COLLECTIVE PROBLEM-SOLVING IN EVOLVING NETWORKS: AN AGENT-BASED MODEL

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ABSTRACT

Research works in collective problem-solving usually assume fixed communication structures and explore effects thereof. In contrast, in real settings, individuals may modify their set of connections in the search of information and feasible solutions. This paper illustrates how groups collectively search for solutions in a space under the presence of dynamic structures and individual-level learning. For that, we built an agent-based computational model. In our model, individuals (i) simultaneously conduct search of solutions over a complex space (i.e. a NK landscape), (ii) are initially connected to each other according to a given network configuration, (iii) are endowed with learning capabilities (through a reinforcement learning algorithm), and (iv) update (i.e. create or sever) their links to other agents according to such learning features. Results reveal conditions under which performance differences are obtained, considering variations in the number of agents, space complexity, agents’ screening capabilities and reinforcement learning.

1 INTRODUCTION

Teams and organizations solve problems whose complexity goes beyond the cognitive capabilities of individuals. Thus, collective problem-solving and decision-making has been long of interest to researchers, both in empirical and theoretical contexts. The success of group performance is contingent upon a variety of factors such as information sharing (Shore et al. 2015), coordination (Knudsen and Srikanth 2014), imitation (Chang and Harrington Jr 2007), communication structure (Lazer and Friedman 2007), individual-level diversity (Estévez-Mujica et al. 2018), and trust (Hagemann and Kluge 2017), among others.

Particular research efforts have been put toward understanding the effect of network structures (e.g. Fang et al. 2010). Most models assume that problem solvers interact according to a given task structure. However, that assumption might not hold particularly in contexts where interactions are dependent on both formal and informal structures (Clement and Puranam 2017). Informal interactions might emerge aside from the formal reporting lines constrained by a given organizational structure. Generally speaking, individuals in organizations seek to identify not only the right actions to take in their tasks, but also with whom to collaborate on these tasks (Clement and Puranam 2017). We refer to this situation as “the search for structure” (i.e. the discovery of valuable patterns of interaction by the members of an organization). The search for structure involves a distinct set of issues.

Learning is also an important feature of individuals in problem-solving settings. Being long known that the “scope of learning” (e.g., from individual to social) decisively affects system-level outcomes (Vriend 2000), extant models have also been developed to investigate how learning mediates the effect of communication networks on team performance (Barkoczi and Galesic 2016). For instance, the results of Barkoczi and Galesic (2016) highlight the importance of individual-level learning strategies. In particular, they found that well-connected networks outperform poorly-connected ones when individuals rely on
copying the most popular solution among their connections. Nonetheless, poorly-connected structures perform best when individuals copy solutions according to the highest observed payoff.

Our paper makes two critical contributions to the literature of collective search models (e.g. Lazer and Friedman 2007) as follows. On the one hand, we relax the assumption that the communication network—over which agents share their solutions—remains unchanged and stable. The evolving network assumption has been explored in the literature in other contexts (Jackson and Watts 2002; Kossinets and Watts 2006; Hanaki et al. 2007; Chang and Harrington Jr 2007). Here, we aim to understand how evolution of the communication network among problem-solvers affects their collective problem-solving capacity. We strictly focus on the implications on group problem-solving under the consideration of (i) changing structures, (ii) limited individual-level capabilities in regard to solution creation and assessment, and (iii) individual-level learning features. We propose an agent-based computational model in which agents perform searches over a solution space, and create or sever links to other agents according to the assessment of valuable interactions. The paper is organized as follows: Section 2 reviews relevant literature to this paper, Section 3 explains the model, Section 4 presents the analysis of results and Section 5 concludes the paper by highlighting the main contributions.

2 LITERATURE REVIEW

2.1 Network Approaches to Collective Problem-Solving

Computational models in the extant literature have explored the implications of network structure in collective problem solving, revealing counterintuitive effects of connectivity. For instance, it has been shown that well-connected networks are more efficient in the short run, but outcompeted by poorly-connected ones in the long run (Lazer and Friedman 2007). However, empirical results have revealed contrasting results (Mason and Watts 2012). In any case, related results reveals how structure can be decisive in the emergence of collective-level exploratory and exploitative adaptive strategies of the organization (March 1991; Uotila 2017). In the same vein, Fang et al. (2010) argue that weakly connected clusters can contribute to balance exploration and exploitation forces. In addition, Xu et al. (2016) find that, under limited individual rationality, network density has a dominant positive effect on knowledge integration.

