# MULTI-SENSOR FAULTDETECTION AND ISOLATION FOR AERO-ENGINE DCS WITH MARKOV TIME DELAY BASED ON H∞UNKNOWN INPUT OBSERVERS

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# ABSTRACT

In this paper, a novel approach is proposed to design a bank of unknown input observers(UIOs) for aeroengine distribute control system(DCS) with multi-sensor fault. First, considering the system with multisensor fault which doesn't satisfy the observer matching condition, a  $H_{\infty}$  UIO is proposed based on the disturbance decoupling principle of unknown input observer(UIO) and  $H_{\infty}$  control theory. The parameter matrixes of UIO are solved in terms of linear matrix inequality(LMI) toolbox. Second, the fault detection and isolation scheme is proposed for DCS with multi-sensor fault and the UIOs are designed for a bank of fault detection and isolation sets which are obtained by reorganizing the control system. The residual signal generated by each observer is robust to the sensor fault in the matching set, and sensitive to the sensor fault out of the matching set. Finally, the residual signals were simulated and the results show the effectiveness of the proposed method.

# **1 INTRODUCTION**

In the aero-engine distribute control system(DCS), the number of intelligent devices such as microprocessors, data transceivers, and data bus interfaces is large (Alireza Behbahani 2010), and engines' working environment is extremely harsh, which makes component failures inevitable. These failures can block data transmission or lead to transmission errors, which affect the dynamic and static performance of the system. If the failures cannot be found and ruled out, they may result in huge loss of property, or even lead to catastrophic consequences (Litt et al. 2005; Xu-sheng et al. 2005).

Statistics indicates that the sensor and actuator faults account for more than 80% of the total number of aircraft engine control system faults (Wang et al. 2011). A series of fruitful researches have been carried out on the aero-engine sensor and actuator fault diagnosis, fault detection (Chen 2014), fault reconstruction (Xu et al. 2012; Rui et al. 2011), fault-tolerant control (Jiang and Chowdhury 2005) and other aspects. On the aspect offault detection and isolation(FDI), one of the widely studied model-based FDI methodologies is to use observers. For adaptive observer-based FDI, parameter fault detection and estimation for nonlinear systems based on an adaptive observer technique is proposed in Zhang et al. (2010); Omar et al. (2015). In Xuemei Zheng et al. (2015), an adaptive backstepping-based non-singular terminal sliding mode(NTSM) control method is proposed for a class of uncertain nonlinear systems in the parameteric-strict feedback form. Moreover, the work in Tzu-Sung et al. (2015) addressed the problem of designing robust observer-based adaptive fuzzy tracking control scheme for MIMO nonlinear systems with plant uncertainties, time delayed uncertainties, and external disturbances. For sliding mode observers, Edwards et al. (2000) considered the application of a particular sliding mode observer to FDI design, and this was extended in

Rios et al. (2015) where fault detection and isolation problems were studied for a certain class of nonlinear systems. For  $H_{\infty}$  observer-based FDI, Mattei et al. (2005) used a bank of  $H_{\infty}$  observers to detect and isolate sensor faults. For descriptor observer-based FDI, Hamdi et al. (2011) considered fault detection, isolation, and reconstruction for linear parameter varying descriptor systems.

