem (VRP) is no exception. For a summary of the existing approaches to solving the classical VRP, we refer the reader to three excellent resources on the topic: Toth and Vigo (2014), Laporte (1992), and Golden et al. (2008). Due to the complexity of large-scale VRP instances, the typical solution techniques mainly belong to the class of heuristic and metaheuristic algorithms. Some of the first attempts to incorporate random sampling techniques into heuristic approaches for solving VRP instances are due to Faulin and Juan (2008b) and Faulin et al. (2008a). The Greedy Randomized Adaptive Search Procedure (GRASP) is a flexible metaheuristic technique proposed in Feo and Resende (1995). GRASP has proven to be successful in a very diverse set of combinatorial optimization problems: from scheduling and routing (Festa and Resende 2009b) to graph coloring and satisfiability (Laguna and Martí 2001; Felici et al. 2017). The versatility shown in solving NP-hard deterministic problems, and the simplicity of its implementation, makes GRASP an excellent solving methodology for the VRPSD. In this work, we discuss a recent extension of this framework, the GRASP with biased randomization (BR-GRASP). In addition, we discuss how BR-GRASP can be hybridized with Monte Carlo simulation in order to obtain reliable and competitive routing plans when considering customers with stochastic demands.

The remainder of the paper is structured as follows: Section 2 summarizes previous approaches to solving stochastic VRPs (SVRPs) and provides a brief survey of the results achieved using the simheuristic paradigm to tackle hard stochastic problems. Section 3 formally introduces the VRP with stochastic demands. The main concepts and implementation strategies of our method are described in Section 4. Section 5 presents test results obtained on a well-established reference benchmark set. Lastly, Section 6 concludes this paper, by presenting the main conclusions and possible future research directions.

2 RELATED WORK

This section reviews two different types of related work. On the one hand, it discusses how exact and metaheuristic approaches have been employed so far to cope with stochastic VRPs. On the other hand, it reviews the basics of simheuristics and different applications of this simulation-optimization methodology.

2.1 Exact and Metaheuristic Methods to Solve Stochastic Routing Problems

The complexity of intractable combinatorial optimization problems, such as the VRP, heavily limits the size of the instances that can be solved with exact methods. Nevertheless, driven by the attractiveness of optimal solutions, several efforts have been made in the literature to design exact algorithms for routing problems. Gendreau et al. (1995) provide optimal solutions to the VRP with stochastic customers and demands by means of an integer L-shaped algorithm. In Laporte et al. (1992), a branch-and-cut method is described for solving the VRP with stochastic travel times. Other examples of exact algorithms that have been investigated are column generation (Taş et al. 2014) and branch-and-price (Christiansen and Lysgaard 2007). At the same time, in order to solve larger instances, a parallel stream of research has focused on heuristic and metaheuristic algorithms. Thus, Gendreau et al. (1996b) propose a Tabu Search algorithm for the VRP with stochastic customers and demands. For the VRP with stochastic demands, the work of Marinakis et al. (2013) describes a particle swarm optimization method, while Bianchi et al. (2006) compare the results achieved by five different well-known metaheuristics.

The common aspect shared by both exact and heuristic methods is the use of a dynamic approach, in which some of the stochastic values are only revealed once vehicles reach customers, at which point solutions can be dynamically re-optimized or updated with recourse actions.

2.2 Simheuristics: Bringing Together Optimization and Simulation

The simheuristic framework is one of the most successful examples of hybrid simulation-optimization approaches. In this paradigm, the typical randomness of real-world optimization scenarios is effortlessly accounted for in the solution strategy, whether it has to be included in a stochastic objective function or a constraint. The main difference between techniques of this family and the majority of stochastic approaches is that simheuristics do not assume that recourse actions are available to correct solutions during the implementation of the solution. On the contrary, they try to achieve a 'reliable' solution by integrating stochastic simulation within the search process.

One essential idea in the simheuristic concept is the strong relation between the strategy devised for solving a classical deterministic problem and the simulation of the random variables. These two different perspectives to the problem are handled respectively by the heuristic (or metaheuristic) core and by a stochastic simulation (in any of its forms). Thus, given a stochastic optimization problem, a first step of a simheuristic approach consists of the transformation of the stochastic variables into their deterministic counterparts. This allows the approach to efficiently obtain a deterministic solution by means of a heuristic algorithm. Subsequently, the quality of the constructed solution is evaluated in a stochastic environment to determine its real value in scenarios with uncertainty. This two-step process establishes a feedback system between the deterministic optimization algorithm and the stochastic simulation. This feedback system is not only able to more accurately evaluate solution quality, but also to better guide the exploration of the search space.