In addition, experimental approaches have explored issues related to structural determinants in regard to different stages of the problem solving process: the search of information, the search of solutions, and the homophilious proclivity in the search of partners to exchange information. For instance, Shore et al. (2015) find that network clustering have different effects depending on the context of exploration, that is, whether such a context refers to the exploration of information or the exploration of solutions. Estévez-Mujica et al. (2018) consider effects of individual-level heterogeneity on the problem solving process and find that the intensity of homophilous interactions might hamper the so-called benefits of group (demographic) diversity. Estévez-Mujica et al. (2018) extend and complement their experimental results with an agent-based computational approach.

2.2 Network Dynamics

In several contexts, individuals might influence the network topology by adding or severing links with other individuals (Jackson 2008). As a result, a number of computational approaches have addressed the fact that networks change over time. (Chang and Harrington Jr 2007) explore the evolving set of connections in innovation networks as agents decide to either imitate or innovate according to a reinforcement learning process. (Son and Rojas 2010) study the dynamics of network formation and knowledge exchange in project-based teams, and find conditions under which teams reach suboptimal states (e.g. early subgroup cohesion). Some few works focus on co-evolutionary processes, where two coupled dynamical systems influence one another and consequently change over time: that of individual-level strategy selection, and that of the network topology (Pacheco et al. 2006; Sayama et al. 2013). Co-evolutionary frameworks have contributed to understand (i) how ”small-world” networks (i.e. those with high clustering and relatively
low average path length) might emerge out of random structures (Luo et al. 2015); (ii) the extent to which post-merger cultural integration between two organizations is heavily affected by within-firm and between-firm links (Yamanoi and Sayama 2013); and (iii) how individual-level characteristics aggregate to collective exploration and exploitation capabilities (Lin 2015).

2.3 Learning on Rugged Landscapes

Computational agents are usually featured with bounded rationality and inductive reasoning. Bounded rationality is usually embodied by solution search in an NK landscape (Levinthal 1997; Felin et al. 2014). Few experimental studies have also contributed to understand peculiarities of human search in complex space search (Mason and Watts 2012; Billinger et al. 2013).

Implications of learning types (i.e. from individual to social), coupled with different communication structures, have also been studied under the presence of NK landscapes (Barkoczi and Galesic 2016). Similarly, the implications of imitative learning in complex spaces have been explored in the strategy research literature (Rivkin 2000), as well as in the study of the comparative performance of both cooperative and independent agents (Fontanari 2015). Learning effects might also be multilevel, implying that either focusing on individual-level autonomous learning or on organizational knowledge assimilation (but not both) might yield equal performance levels (Hanaki and Owan 2013).

3 AN AGENT-BASED MODEL

We model problem-solving as collective searches, that is, carried out by a group of agents (e.g. decision-makers, employees). Agents are assumed to have imperfect capability in solution generation and evaluation due to bounded rationality (e.g. March and Simon 1958). They also engage in a social learning process of modifying their social ties.

In our model, we make four assumptions: (i) there are a large number of agents so that each has a particular screening capability level, and conducts searches over a performance landscape, (ii) agents engage in either individual problem-solving (i.e. local search) or social learning (i.e. by imitating other agents’ solutions), (iii) agents are embedded in a communication network through which they communicate their solutions, and (iv) the communication network evolves as agents break or create ties through a reinforcement learning process. These assumptions are detailed in the following sections.

3.1 The Solution Landscape

We model the performance landscape payoff function using an NK landscape (Kauffman 1993). In the pay-off function, there are \( N \) elements (i.e. dimensions) on which the agents must decide. These decisions are conceptualized as binary choices. Consequently, the problem search space consists of a total of \( 2^N \) possible solution alternatives.

We briefly discuss here NK performance model, as the reader is referred to the extensive literature on the application of the NK model in organizational research (e.g. Levinthal 1997; Gavetti and Levinthal 2000). Let us assume a vector \( a = (a_1, a_2, \ldots, a_N) \) representing one of \( 2^N \) possible solution alternatives, and that such a solution results in payoff / fitness value \( f(a) \). This fitness value is defined as the average of the contributions of all solution elements as shown in below. The contribution of each element \( a_j \) when the choice vector is \( a \), is represented by \( C(a_j|a) \), so that:

\[
f(a) = \frac{1}{N} \sum_{j=1}^{N} C(a_j|a)
\]

In the NK landscape model, parameter \( K \) indicates the degree of interaction among elements (or more generally, the degree of landscape ruggedness), which means that the contribution of an element \( C(a_j|a) \) in equation 1 depends on the state of \( K \) other randomly selected elements. Often, the set of \( K \) elements
that interacts with element $a_j$ is specified randomly, and then, the landscape function is generated. Also, the contribution values associated to each element $C(a_j|a)$ are drawn from a uniform distribution $([0,1])$.