One of the most promising approaches for designing robust fault detection observers (FDOs) is to use the unknown input decoupling principle. Considerable attention has been paid to the design of unknown input observers (UIOs) in the field of FDI. As described in Frank and Wunnenberg (1989); Hou and Muller (1994), FDOs based on UIO theory were designed to be insensitive to unknown inputs while sensitive to the faults of interest. In JafarZarei and Ehsan (2014); Saeed Ahmadizadeh et al. (2014), a new method is proposed to design a Nonlinear Unknown Input Observer for robust sensor fault detection. The necessary and sufficient condition to construct a UIO or an FDO is that the invariant zeros of the system must lie in the open left hand complex plane, and the observer matching condition (the rank condition between an output matrix and an unknown input matrix) is satisfied (Shenghui Guo et al. 2015; Fridman et al. 2007). The aforementioned works for the design of UIOs or FDOs are all accomplished under the circumstances that the system meets the two conditions. However, the observer matching condition is sometimes too restrictive for the design of UIOs or FDOs for practical systems because many physical systems do not satisfy the observer matching condition, especially for the system with multiple faults occurred. And the literature review showed that previous research has evaluated the capability of FDI method to handle single fault occurred in system, but generally has not investigated the ability of the FDI method to deal with multiple simultaneous sensors fault. Besides, for the aero-engine DCS, the signal is transmitted in the network, so when designing FDO, the influence of Markov delay should betaken into account. Therefore, this paper Consider the aero-engine DCS with Markov delay and multi-sensor fault which doesn't satisfy the observer matching condition, a  $H_{\infty}$  unknown input observer-based fault diagnosis scheme is proposed. The structure of the paper is as follows. In Section 2, the aero-engine DCS model is described, and time delay is modeled as finite state Markov random. In Section 3, the  $H_{\infty}$  UIO-based fault diagnosis scheme is proposed, and the parameter matrixes of UIO are solved by linear matrix inequality(LMI) toolbox. In Section 4, the FDI scheme for system with multi-sensor fault occurred is proposed. Finally in Section 5, some numerical simulation examples are given to illustrate the effectiveness of proposed method, and some conclusions are summarized in Section 6.

## 2 AERO-ENGINE DCS DESCRIPTION

The aero-engine DCS is shown in Figure 1, where  $\tau^i(sc)$  is the i<sup>th</sup> sensor-controller delay,  $\tau^i(ca)$  is the i<sup>th</sup> controller-actuator delay.



Figure 1: Structure of aero-engine DCS.

For the aero-engine DCS, the following assumptions are given:

1. The sensors and actuators are time-driven, while the central controller is event-driven;

- Data has timestamp and is transmitted in single-packet, and wrong order of data packet does not exist;
- Data packet dropout is considered. Data obtained in the previous sampling point is followed when data packet dropout exists;
- 4. Both controller-actuator delay and sensor-controller delay have upper bound.

The output of all sensors has the same priority when signal is transmitted in the network. Therefore, all the sensor-controller time delay can be approximately regarded as the same, which is expressed as  $\tau(sc)$ . In the same way, all the controller-actuator time delay can be approximately regarded as the same, which is expressed as  $\tau(ca)$ . According to Qixin and Shousong (2003), based on Assumption 1 and Assumption 3, sensor-controller delay and controller-actuator delay can be combined as  $\tau(t)$ . The system model can be described as follows:

$$\begin{cases} \mathbf{x}(t) = A\mathbf{x}(t) + A_d \mathbf{x}(t - \tau(t)) + B\mathbf{u}(t) + E\mathbf{w}(t) \\ \mathbf{y}(t) = C\mathbf{x}(t) \end{cases}$$
(1)

where  $\mathbf{x}(t) = \begin{bmatrix} \mathbf{n}_{L_T} & \mathbf{n}_H & \mathbf{p}_{31} \end{bmatrix}^T \in \mathbf{R}^n$  is state vector,  $\mathbf{u}(t) = \begin{bmatrix} \mathbf{m}_f & \mathbf{A}_8 \end{bmatrix}^T \in \mathbf{R}^m$  is control input vector,  $\mathbf{y}(t) = \begin{bmatrix} \mathbf{n}_L & \mathbf{n}_H & \pi_T \end{bmatrix}^T \in \mathbf{R}^n$  is output vector,  $\mathbf{n}_L$  is the low pressure rotor speed,  $\mathbf{n}_H$  is high pressure compressor rotor speed,  $\mathbf{p}_{31}$  is compressor outlet total pressure,  $\mathbf{m}_f$  is the main fuel mass flow,  $A_8$  is the throat area,  $\pi_T$  is the turbine expansion ratio,  $\mathbf{w}(t)$  is external disturbance, matrices  $\mathbf{A} \in \mathbf{R}^{n \times n}$ ,  $\mathbf{A}_d \in \mathbf{R}^{n \times n}$ ,  $\mathbf{B} \in \mathbf{R}^{n \times m}$ ,  $\mathbf{C} \in \mathbf{R}^{n \times n}$  and  $\mathbf{E} \in \mathbf{R}^{n \times d}$  are matrices with appropriate dimensions. Time delay and data packet dropout are both related to the bus load in a certain period, which means they don't change randomly. So, time delay  $\{\tau(t) \mid 0 < \tau(t) \le \tau_m(t) < \infty\}$  can be modeled as finite state Markov random process. The state space of  $\tau(t)$  is  $\mathbf{\Omega} = \{1, 2\}$ , and the time delay state transition matrix of  $\tau(t)$  is  $\mathbf{\Pi} = \pi(i, j), (i, j \in \mathbf{\Omega}), \pi(i, j) = P\{\mathbf{x}(t) = j \mid \tau(t) = i\} \ge 0, \pi_{ij} = 0(if j > i + 1)$ .