The scientific literature has recently seen several successful applications of simheuristic algorithms to a broad variety of stochastic combinatorial optimization problems. In Guimarans et al. (2016), a hybrid simheuristic is proposed to solve a variant of the capacitated VRP. This variant combines vehicle routing and packing constraints, and so it is known as the two-dimensional VRP with stochastic travel times. The multi-start randomized simheuristic, presented in Juan et al. (2014b) for the single-period and stochastic Inventory Routing Problem with stock-outs, shows how personalized refill policies can be crucial to enable significant cost reductions in comparison to what can be achieved by other algorithms employing standard refill policies. Gonzalez-Martin et al. (2018) tackle the Arc Routing Problem (ARP) with stochastic demands by embedding Monte Carlo simulation into a classical strategy for the capacitated ARP. Without being exhaustive in our review, other notable efforts in the literature can be found in the area of Permutation Flow Shop Problems (Juan et al. 2014a; Ferone et al. 2016a; Gonzalez-Neira et al. 2017), and facility location problems (de Armas et al. 2017).

3 THE VEHICLE ROUTING PROBLEM WITH STOCHASTIC DEMANDS

As in the case of other notable stochastic combinatorial optimization problems, a Stochastic Vehicle Routing Problem (SVRP) emerges whenever a random component is included in the mathematical model of a classic VRP formulation. The most-common examples of such random components are mainly related to the stochasticity of customer availability, customer demands, and travel times.

Given the critical importance of understanding the influence of uncertainty over decisions, which is a crucial matter in logistics and operations management, solving SVRPs is vitally important to both the simulation-optimization community and industry (Gendreau et al. 1996a). One of the initial contributions to the VRP with stochastic demands can be found in Bertsimas (1992), where the author tries to: *(i)* determine a fixed routing sequence covering all potential customers; and then *(ii)* update the solution according to two different criteria once the information regarding customer demands becomes available. The approach of Bertsimas (1992) is to generate a deterministic solution and to re-optimize it in real-time as uncertainties materialize. Such a strategy has clear advantages for tractable problems, as in the case of Shortest Path Problems (Ferone et al. 2016b; Ferone et al. 2017). However, for problems that are characterized by their hard complexity or whenever a reduced amount of computing resources are available to complete a reactive strategy, then allowing for stochasticity in the initial solution process is beneficial. This is the approach taken in this work, whose aim is to generate a 'reliable' or 'stable' solution for the VRPSD, i.e., a set of routes that can achieve a good performance not only in terms of average cost but also by being able to support moderate levels of variability in the values of the random customer demands.

The VRPSD is formally stated on a graph $G = (V,E)$, where the set of nodes includes both the depot, usually denoted by 0, and the location of customers, $V \setminus \{0\}$. A fleet of vehicles with restricted capacity is available at the depot, which is the starting and ending point for each trip. Moreover, a non-negative cost function $c : E \to \mathbb{R}^+$ associates a cost (either based on distance or on travel time) to each edge in E. For each $i \in V \setminus \{0\}$, a random variable x_i indicates the demand of goods that customer i requires. The goal of the problem is to find a set of routes with minimal total expected cost while serving all customers and satisfying the capacity constraints.

4 OUR SIMULATION-OPTIMIZATION APPROACH

This section begins by discussing the classical GRASP metaheuristic framework. Then, it analyzes how this basic GRASP version can be easily enhanced by making use of biased-randomization concepts. Finally, the BR-GRASP is hybridized with Monte Carlo simulation in order to address the stochastic component of the VRPSD that is considered in this paper.

4.1 GRASP Framework

GRASP is a well-known metaheuristic framework for hard combinatorial optimization problems. Initially proposed in Feo and Resende (1995), it has achieved many benchmark results for large-scale instances of NP-hard problems. The reader is referred to Festa and Resende (2009a) and Festa and Resende (2009b) for a detailed analysis of its theoretical foundations as well as some of its most popular applications.

The classical GRASP version is made up by two main components (Algorithm 1): a constructive phase and a local search. In the former, a solution is built iteratively by adding one component at a time from a candidate set. In each iteration, the element to be added to the solution under construction is selected uniformly at random among the elements of a *restricted candidate list* (RCL). The RCL collects all the best insertion candidates according to some greedy score function. It includes all the elements whose score is better than a user-specified percentage of the best score. This adaptive thresholding process is guided by a parameter, α, which tunes the greediness of the constructive phase.
Upon completion of the first phase, a feasible solution is obtained. At this point, the local search attempts to reach a local optimum with respect to some suitably-defined neighborhood structure. Given the randomness of this framework, the two phases are repeatedly applied in a multi-start fashion. At the end of the process, the best solution found is returned as the final result.

4.2 GRASP with Biased Randomization

Our solution method relies on a recent extension of the classical GRASP paradigm: GRASP with biased randomization or BR-GRASP (Algorithm 2).