### 3.2 Individual and Social Learning

Agents have varying degrees of screening capability. Following Knudsen and Levinthal (2007), we use a screening function to model agents’ imperfect search capability. This is shown in Figure 1. The horizontal axis represents the actual fitness difference between the current solution of agent $i$, and a new solution $a'$ (that is being evaluated for potential adoption). That difference is calculated as current solution fitness minus new solution fitness, and hence, it ranges in $[-1,1]$. The vertical axis indicates the probability to adopt the new solution alternative $a'$.

![Agent screening function, adapted from Knudsen and Levinthal (2007).](image)

**Figure 1**: Agent screening function, adapted from Knudsen and Levinthal (2007).

Similar to Knudsen and Levinthal (2007), we use a linear screening function $J(x)$ whose input $x$ is the difference between the fitness of the new solution $a'$ minus that of the current solution $a'$: $f(x) = 0.5 + \lambda x = 0.5 + \hat{\lambda} (f_i^t(a') - f_{i-1}^t(a))$. It can be noticed that, as the slope turns upward, the agent becomes more “intelligent”, implying that superior alternatives are more likely to be accepted than inferior alternatives.

We assume each agent $i$ has a screening function equal to $J_i(x) = 0.5 + \lambda_i x$, whose parameter $\lambda_i$ is calculated as follows:

$$\lambda_i = LM + \left(\frac{HM - LM}{n_g}\right)$$

The terms $LM$ and $HM$ in equation 2 determine the value range of agents’ screening capability.

We formalize the agents’ parallel collective search process as follows. At time $t$, each agent $i$’s solution, which is represented by $a_i^t$, corresponds to a position over the solution landscape with $N$ decisions. At time $t = 0$, each agent $i$ is randomly assigned a state (i.e. $a_i^0$), and her fitness value (i.e. performance) is calculated according to equation 1. At each subsequent time $t$, each agent $i$ engages in the learning process—that is a combination of both social and individual learning processes (Lazer and Friedman 2007; Fang et al. 2010; Barkoczi and Galesic 2016).

According to the learning process, a focal agent initially attempts to improve her own solution by a local search (by modifying her own current solution). Then, agent $i$ examines the resulting payoff of modifying a single digit of her current solution (e.g. $a = (0,0,0,1,0,0,0,0)$ in the case of $N = 8$). Afterwards, agent $i$ adopts the resulting new solution, if it brings a higher payoff. Otherwise, the agent conducts a social learning process.

In the social learning process, the agent considers solutions of those agents with whom she has connections (according to the interaction network). Then, the focal agent identifies those with superior performance solutions (i.e. those with higher fitness values), according to her screening capability.
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After the identification of superior agents, the focal agent uses a decision rule to select one agent and imitate the solution of that selected agent. We consider two decision rules. One is “random selection”, by which one of the perceived superior agents is randomly selected by the focal agent and the corresponding solution imitated (We emphasize that those agents are just “perceived” to be superior by the focal agent, as the focal agent-like all the other ones- has imperfect screening capability). The other decision rule is "conformity", by which an agent selects the most popular solution among those solutions that are perceived to be superior.

Consider the following example with $n_g = 10$ agents. The solution space includes $N = 4$ decision elements. At time $t = 1$, assume agent $i = 2$ has solution choice $a_2^1 = [1, 1, 1, 1]$, and its fitness is $f(a_2^1) = 0.9$. Assume this agent’s local search at time $t = 1$ has not provided a superior solution, so she attempts to socially learn (as described earlier). According to the communication network (see Figure 2), this agent has interactions with two other problem-solvers, $i' = 1$ and $i'' = 7$. Their choice possibilities, fitness and screening capability coefficient values, $f(a_i)$ and $\lambda_i$, are shown in Figure 2. With probability $0.5 + 0.6(0.2 - 0.9) = 0.08$, agent $i = 2$ considers the solution of agent $i' = 1$ (i.e. $[0, 1, 0, 1]$) as superior solution (despite the fact that it is an inferior solution with a lower fitness value of $f_1^1(a) = 0.9 > f_1^1(a) = 0.2$). Similarly, agent $i = 2$ perceives agent $i'' = 7$ solution as superior with probability $0.5 + 0.6(0.8 - 0.9) = 0.44$. Therefore, with probability $0.44 + 0.08 = 0.52$, agent $i = 2$ considers the solutions of either of her neighboring agents as superior ones, despite the fact that her own solution is superior. In the next step, depending on the applied decision rule, she might select one of these agents and imitate the corresponding solution.