### 3 THE $H_{\infty}$ UNKNOWN INPUT OBSERVER DESIGN

The FDO design problem for the DCS (1) is to construct an observer that is sensitive to the sensor fault, while insensitive to the unknown inputs w(t). When rank(CE)=rank(E), we say that the system (1) meets the observer matching condition. As mentioned before, many methods were suggested to design the UIO when the observer matching condition is met. Recently, some methods (Fu 2015; Kee-Sang Lee 2015; Kalsi et al. 2009; Loza et al. 2009) were proposed to circumvent the observer matching condition. In these methods, the observer is designed based on the estimation of the auxiliary outputs. In this paper, the  $H_{\infty}$  UIO-based fault diagnosis scheme for system with multiple fault occurred when systems do not meet the observer matching condition is proposed, unlike the works of Fu (2015); Kee-Sang Lee (2015) ; Kalsi et al. (2009); Loza et al. (2009) that need to estimate the auxiliary outputs.

In order to isolate the multiple possible fault, it is necessary to reorganize the system. Define the sets as follows:

$$\boldsymbol{A}_{\alpha_{g}} = \begin{bmatrix} a_{\alpha_{1}} \boldsymbol{L} & a_{\alpha_{g}} \end{bmatrix}, \ \boldsymbol{A}_{d\alpha_{g}} = \begin{bmatrix} a_{d\alpha_{1}} \boldsymbol{L} & a_{d\alpha_{g}} \end{bmatrix}, \ \boldsymbol{B}_{\alpha_{g}} = \begin{bmatrix} b_{\alpha_{1}} \boldsymbol{L} & b_{\alpha_{g}} \end{bmatrix}, \\ \boldsymbol{C}_{\alpha_{g}} = \begin{bmatrix} c_{\alpha_{1}} \boldsymbol{L} & c_{\alpha_{g}} \end{bmatrix}, \ \boldsymbol{E}_{\alpha_{g}} = \begin{bmatrix} e_{\alpha_{1}} \boldsymbol{L} & e_{\alpha_{g}} \end{bmatrix}, \ \boldsymbol{x}_{\alpha_{g}} = \begin{bmatrix} \boldsymbol{x}_{\alpha_{1}} \boldsymbol{L} & \boldsymbol{x}_{\alpha_{g}} \end{bmatrix}^{T}$$

The multi-sensor fault detection and isolation scheme is shown in Figure 2.

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Figure 2: The multi-sensor fault detection and isolation scheme.

Then the DCS (1)can be described as:

$$\begin{cases} \mathbf{x}_{\alpha_g}^{\mathbf{x}}(t) = \overline{A}_{\alpha_g} \, \mathbf{x}_{\alpha_g}(t) + \overline{A}_{d\alpha_g} \, \mathbf{x}_{\alpha_g}(t - \tau(t)) + \overline{B}_{\alpha_g} \, \mathbf{u}(t) + \overline{E}_{\alpha_g} \, \mathbf{w}(t) \\ \mathbf{y}(t) = \mathbf{C}_{\alpha_g} \, \mathbf{x}_{\alpha_g}(t) + \overline{\mathbf{C}}_{\alpha_g} \, \mathbf{x}_{\alpha_g}(t) \end{cases}$$
(2)