Algorithm 1: Construction phase with RCL

Input: $\alpha \in [0,1]$
$s \leftarrow \emptyset$
initialize candidate set: CL $\leftarrow$ E
order CL according to $c(\cdot)$
while *solution s is not complete* **do**
  $c_{\min} \leftarrow \min_{x \in CL}\{c(x)\}$
  $c_{\max} \leftarrow \max_{x \in CL}\{c(x)\}$
  $thr \leftarrow c_{\min} + \alpha(c_{\max} - c_{\min})$
  $RCLsize \leftarrow |\{x \in CL\colon c(x) \leq thr\}|$
  $pos \leftarrow$ $\texttt{UnifRand}(1, 2, \ldots, RCLsize)$
  $s \leftarrow s \cup \{CL[pos]\}$
  $CL \leftarrow CL \setminus \{CL[pos]\}$
  Reorder CL
end
return s

Algorithm 2: Construction phase with Biased Randomization

Input: Distribution D; Parameter β
$s \leftarrow \emptyset$
initialize candidate set: CL $\leftarrow$ E
order CL according to $c(\cdot)$
while *solution s is not complete* **do**
  Randomly select $pos \in \{1, \ldots, |CL|\}$ according to distribution $D(\beta)$
  $s \leftarrow s \cup \{CL[pos]\}$
  $CL \leftarrow CL \setminus \{CL[pos]\}$
  Reorder CL
end
return s

The aim of the BR-GRASP is to guide the construction process towards more favorable regions of the search space, with the goal of achieving diversification without excluding any potentially good candidate elements. More specifically, while in the classical GRASP the RCL implementation is obtained by the two-step process described above (thresholding plus uniform selection), this new paradigm guides the constructive phase using a skewed (non-uniform) probability distribution and without using a RCL.

Different skewed probability distributions can be considered within this extension. As suggested in Grasas et al. (2017), we use a geometric distribution with parameter β. The result of this process is a constructive phase that is able to balance greedy bias and search diversification. A depiction of the differences among the selection processes of the classical GRASP and the BR-GRASP can be found in Figure 1.

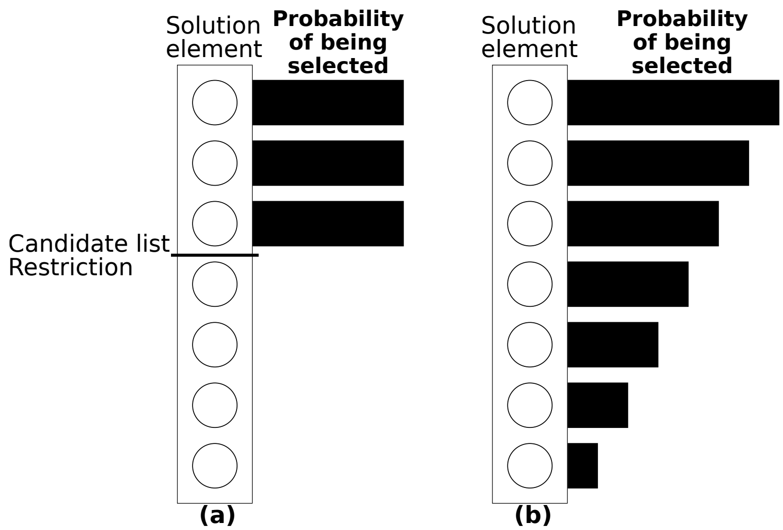

Figure 1: Differences in the selection processes of the classic GRASP *(a)* and the BR-GRASP *(b)*.

Biased-randomization techniques have been successfully used in the past to solve different VRP variants, including the two-dimensional loading VRP (Dominguez et al. 2014; Dominguez et al. 2016) or the multi-depot VRP (Juan et al. 2015).

4.3 Our SimGRASP Approach for the VRPSD

Metaheuristic frameworks were originally conceived for tackling deterministic problems. Accordingly, the classic GRASP is not designed to consider uncertainty within the input data. From a technical perspective, this represents a major drawback when dealing with real-life scenarios, for which the use of a deterministic model could represent an oversimplification which leads to suboptimal solutions (Lucas et al. 2015).

With the aim of allowing for the inherent uncertainty within logistics and transportation problems, our algorithm embraces the paradigm of simheuristics (Grasas et al. 2016). The resulting framework, the SimGRASP, incorporates Monte Carlo simulation at two different stages of the BR-GRASP approach. Incorporating simulation within an optimization algorithm is not on its own enough to guarantee a reliable solution. The performance of the stochastic solution obtained by the simheuristic is tightly linked to the quality of the deterministic optimization algorithm, that is, both components are complementary to the quality of the final solution. In this work we use BR-GRASP as the deterministic optimization component. The operation flow of our simulation-optimization technique is summarized in Figure 2.

As in the deterministic case, the algorithm begins with a biased-randomized constructive phase. The output is a first feasible solution s. This first phase is then followed by a local search, with the aim of improving s to achieve a deterministic local optimal solution, s^*, according to a two-opt exchange neighborhood structure $N(x)$.