Figure 2: An example that illustrates how the social learning process is conducted by agents with imperfect screening capability.

### 3.3 Problem-solvers, Learning Process and Network Evolution

Unlike the extant collective search models (e.g. Lazer and Friedman 2007; Barkoczi and Galesic 2016), our approach features a communication network that evolves according to the decisions of agents. Such evolution is guided by agents’ learning process. There are a number $n_g$ of agents conducting searches over a performance landscape. They also alter their interaction network over time. An agent $i$ has two actions to make. One is creating a new tie; the other is breaking an existing one. Following previous research (Clement and Puranam 2017), and consistent with the principle of problematic search, we assume that agents engage in exploration for new partners (Baum et al. 2005) when their fitness falls below an aspiration level, $\beta_i$. This parameter is set using the following formula:

$$\beta_i = LB + \left(\frac{HB - LB}{n_g}\right)$$

The terms $LB$ and $HB$ in equation 3 are used to determine the value range of agents’ aspiration levels. We use the well-known reinforcement learning model (Sutton and Barto 1998) to represent agents’ selection
among the two actions (i.e. creating or breaking a tie). Let \( A_{i,t} = \{1, 2\} \) indicate the action type of agent \( i \) at time \( t \), with \( A_{i,t} = 1 \) showing link creation, and with \( A_{i,t} = 2 \) showing link deletion. We define the probability of selecting one of these two actions as a function of the expected payoff of that action, relative to the aggregate expected payoff for both actions. We make use of the Softmax functional form:

\[
p_{A_{i,t}} = \frac{\exp(\pi_{A_{i,t}}/\tau)}{\sum_{A'} \exp(\pi_{A'/i}/\tau)}
\]  

(4)

In equation 4, \( p_{A_{i,t}} \) is the probability by which agent \( i \) selects action \( A_{i,t} \) at time \( t \); \( \pi_{A_{i,t}} \) stands for the expected payoff for agent \( i \) for taking action \( A_{i,t} \); \( \tau \) represents the discrimination power among competing actions. Low values of \( \tau \) result in higher probabilities of selecting actions with higher expected payoffs, whereas high values of \( \tau \) provide equal selection likelihoods to link creation / deletion.

Feedback is another critical part of learning process as each agent learns from it, and modifies its beliefs about the expected payoff from taking a particular action (e.g., creating a new tie). To materialize this learning process, we assume that agents engage in reinforcement learning over the simulation time period (Sutton and Barto 1998):

\[
\pi_{A_{i,t}} = \pi_{A_{i,t-1}} + \phi \left[ (f_{t}^{i}(a'') - f_{t-1}^{i}(a')) - \pi_{A_{i,t-1}} \right], \quad f_{t}^{i}(a') < \beta_i
\]

(5)

An important part of equation 5 is the difference between agent \( i \)'s fitness values between that of the current time (\( t \)) and that of the previous time period (\( t-1 \)). Mathematically, this is \( (f_{t}^{i}(a'') - f_{t-1}^{i}(a')) \) (the term \( f_{t}^{i}(a') \) represents the fitness value of agent \( i \) at time \( t \) when she has solution \( a' \)). This expression embodies a positive or a negative reward. A critical parameter in the reinforcement learning in equation 5 is \( \phi \), which specifies the rate at which expected payoffs are adjusted (both upward and downward) based on perceived rewards. A high value for \( \phi \) indicates an agent having a capability to recognize and adapt rapidly to both positive and negative feedback.

4 SIMULATION EXPERIMENTS AND RESULTS

Two general collective problem solving settings are considered: one with \( n_g = 40 \) and another with \( n_g = 80 \) agents. Each scenario explores implications of value combinations of the following parameters: \( K, \phi, \tau, \) and decision rule type. For each scenario we ran 30 simulations. Relevant results are reported next.

We set the number of decision elements of the problem space to be \( N = 15 \). The agents’ aspiration level parameters are assumed to be in the following range \([LB, HB] = [-1, 5]\). The screening capability parameter range is established with \( LM = 0.4 \), and \( HM \in [0.6, 0.8] \).

We set up two levels for all the other parameters of the model. To explore model behavior for problem spaces with both low and high complexity levels (i.e. smooth and rugged landscapes), we assume \( K \in \{2, 10\} \). In regard to the learning related parameters, we assume that \( \phi \in \{0.1, 0.8\} \), and \( \tau \in \{0.01, 0.2\} \).