For the system (2), an UIO can be constructed to be insensitive to unknown inputs, while being sensitive to the faults of interest. The design of full order UIO for a class of time delay systems can be represented as follows:

$$\begin{cases} \boldsymbol{\xi}(t) = \boldsymbol{F}_{k}\boldsymbol{\xi}(t) + \boldsymbol{F}_{dk}\boldsymbol{\xi}(t-\tau(t)) + \boldsymbol{D}_{k}\boldsymbol{y}(t) + \boldsymbol{D}_{dk}\boldsymbol{y}(t-\tau(t)) + \boldsymbol{G}_{k}\boldsymbol{u}(t) \\ \boldsymbol{x}_{a_{g}}(t) = \boldsymbol{\xi}(t) + \boldsymbol{H}_{k}\boldsymbol{y}(t) \end{cases}$$
(3)

From Equations (2) and (3), define  $e(t) = \bar{x}(t) - \frac{1}{\bar{x}}(t)$ , we can obtain the error system is

$$\boldsymbol{\mathscr{E}}(t) = \boldsymbol{F}_{k}\boldsymbol{e}(t) + \boldsymbol{F}_{dk}\boldsymbol{e}(t-\tau(t)) + \left(\boldsymbol{\overline{A}}_{d\alpha_{g}} - \boldsymbol{F}_{dk} - \boldsymbol{L}_{1k}\boldsymbol{\overline{C}}_{\alpha_{g}}\boldsymbol{\overline{A}}_{d\alpha_{g}} - \boldsymbol{L}_{3k}\boldsymbol{\overline{C}}_{\alpha_{g}}\right)\boldsymbol{\overline{x}}_{\alpha_{g}}\left(t-\tau(t)\right) \\ + \left(\boldsymbol{\overline{B}}_{\alpha_{g}} - \boldsymbol{G}_{k} - \boldsymbol{L}_{1k}\boldsymbol{C}_{\alpha_{g}}\boldsymbol{\overline{B}}_{\alpha_{g}} - \boldsymbol{L}_{1k}\boldsymbol{\overline{C}}_{\alpha_{g}}\boldsymbol{\overline{B}}_{\alpha_{g}}\right)\boldsymbol{u}(t) + \boldsymbol{\phi}_{k}\boldsymbol{\eta}(t) + \left(\boldsymbol{\overline{A}}_{\alpha_{g}} - \boldsymbol{F}_{k} - \boldsymbol{L}_{1k}\boldsymbol{\overline{C}}_{\alpha_{g}}\boldsymbol{\overline{A}}_{\alpha_{g}} - \boldsymbol{L}_{2k}\boldsymbol{\overline{C}}_{\alpha_{g}}\right)\boldsymbol{\overline{x}}_{\alpha_{g}}(t)$$

$$(4)$$

where  $\phi_k = \begin{bmatrix} \phi_{1k} & \phi_{2k} & \phi_{3k} \end{bmatrix}$ ,  $\eta(t) = \begin{bmatrix} w(t) & x_{\alpha_g}(t) & x_{d\alpha_g}(t-\tau(t)) \end{bmatrix}^T$ ,  $\phi_{1k} = \overline{E}_{\alpha_g} - L_{1k}C_{\alpha_g}E_{\alpha_g} - L_{1k}\overline{C}_{\alpha_g}\overline{E}_{\alpha_g}$ ,  $\phi_{2k} = -L_{1k}C_{\alpha_g}A_{\alpha_g} - L_{2k}C_{\alpha_g}\phi_{3k} = -L_{3k}C_{\alpha_g} - L_{1k}C_{\alpha_g}A_{d\alpha_g}$ .