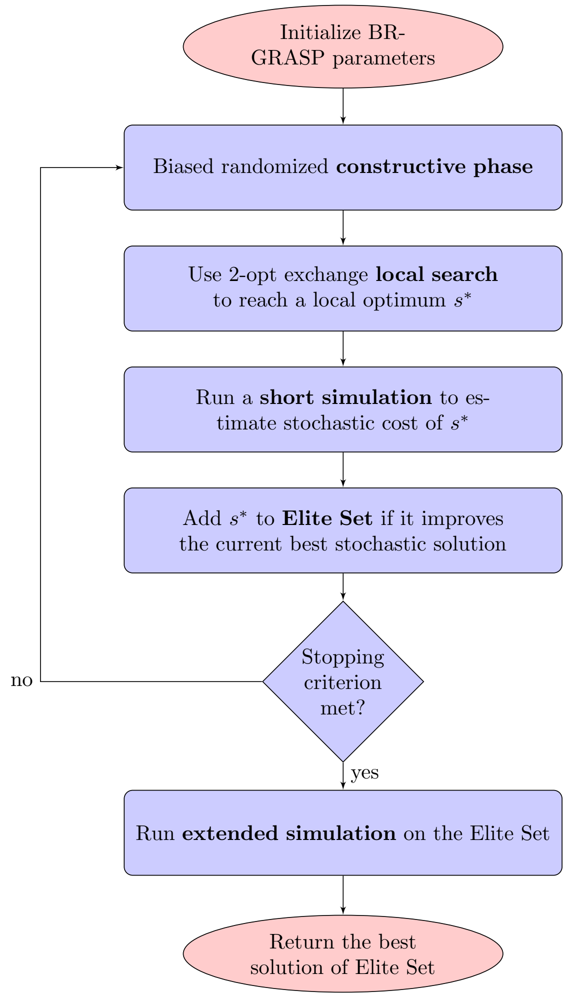

Figure 2: Flowchart of our SimGRASP algorithm.

The deterministic local optimum s^* is then evaluated in a stochastic environment, which represents the first of two applications of Monte Carlo simulation, in which a limited number of simulation replications are used. Within each single run of the simulation, all customers' demands are randomly generated accordingly to the corresponding probability distributions. The number of runs performed in this first simulation stage is denoted $nIter_{short}$. This use of a limited number of simulations runs provides a mechanism for controlling the trade-off between run times and accuracy. Subsequently, the initial solution s^* is set as the current best solution, and the 'elite set' (ES) is initialized with s^* as its only element.

Thereafter, as in the classical GRASP paradigm, the procedure is repeated in a multi-start fashion until the stopping criterion is satisfied. In each iteration, new deterministic solutions are generated and consequently improved by means of the local search, hence obtaining new deterministic local optima s^{**}. Each of the locally optimal solutions is evaluated in $nIter_{short}$ simulation runs. If in this stochastic analysis it is found that the performance of s^{**} is better than those obtained by the current best solution, then s^{**} is added to ES and the current best solution is accordingly replaced.

Once the multi-start process has finalized, all the solutions belonging to ES are evaluated through a second simulation stage, this time using a higher number of simulation runs, $nIter_{long}$, for each evaluation of a candidate solution. This second simulation stage is only applied to a reduced set of 'promising' solutions, which limits the necessary computing time. The aim of this second simulation stage is to thoroughly assess, with a higher degree of accuracy, the stochastic quality of the solution generated during the heuristic search process.

5 COMPUTATIONAL EXPERIMENTS

In this section, a series of computational experiments are described and their results analyzed. These numerical experiments contribute to verify and validate the proposed simulation-optimization methodology.

5.1 Experimental Settings and Benchmarks

An assessment of the performance of our SimGRASP was obtained by comparison with the multi-start simheuristic discussed in Juan et al. (2013). In this paper, the authors use a multi-start algorithm that is later combined with simulation to estimate the expected distribution costs. Both the multi-start algorithm and our SimGRASP make use of a geometric probability distribution to benefit from biased-randomization techniques (Grasas et al. 2017). In addition, the multi-start algorithm (SimMultiStart) uses a splitting strategy, which tries to improve each solution by dividing it into smaller problems. The SimMultiStart approach considers the concept of safety stocks to prevent running out of goods due to the occurrence of higher-than-expected demands. Finally, the local search employed in the SimMultiStart algorithm implements a cache-memory to store past routes, while the local search in our BR-GRASP incorporates a 2-opt exchange operator. In the interest of a fair analysis, the simulation paradigm considered for the two algorithms follows the same construction mechanism. The demands d_i used in the deterministic benchmark set are replaced by their stochastic counterpart, D_i, according to a Log-Normal probability distribution, with $E[D_i] = d_i$. Also, $Var[D_i] = k \cdot d_i$, where $k > 0$ is a design parameter. The experiment results reported here were obtained with a variance level of $k = 0.25$. Consequently, the Log-Normal distribution can be described through the location (μ_i) and scale (σ_i) parameters as follows:

$$\mu_i = \ln(E[D_i]) - \frac{1}{2} \cdot \ln\left(1 + \frac{Var[D_i]}{E[D_i]^2}\right)$$

$$\sigma_i = \left| \sqrt{\ln\left(1 + \frac{Var[D_i]}{E[D_i]^2}\right)} \right|$$