In the first set of the following graphs, we illustrate the effects of high and low learning capability of agents on their collective performance. Figures 3 and 4 show the average fitness of solutions of all problem-solvers. The top panels in these figures represent the results for collective searches where the problem space is simple (\( K = 2 \)). The bottom panels report the results for complex spaces (\( K = 10 \)). In these and the following figures, left panels illustrate scenarios with \( \tau = 0.01 \), while the right ones indicate scenarios with \( \tau = 0.2 \). Also, all of them correspond to box-plots over simulation time. These box plots provide, at each simulation time, variations across 30 runs.

According to Figures 3 and 4, a higher learning rate (\( \phi = 0.8 \)) results in higher average performance only under a high discrimination power between the benefits of link addition / deletion (\( \tau = 0.01 \)). That is, when the following conditions hold, solutions of all agents, on average, are enhanced in the presence of
Figure 3: Box-plots of average fitness values of all agents for every 50 time intervals of simulation time span. In this graph, $n_g = 80$, $HB = 0.6$, and decision rule is "random selection".

Figure 4: Box-plots of average fitness values of all agents for every 50 time intervals of simulation time span. In this graph, $n_g = 40$, $HB = 0.8$, and decision rule is "random selection".

A high learning rate: (i) decision rule is "random selection", (ii) the search landscape is rugged ($K = 10$), and (iii) value discrimination in link creation / deletion is high ($\tau = 0.01$). It is also worth noting that, under low aspiration levels (i.e., $HB = 0.6$ in Figure 3), performance differences of different learning rates
tends to be more salient in complex problem spaces than in simple problem spaces. However, with higher aspiration levels $HB = 0.8$, this pattern changes (see Figure 4).

Panels in Figure 5 represent also the average performance of all agents for the same settings as in Figure 3, but using a “conformity” decision rule rather than a “random selection” one. Interestingly, for these cases, higher learning rate does not seem to be an influential factor. Thus, problem-solvers in such environment are unlikely to find better solutions if they have a higher learning rate.

![Box-plots of average fitness values of all agents for every 50 time intervals of simulation time span. In this graph, $n_g = 80$, $HB = 0.6$, and decision rule is “conformity”.

In addition to the average fitness value of agents’ solutions, we have also investigated the evolving pattern of the network structure. Figures 6 and 7 illustrate the average clustering coefficient of interaction networks over time for the same settings shown in Figures 3 and 4. Interestingly, we observe that the previously observed difference in performance emerges from changes in the interaction network. In particular, higher learning rates about social ties results in lower network clustering coefficient values.

5 CONCLUDING REMARKS

Our model contributes to the problem-solving literature in regard to the consideration of evolving network structures. Evolution of agents’ communication network is a critical part of our simulation model. While such an assumption has not been incorporated in all collective search models (e.g., Lazer and Friedman 2007) scholars have observed emerging patterns for online collective problem solvers networks (Mitrović and Tadić 2012; Dankulov et al. 2015; Andjelković et al. 2016). Our results reveal that performance differences are contingent upon nontrivial combinations of decision rule types, aspiration levels, landscape complexity, link creation / deletion value discrimination, and learning rates.

Results also show the emergence of some kind of order (e.g. high clustering) due to the agents’ perceived marginal benefits and costs of making changes in their communication network (see Chang and Harrington Jr 2007; Hanaki et al. 2007).

We conceptualize agents’ decision process of link update to follow a reinforcement learning model (Sutton and Barto 1998). In the learning model, agents classify their ties into two categories, one containing

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those who have been seen superior in the last time period (and according to their imperfect evaluation function), and the other one including those ties that are found inferior in the last iteration. Therefore, we aimed to investigate how the learning capability of agents in evolving their social ties affects the performance
of collective searches. Another important aspect of the evolving structural pattern is that agents have limited and heterogeneous capabilities in evaluating problem solutions. We use the NK rendering for complex landscapes and the screening function (Knudsen and Levinthal 2007) by which agents imperfectly assess alternative solutions.

Model outcomes pose important managerial implications on group performance. For instance, improvement of individual learning is only salient when there are clear perceived benefits for link exploration (i.e. low aspiration levels and high link creation/deletion utility discrimination). High aspiration levels impedes the link exploration process; in such a case, performance gains due to individual learning are only evident under simple spaces. In summary, learning improves performance either when the benefits for link update are salient, or when problems are simple otherwise.

All of the above results are obtained under the assumption of a “random selection” decision rule. Major differences in parameter value combination are not observed under the “conformity” decision rule. This means that selecting the most popular solution may inhibit any benefit derived from a high learning rate.

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AUTHOR BIOGRAPHIES

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