Lemma 1(Fu 2004) Observer in the form of (3) is an UIO for system (2) if and only if the following relations are satisfied

$$\boldsymbol{\mathscr{E}}(t) = \boldsymbol{F}\boldsymbol{e}(t) + \boldsymbol{F}_{d}\boldsymbol{e}(t-\tau(t)).$$
$$\boldsymbol{\bar{A}}_{\alpha_{g}} - \boldsymbol{L}_{1k}\boldsymbol{\bar{C}}_{\alpha_{g}}\boldsymbol{\bar{A}}_{\alpha_{g}} - \boldsymbol{L}_{2k}\boldsymbol{\bar{C}}_{\alpha_{g}} = \boldsymbol{F}_{k}.$$
$$\boldsymbol{\bar{A}}_{d\alpha_{g}} - \boldsymbol{L}_{1k}\boldsymbol{\bar{C}}_{\alpha_{g}}\boldsymbol{\bar{A}}_{d\alpha_{g}} - \boldsymbol{L}_{3k}\boldsymbol{\bar{C}}_{\alpha_{g}} = \boldsymbol{F}_{dk}.$$
$$\boldsymbol{\bar{B}}_{\alpha_{g}} - \boldsymbol{L}_{1k}\boldsymbol{C}_{\alpha_{g}}\boldsymbol{B}_{\alpha_{g}} - \boldsymbol{L}_{1k}\boldsymbol{\bar{C}}_{\alpha_{g}}\boldsymbol{\bar{B}}_{\alpha_{g}} = \boldsymbol{G}_{k} \boldsymbol{\phi}_{k} = 0$$

and the error system  $\mathscr{E}(t)$  is asymptotically stable.

Based on Lemma 1, we can obtain the following error equation.

$$\boldsymbol{\mathscr{E}}(t) = \boldsymbol{F}_{k}\boldsymbol{e}(t) + \boldsymbol{F}_{dk}\boldsymbol{e}(t-\tau(t)) + \boldsymbol{\phi}_{k}\boldsymbol{\eta}(t) \Box.$$
(5)

Obviously, the new unknown input  $\eta(t)$  is composed by the system state value with time delay  $\tau(t)$  and the unknown input w(t), and the effect of mismatched unknown inputs  $\eta(t)$  cannot be algebraically removed. Therefore, it should be treated in some other way so that the UIOs provide the correct state estimates with regard of the unknown inputs. In this study, the UIOs for the time delay system is designed to minimize the negative impact of  $\phi_k \eta(t)$ . The problem of design unknown input observer is described as follows.

**Problem**  $H_{\infty}$  **UIO**: Given system(2) and the observer (4), designing the observer parameters  $F_k$ ,  $F_{dk}$ ,  $G_k$ ,  $L_{1k}$ ,  $L_{2k}$  and  $L_{3k}$  such that the following properties are satisfied: (1)  $\lim e(t) \to 0$  for  $\eta(t) = 0$ ; (2)  $||T|| \leq \gamma$  for  $\eta(t) \neq 0$ , where  $\gamma > 0$  is the unknown input attenuation level. To solve this problem, the theorem is proposed as follow:

**Theorem 1.** Consider the system (2), if there exists  $P_k > 0$ ,  $Q_k > 0$ ,  $R_k > 0$ , and  $\gamma > 0$  satisfying the following the optimization conditions (6)

$$\min \gamma$$

$$\min \gamma$$

$$\int_{s.t.} \begin{bmatrix} \boldsymbol{\Theta}_{11} & \boldsymbol{\Theta}_{12} & \boldsymbol{P}_{k} \boldsymbol{\phi}_{k} \\ \boldsymbol{\Theta}_{12}^{T} & -(1 - \tau_{d}(t)) \boldsymbol{Q}_{k} & \boldsymbol{0} \\ \boldsymbol{\phi}_{k}^{T} \boldsymbol{P}_{k} & \boldsymbol{0} & -\gamma^{2} \boldsymbol{I} \end{bmatrix} < 0;$$

$$\sum_{j=1}^{s} \pi_{ij} \boldsymbol{Q}_{j} - (1 - \tau_{d}(t)) \boldsymbol{R}_{k} \leq 0$$

$$\boldsymbol{P}_{k} > 0, \boldsymbol{Q}_{k} > 0, \boldsymbol{R}_{k} > 0, \gamma > 0$$
(6)