In order to obtain a reliable evaluation, the method presented here was applied to the well-established benchmark instances of types A and B that were proposed in Augerat et al. (1998) for the capacitated VRP. The SimGRASP was implemented as a Java application and run directly on Eclipse using a personal computer with an Intel i7 Quad core, 2.67 GHz clock, and 6 GB RAM. After a quick fine-tuning of the BR-GRASP parameters, the following values were set: for the threshold parameter in Algorithm 1, $\alpha = 0.3$; for the Geometric distribution parameter in Algorithm 2, $\beta = 0.5$. The algorithm was executed with a time limit of 10 seconds.

5.2 Analysis of Results

Table 1 summarizes the results generated in the testing phase. The comparison of the solutions obtained by: *(i)* the coupling of a multi-start heuristic and simulation; and *(ii)* the SimGRASP approach shows how this new technique outperforms the multi-start one in each instance. The measured gap, indeed, is negative for all instances in the benchmark set, with an average value of -9.81% over all instances. These good results are mainly due to the optimization paradigm used in the implementation of our BR-SimGRASP, mainly due to the incorporation of an enhanced local search operator.

Table 1: Performance of BR-GRASP and SimGRASP for the VRP.

Instance	SimMultiStart (1)	SimGRASP (2)	% Gap (1)-(2)
A-n32-k5	993.20	890.95	-10.30
A-n33-k5	815.40	750.63	-7.94
A-n33-k6	912.60	837.63	-8.21
A-n37-k5	795.00	734.44	-7.62
A-n38-k5	885.10	824.37	-6.86
A-n39-k6	1010.60	926.11	-8.36
A-n45-k6	1184.30	1091.36	-7.85
A-n45-k7	1502.00	1336.08	-11.05
A-n55-k9	1408.40	1258.72	-10.63
A-n60-k9	1795.70	1579.79	-12.02
A-n61-k9	1330.60	1224.44	-7.98
A-n63-k9	2203.70	1897.29	-13.90
A-n65-k9	1555.30	1437.84	-7.55
A-n80-k10	2328.40	2177.10	-6.50
B-n31-k5	855.70	757.60	-11.46
B-n35-k5	1255.50	1098.00	-12.54
B-n39-k5	695.90	621.34	-10.71
B-n41-k6	1103.20	1005.62	-8.84
B-n45-k5	904.60	828.04	-8.46
B-n50-k7	945.80	859.48	-9.13
B-n52-k7	944.40	848.71	-10.13
B-n56-k7	920.00	845.37	-8.11
B-n57-k9	2199.70	1885.69	-14.27
B-n64-k9	1179.60	1064.53	-9.75
B-n67-k10	1404.50	1247.67	-11.17
B-n68-k9	1754.70	1515.88	-13.61
Average			***-9.81***

Furthermore, the results show that the performance of the stochastic solutions are enhanced by the biased-randomized construction with respect to the classical GRASP paradigm. Another example of the effect of the quality of the solution generated in the construction phase and stochastic value of the final solution can be found in the radar chart presented in Figure 3. This plot illustrates the solution values obtained, for instances A-n33-k5 and B-n31-k5, by three different algorithmic approaches: SimMultiStart, SimGRASP, and SimGRASP with BR. Thus, values closer to the center of the triangle represent a lower-cost value. Figure 3 shows how the biased-randomized SimGRASP achieves the best results among the three approaches.

6 CONCLUSIONS AND FURTHER POSSIBLE EXTENSIONS

This work is motivated by the benefits of accounting for stochasticity when solving hard combinatorial optimization problems. In our view, successfully incorporating stochasticity within deterministic optimization approaches depends on a careful integration of well-performing optimization tools in industrial settings. More specifically, we have proposed the use of an approach that combines simulation and metaheuristic optimization to solve one of the most difficult and widely studied stochastic optimization problems: the Vehicle Routing Problem with Stochastic Demands.

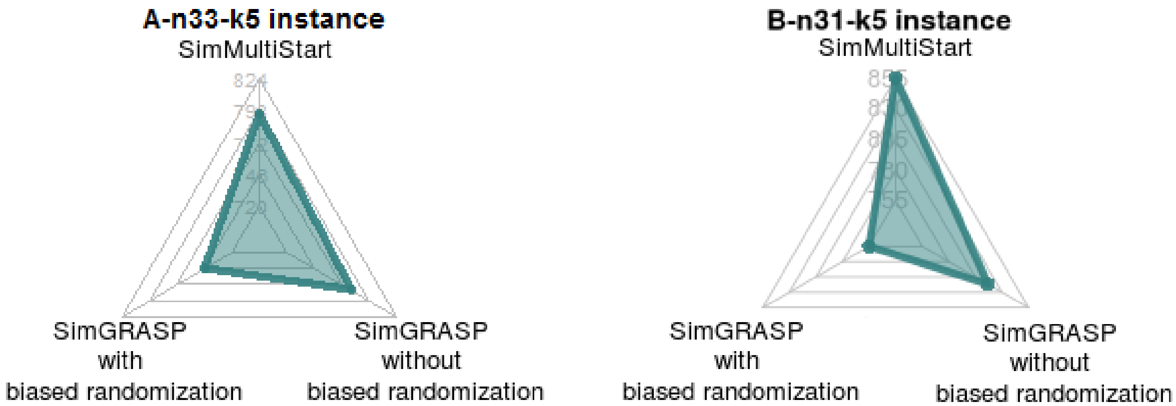

Figure 3: Comparison of a MS simheuristic, SimGRASP with BR, and SimGRASP without BR.

The simheuristic proposed here is based on a recent extension of the classical GRASP framework, the GRASP with biased randomization. This extension has been shown to be a successful meeting point between performance and simplicity of implementation. The algorithm has been tested over a set of stochastic instances obtained from an established benchmark set for the deterministic version of the problem. The results show that our method is able to attain high-quality solutions over all instances in reasonably low computing times. In particular, our results provide results supporting the idea that high-quality deterministic local optimal solutions are complementary to the quality of the robust solutions that are derived from the proposed hybridization of BR-GRASP and Monte Carlo Simulation.

Future extensions of the present work are two-fold: a first stream of investigation can be based on improving implementation features that enhance the performance of the heuristic core of our algorithm. Path relinking, for example, is a feature that has been successfully embedded in multiple GRASP frameworks (Laguna and Marti 1999; Resendel and Ribeiro 2005; Ferone et al. 2016c). On the other hand, to improve the quality of the solutions generated specifically for the vehicle routing problem, some problem-specific intensification-diversification strategies can be devised, such as granular search, or the restricted neighborhood structure as applied in Toth and Vigo (2003).

REFERENCES

Augerat, P., J. Belenguer, E. Benavent, A. Corberán, D. Naddef, and G. Rinaldi. 1998. “Computational Results with a Branch-and-cut Code for the Capacitated Vehicle Routing Problem”. Technical Report R.495, IASI-CNR.

Bertsimas, D. J. 1992. “A Vehicle Routing Problem with Stochastic Demand”. *Operations Research* 40(3):574–585.

Bianchi, L., M. Birattari, M. Chiarandini, M. Manfrin, M. Mastrolilli, L. Paquete, O. Rossi-Doria, and T. Schiavinotto. 2006. “Hybrid Metaheuristics for the Vehicle Routing Problem with Stochastic Demands”. *Journal of Mathematical Modelling and Algorithms* 5(1):91–110.

Christiansen, C. H., and J. Lysgaard. 2007. “A Branch-and-price Algorithm for the Capacitated Vehicle Routing Problem with Stochastic Demands”. *Operations Research Letters* 35(6):773–781.

de Armas, J., A. A. Juan, J. M. Marquès, and J. P. Pedroso. 2017. “Solving the Deterministic and Stochastic Uncapacitated Facility Location Problem: From a Heuristic to a Simheuristic”. *Journal of the Operational Research Society* 68(10):1161–1176.

Dominguez, O., A. A. Juan, B. Barrios, J. Faulin, and A. Agustin. 2016. “Using Biased Randomization for Solving the Two-dimensional Loading Vehicle Routing Problem with Heterogeneous Fleet”. *Annals of Operations Research* 236(2):383–404.

Dominguez, O., A. A. Juan, and J. Faulin. 2014. “A Biased-randomized Algorithm for the Two-dimensional Vehicle Routing Problem with and without Item Rotations”. *International Transactions in Operational Research* 21(3):375–398.

Faulin, J., M. Gilibert, A. A. Juan, X. Vilajosana, and R. Ruiz. 2008a. “SR-1: A Simulation-based Algorithm for the Capacitated Vehicle Routing Problem”. In *Proceedings of the 2008 Winter Simulation Conference*, edited by S. Mason et al., 2708–2716. Piscataway, New Jersey: IEEE.

Faulin, J., and A. A. Juan. 2008b. “The ALGACEA-1 Method for the Capacitated Vehicle Routing Problem”. *International Transactions in Operational Research* 15(5):599–621.