Where

$$\boldsymbol{\Theta}_{11} = \bar{\boldsymbol{A}}_{\alpha_g}{}^T \boldsymbol{P}_k - \bar{\boldsymbol{A}}_{\alpha_g}{}^T \bar{\boldsymbol{C}}_{\alpha_g}{}^T \boldsymbol{W}_k{}^T - \bar{\boldsymbol{C}}_{\alpha_g}{}^T \boldsymbol{Y}_k{}^T + \boldsymbol{P}_k \bar{\boldsymbol{A}}_{\alpha_g} - \boldsymbol{W}_k \bar{\boldsymbol{C}}_{\alpha_g} \bar{\boldsymbol{A}}_{\alpha_g} - \boldsymbol{Y}_k \bar{\boldsymbol{C}}_{\alpha_g} + \boldsymbol{\tau}_m(t) \boldsymbol{R}_k + \boldsymbol{Q}_k + \sum_{j=1}^s \boldsymbol{\pi}_{ij} \boldsymbol{P}_j + \boldsymbol{I}$$
$$\boldsymbol{\Theta}_{12} = \boldsymbol{P}_k \bar{\boldsymbol{A}}_{d\alpha_g} - \boldsymbol{W}_k \bar{\boldsymbol{C}}_{\alpha_g} \bar{\boldsymbol{A}}_{d\alpha_g} - \boldsymbol{Z}_k \bar{\boldsymbol{C}}_{\alpha_g} \,.$$

Then we can conclude that: (a) The observer(4) is an asymptotically stable estimator of the system(2), (b)  $||T|| \le \gamma$ , and the observer gain parameters are given by

$$\boldsymbol{L}_{1k} = \boldsymbol{P}_k^{-1} \boldsymbol{W}_k, \, \boldsymbol{L}_{2k} = \boldsymbol{P}_k^{-1} \boldsymbol{Y}_k, \, \boldsymbol{L}_{3k} = \boldsymbol{P}_k^{-1} \boldsymbol{Z}_k.$$

## 4 MULTI SENSOR FAULT DETECTION AND ISOLATION

There are many approaches to choose a decision-making function. A suitable approach to determine the threshold  $T_R$  is to apply the statistical properties of residual signal R(t). Here threshold test is written as follows (Kee-Sang 2015):

$$\begin{cases} \overline{\mathbf{R}} - \lambda \sigma_r \leq \mathbf{R} \leq \overline{\mathbf{R}} + \lambda \sigma_r & \text{faulty free} \\ R < \overline{\mathbf{R}} - \lambda \sigma_r & \text{or } \mathbf{R} > \overline{\mathbf{R}} + \lambda \sigma_r & \text{faulty} \end{cases}$$
(7)

The tolerance parameter  $\lambda$ , normally  $\lambda > 3$ , is selected to have a good trade-off between the maximization of fault detection probability and minimization of false alarm occurrence.

**Remark 1.**In order to isolate the multiple possible faults, several residuals with different fault sensitivities should be designed using a bank of observers. Here, the FDI observers is designed to be only sensitive to the sensor fault out of the matching set  $\Omega^i$ , and insensitive to the sensor fault in the matching set.

**Remark 2.** A fault isolation index S(FDI) is introduced, which is defined as the maximum number of faults that can be isolated. It could be easily obtained that  $0 \le S(FDI) \le m-1$ . If the number of sensor faults is k, then there are  $l_{num} = C_{m-k}^{S_{FDI}-k}$  residual signals which sensitive to the sensor fault. Define a set  $\Omega^{i} = [\alpha_{1}^{i}, L, \alpha_{S_{FDI}}^{i}], i = 1, L, l_{num}$ , and

$$\Omega_F = \prod_{I=1}^{l_{num}} \Omega^i ,$$

so the number of elements in the set  $\Omega_F$  is the number of sensor faults, and each element is corresponding to a certain sensor fault.

## 5 NUMERICAL SIMULATIONS

In order to investigate the effectiveness of the proposed algorithm, several cases are considered. Consider the system with multi-sensor fault occurred which is defined by (8) with the following matrices:

$$\begin{cases} \mathbf{\hat{x}}_{a_g}(t) = \overline{A}_{a_g} \, \overline{\mathbf{x}}_{a_g}(t) + \overline{A}_{da_g} \, \overline{\mathbf{x}}_{a_g}(t - \tau(t)) + \overline{B}_{a_g} \, \mathbf{u}(t) + \overline{E}_{a_g} \, \mathbf{w}(t) \\ \mathbf{y}(t) = \mathbf{C}_{a_g} \, \mathbf{x}_{a_g}(t) + \overline{\mathbf{C}}_{a_g} \, \overline{\mathbf{x}}_{a_g}(t) + \mathbf{F}_s f(t) \end{cases}$$
(8)

where f(t) is the sensor fault,  $F_s$  is fault distribution matrix. And

$$\boldsymbol{A} = \begin{bmatrix} -3.8 & 1.5 & -0.5 \\ 0.5 & -3 & 1 \\ -0.3 & 0.7 & -2.4 \end{bmatrix}, \ \boldsymbol{A}_{d} = \begin{bmatrix} 0.4 & 0.1 & -0.2 \\ 0.1 & -0.8 & 0.2 \\ 0.7 & -0.1 & 0.5 \end{bmatrix}, \ \boldsymbol{B} = \begin{bmatrix} 0.1 & 0.3 \\ 0.2 & 0.1 \\ -0.4 & -0.1 \end{bmatrix}^{T}, \ \boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ \boldsymbol{E} = \begin{bmatrix} -0.4 \\ 0.1 \\ -0.3 \end{bmatrix}.$$

The results presents in Theorem 1 are used to design observers with different time delay k, after solving the optimization problem under LMI constraints. The parameter matrixes of UIO are solved in terms of linear matrix inequality(LMI) toolbox. The LMI toolbox is a high performance for solving general LMI problems. It blends simple tools for the specification and manipulation of LMI problems. The high performance is achieved through C-MEX implementation and by taking advantage of the particular structure of each LMI. The gains of observers are obtained by

$$\boldsymbol{L}_{1k} = \boldsymbol{P}_{k}^{-1} \boldsymbol{W}_{k}, \, \boldsymbol{L}_{2k} = \boldsymbol{P}_{k}^{-1} \boldsymbol{Y}_{k}, \, \boldsymbol{L}_{3k} = \boldsymbol{P}_{k}^{-1} \boldsymbol{Z}_{k}.$$
(9)

Then we can obtained

$$\boldsymbol{L}_{11} = \begin{bmatrix} 0.5321 & 0.1235 & 0.8974 \\ -0.2504 & 0.6840 & -0.2581 \\ 0.7894 & -0.1560 & 0.2648 \end{bmatrix}, \ \boldsymbol{L}_{12} = \begin{bmatrix} 0.5014 & 0.1147 & 0.8560 \\ -0.2023 & 0.6976 & -0.2147 \\ 0.7456 & -0.1170 & 0.2247 \end{bmatrix}, \ \boldsymbol{L}_{21} = \begin{bmatrix} 1.2243 & 0.9446 & -0.2545 \\ 0.2341 & 0.4140 & 1.4870 \\ 0.1789 & -1.5044 & 0.1548 \end{bmatrix}, \ \boldsymbol{L}_{22} = \begin{bmatrix} 1.0145 & 0.7457 & -0.2107 \\ 0.2021 & 0.4470 & 1.2477 \\ 0.1120 & -1.5474 & 0.1145 \end{bmatrix}, \ \boldsymbol{L}_{31} = \begin{bmatrix} 0.1412 & -0.4312 & 0.1252 \\ -0.3465 & 0.1231 & 0.7813 \\ 0.2841 & 0.6131 & -0.7832 \end{bmatrix}, \ \boldsymbol{L}_{32} = \begin{bmatrix} 0.1120 & -0.4012 & 0.1042 \\ -0.3060 & 0.1231 & 0.7143 \\ 0.2074 & 0.6021 & -0.7571 \end{bmatrix}.$$

The optimized parameter is  $\gamma_1 = 0.5016$ ,  $\gamma_2 = 0.5634$ . The transition matrix of time delay is defined as:

$$\boldsymbol{\Pi} = \begin{bmatrix} 0.3 & 0.7\\ 0.6 & 0.4 \end{bmatrix}. \tag{10}$$

and the time delay distribution is shown in Figure 3.