Felici, G., D. Ferone, P. Festa, A. Napoletano, and T. Pastore. 2017. “A GRASP for the Minimum Cost SAT Problem”. In *Learning and Intelligent Optimization*, edited by R. Battiti et al., 64–78. Cham, Switzerland: Springer International Publishing.

Feo, T. A., and M. G. Resende. 1995. “Greedy Randomized Adaptive Search Procedures”. *Journal of Global Optimization* 6(2):109–133.

Ferone, D., P. Festa, A. Gruler, and A. A. Juan. 2016a. “Combining Simulation with a GRASP Metaheuristic for Solving the Permutation Flow-shop Problem with Stochastic Processing Times”. In *Proceedings of the 2016 Winter Simulation Conference*, edited by T. M. K. Roeder et al., 2205–2215. Piscataway, New Jersey: IEEE.

Ferone, D., P. Festa, A. Napoletano, and T. Pastore. 2016b. “Reoptimizing Shortest Paths: From State of the Art to New Recent Perspectives”. In *18th International Conference on Transparent Optical Networks (ICTON)*, edited by M. Ferrari et al., without page numbers. Piscataway, New Jersey: IEEE.

Ferone, D., P. Festa, A. Napoletano, and T. Pastore. 2017. “Shortest Paths on Dynamic Graphs: A Survey”. *Pesquisa Operacional* 37(3):487–508.

Ferone, D., P. Festa, and M. G. Resende. 2016c. “Hybridizations of GRASP with Path Relinking for the Far from Most String Problem”. *International Transactions in Operational Research* 23(3):481–506.

Festa, P., and M. G. C. Resende. 2009a. “An Annotated Bibliography of GRASP – Part II: Applications”. *International Transactions in Operational Research* 16(2):131–172.

Festa, P., and M. G. C. Resende. 2009b. “An Annotated Bibliography of GRASP - Part I: Algorithms”. *International Transactions in Operational Research* 16(1):1–24.

Gendreau, M., G. Laporte, and R. Séguin. 1995. “An Exact Algorithm for the Vehicle Routing Problem with Stochastic Demands and Customers”. *Transportation Science* 29(2):143–155.

Gendreau, M., G. Laporte, and R. Séguin. 1996a. “Stochastic Vehicle Routing”. *European Journal of Operational Research* 88(1):3–12.

Gendreau, M., G. Laporte, and R. Séguin. 1996b. “A Tabu Search Heuristic for the Vehicle Routing Problem with Stochastic Demands and Customers”. *Operations Research* 44(3):469–477.

Golden, B. L., S. Raghavan, and E. A. Wasil. 2008. *The Vehicle Routing Problem: Latest Advances and New Challenges*. New York: Springer Science & Business Media.

Gonzalez-Martin, S., A. A. Juan, D. Riera, M. G. Elizondo, and J. J. Ramos. 2018. “A Simheuristic Algorithm for Solving the Arc Routing Problem with Stochastic Demands”. *Journal of Simulation* 12(1):53–66.

Gonzalez-Neira, E. M., D. Ferone, S. Hatami, and A. A. Juan. 2017. “A Biased-randomized Simheuristic for the Distributed Assembly Permutation Flowshop Problem with Stochastic Processing Times”. *Simulation Modelling Practice and Theory* 79:23–36.

Grasas, A., A. A. Juan, J. Faulin, J. de Armas, and H. Ramalhinho. 2017. “Biased Randomization of Heuristics using Skewed Probability Distributions: A Survey and Some Applications”. *Computers & Industrial Engineering* 110:216–228.

Grasas, A., A. A. Juan, and H. R. Lourenço. 2016. “SimILS: A Simulation-based Extension of the Iterated Local Search Metaheuristic for Stochastic Combinatorial Optimization”. *Journal of Simulation* 10(1):69–77.

Guimarans, D., O. Dominguez, A. A. Juan, and E. Martinez. 2016. "A Multi-start Simheuristic for the Stochastic Two-dimensional Vehicle Routing Problem". In *Proceedings of the 2016 Winter Simulation Conference*, edited by T. M. K. Roeder et al., 2326–2334. Piscataway, New Jersey: IEEE.

Juan, A. A., B. B. Barrios, E. Vallada, D. Riera, and J. Jorba. 2014a. "A Simheuristic Algorithm for Solving the Permutation Flow Shop Problem with Stochastic Processing Times". *Simulation Modelling Practice and Theory* 46:101–117.

Juan, A. A., J. Faulin, J. Jorba, J. Caceres, and J. M. Marquès. 2013. "Using Parallel & Distributed Computing for Real-time Solving of Vehicle Routing Problems with Stochastic Demands". *Annals of Operations Research* 207(1):43–65.

Juan, A. A., S. E. Grasman, J. Caceres-Cruz, and T. Bektaş. 2014b. "A Simheuristic Algorithm for the Single-period Stochastic Inventory-routing Problem with Stock-outs". *Simulation Modelling Practice and Theory* 46:40–52.