Figure 3: Distribution of time delay  $\tau$ .

By the proposed algorithm, we can isolate two sensors faults occurred in system. That is S(FDI) = 2. Then design three UIOs for sets  $\Omega^1 = \{1,2\}, \Omega^2 = \{1,3\}$  and  $\Omega^3 = \{2,3\}$ , respectively. To show the effectiveness of the designed FDI system, three types of fault were exerted on the system.

Case 1: In the first case, an abrupt fault occurred in the first and the third sensor from 40s to 60s, that is

$$\boldsymbol{F} = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}^T, \ \boldsymbol{f}(t) = \begin{cases} 1 & 40 \le t \le 60\\ 0 & otherwise \end{cases}.$$
 (11)

the residual signals  $r\{1,2\}$ ,  $r\{1,3\}$  and  $r\{2,3\}$  are shown in Figure 4 to 6. The threshold  $T_R$  is the red line which is obtained by (Kee-Sang Lee 2015). It can be seen that the residual signals  $r\{1,2\}$  and  $r\{2,3\}$  change when faults occurs. However, the residual signals  $r\{1,3\}$  show no sensitivity to the faults. So according to the proposed algorithm, it can be concluded that the faults occurred in the first and the third sensor.





Figure 6: The residual signals  $r\{2,3\}$ .

Case 2: In the second case, an incipient fault occurred in the first and the second sensor. That is

$$\boldsymbol{F} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}^{T}, \ f(t) = \begin{cases} 1 - 0.1 \times |t - 50| & 40 \le t \le 60 \\ 0 & otherwise \end{cases}.$$
 (12)

The residual signals are depicted in Figure 7, Figure 8 and 9, respectively. It can be seen that the residual signals  $r\{1,3\}$  and  $r\{2,3\}$  show enough sensitivity to the occurrence of the fault. However, the residual signals  $r\{1,2\}$  show no sensitivity to the faults. Then we can obtain the set  $\Omega_F = \{1,2\}$ , so the fault occurred in the first sensor and the second sensor by the proposed algorithm.



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Figure 9: The residual signal  $r\{1,2\}$ .

Case 3: The developed approach can also be used to design the observer for the single-sensor-fault case. For the case where an abrupt fault occurs in the third sensor from 40s to 60s,

$$F = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^{T}, \ f(t) = \begin{cases} 1 - 0.1 \times |t - 50| & 40 \le t \le 60\\ 0 & otherwise \end{cases}.$$
 (13)

The residual signals  $r\{1,2\}$ ,  $r\{1,3\}$  and  $r\{2,3\}$  are shown in Figure 10 to 12. It can be seen that only the residual signals  $r\{1,2\}$  change when the faults occurs. Then we can obtain the set  $\Omega_F = \{1,3\} I \{2,3\} = \{3\}$ , so the faults occurred in the third sensor.



Figure 12: The residual signal  $r\{1,2\}$ .

Through the simulation analysis of the above three cases, we can see that the FDI method proposed in this paper not only has the ability of identifying multi-sensor faults, but also can isolate different faults despite the presence of Markov time delay and external disturbance. And the method can also be applies to single sensor fault detection and isolation. However, this method also has limitations, i.e., when fault occurs in all sensors, fault detection and isolation cannot be realized, but such situation rarely occurs.

## 6 CONCLUSION

In this paper, a novel approach is developed to design fault detection system for aero-engine DCS. The main contribution of paper is the design of fault detection and isolation system for the case where multisensor fault occurred in DCS using a bank of  $H_{\infty}$  unknown input observers. This is achieved by developing a bank of fault detection and isolation sets which are obtained by reorganizing the controllers. Based on this,  $H_{\infty}$  unknown input observers for each set are designed, and the residual generated by each observer is robust to the sensor fault in the matching set, and sensitive to the sensor fault out of the matching set. In order to verify the performance of the proposed method, a numerical example is investigated. The simulation results confirm the robustness and effectiveness of the proposed scheme for the system with multi-sensor fault. In the future work, this approach can be applied on fault estimation and fault tolerant control strategies for aero-engine DCS.

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