Juan, A. A., I. Pascual, D. Guimarans, and B. Barrios. 2015. "Combining Biased Randomization with Iterated Local Search for Solving the Multidepot Vehicle Routing Problem". *International Transactions in Operational Research* 22(4):647–667.

Laguna, M., and R. Marti. 1999. "GRASP and Path Relinking for 2-layer Straight Line Crossing Minimization". *INFORMS Journal on Computing* 11(1):44–52.

Laguna, M., and R. Martí. 2001. "A GRASP for Coloring Sparse Graphs". *Computational Optimization and Applications* 19(2):165–178.

Laporte, G. 1992. "The Vehicle Routing Problem: An Overview of Exact and Approximate Algorithms". *European Journal of Operational Research* 59(3):345–358.

Laporte, G., F. Louveaux, and H. Mercure. 1992. "The Vehicle Routing Problem with Stochastic Travel Times". *Transportation Science* 26(3):161–170.

Lucas, T. W., W. D. Kelton, P. J. Sanchez, S. M. Sanchez, and B. L. Anderson. 2015. "Changing the Paradigm: Simulation, Now a Method of First Resort". *Naval Research Logistics (NRL)* 62(4):293–303.

Marinakis, Y., G.-R. Iordanidou, and M. Marinaki. 2013. "Particle Swarm Optimization for the Vehicle Routing Problem with Stochastic Demands". *Applied Soft Computing* 13(4):1693–1704.

Resendel, M. G., and C. C. Ribeiro. 2005. "GRASP with Path-relinking: Recent Advances and Applications". In *Metaheuristics: Progress as Real Problem Solvers*, edited by T. Ibaraki et al., 29–63. New York, NY, USA: Springer.

Taş, D., M. Gendreau, N. Dellaert, T. Van Woensel, and A. De Kok. 2014. "Vehicle Routing with Soft Time Windows and Stochastic Travel Times: A Column Generation and Branch-and-price Solution Approach". *European Journal of Operational Research* 236(3):789–799.

Toth, P., and D. Vigo. 2003. "The Granular Tabu Search and its Application to the Vehicle-routing Problem". *Informs Journal on computing* 15(4):333–346.

Toth, P., and D. Vigo. 2014. *Vehicle Routing: Problems, Methods and Applications*. Philadelphia, PA: Society for Industrial and Applied Mathematics.

AUTHOR BIOGRAPHIES

PAOLA FESTA obtained her Ph.D. degree in Operations Research in 2000 and is currently Associate Professor in Operations Research at the University of Napoli FEDERICO II (UNFII). She is in the scientific committee of the Ph.D. Programs in Operations Research (University of Calabria) and in Mathematics & Computer Science (UNFII). Since 1999, she has been frequently and regularly Research Scholar at several national and international research institutions, including CNRs, MIT Lab. for Information and Decision Systems (USA), AT&T Labs Research (USA), and Department of Industrial and Systems Engineering of the University of Florida (USA). She is Associate Editor of several international journals. She has been chair or member of the scientific committee of more than 50 international conferences and workshops. She is author/co-author of more than 100 publications. Her website address is http://www.dma.unina.it/festa

and her email address is paola.festa@unina.it.

DANIELE FERONE is a post-doctoral researcher in the ICSO group at the IN3 – Universitat Oberta de Catalunya. His main research interests include network optimization, constrained shortest path problems, and logistics optimization. His email address is danieleferone@gmail.com.

TOMMASO PASTORE is a Ph.D candidate in Operations Research at the University of Naples FEDERICO II, Italy. He holds a Master Degree in Mathematics. His main research interests include network optimization, graph drawing, and boolean learning problems. His email address is tommaso.pastore@unina.it.

ANGEL A. JUAN is Professor of Operations Research & Industrial Engineering in the Computer Science Dept. at the Universitat Oberta de Catalunya (Spain). He is also the coordinator of the ICSO research group at the IN3. Dr. Juan holds a Ph.D. in Industrial Engineering and an M.S. in Mathematics. He completed a predoctoral internship at Harvard University and postdoctoral internships at Massachusetts Institute of Technology and Georgia Institute of Technology. His main research interests include applied optimization and simulation (metaheuristics and simheuristics) in computational transportation & logistics, Internet computing, and computational finance. He has published more than 65 articles in JCR-indexed journals and more than 170 documents indexed in Scopus. His website address is http://ajuanp.wordpress.com and his email address is ajuanp@uoc.edu.

CHRISTOPHER BAYLISS is a post-doctoral researcher in the ICSO group at the IN3 – Universitat Oberta de Catalunya. His main research interests include metaheuristics, simulation optimization, revenue management, packing problems, airline scheduling, and logistics optimization. His email address is cbayliss@uoc.edu.