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ON-LINE PARAMETER ESTIMATION OF REDUCED-ORDER MODELS FOR BUILDINGS ENERGY DYNAMICS USING THE MODULATING FUNCTION METHOD

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ABSTRACT

This paper considers parameter estimation of a reduced-order model (ROM) for building energy dynamics using the modulating function (MF) method. After briefly presenting a model of building dynamics, we recall the MF method and present a way of determining a modulating function directly from the available data instead of defining it a priori. Then, we look at the application of this method in a case study, where actual weather-based data (solar radiation and outdoor ambient temperature) is used. The results together with the advantages of the MF underline the potential of the proposed algorithm.

1 INTRODUCTION

Efforts towards making buildings more energy efficient have increased considerably in the last years due to higher rates of energy consumption and CO_2 emissions (Cao et al. 2016). For analyzing the performance and improving the overall control strategy to achieve the new energy goals (D'Agostino et al. 2017), a good building energy model is essential (Li and Wen 2014).

In the existing literature, the methods for modeling buildings are divided into three main categories: physical models, data-driven models and gray-box models (Foucquier et al. 2013).

The specialized building simulation tools available for physical modeling (for example EnergyPlus, TrnSys, IDA-ICE, BSim) (Crawley et al. 2008) offer a high variety of possibilities in analyzing the dynamics and creating decent models, but with a considerable implementation time and computation power costs (Ionesi et al. 2015). Even if the results can be interpreted in physical terms and no training data is required to develop such a model, the main drawback is that it requires a full description of the buildings' geometry, construction materials, services, and operations details which are not always available.

On the other hand, the so-called statistical or data-driven models are in many cases generated only from recorded data by using regression (Aranda et al. 2012) and optimization techniques (Kalogirou 2001). While the shortcomings of a precise building description is circumvented, a high amount of training data is now necessary. A review of various data-driven models for energy prediction in buildings can be found in Wei et al. (2018).

In order to overcome the specific issues related to both physical and statistical methods, the so-called gray-box models or reduced-order models (ROM) were introduced (Harb et al. 2016). This approach combines the previously mentioned techniques, using a relatively light description of building physics while requiring a comparatively low amount of measured data to generate a very simple model. Many studies are done for evaluating the optimal configuration of the gray-box models (Bacher and Madsen 2011; Harb et al. 2016) for applications in energy predictions, control and demand response problems. One of the simplest and most commonly used model in the literature is a 2nd order model which is said to capture sufficiently the dynamics of the building in order to predict the heating and cooling demand, as well as indoor air temperature (Berthou et al. 2014; Harb et al. 2016; Fonti et al. 2017). In Mejri et al. (2011) it is argued that increasing the order model beyond 2 would not significantly improve the outcome, while in Barrio et al. (2000) it is mentioned that the indoor air temperature can be better predicted when using a 4th order model. Increasing the model order further would lead to over-parametrization problems (Reynders et al. 2014; Mejri et al. 2011).

An essential part of the ROM approach is concerned with the estimation of the lumped parameters. Techniques such as genetic algorithms (Wang and Xu 2006), Kalman filter (Radecki and Hencey 2012), Least Square variants (Bengea et al. 2011) are some of the most used today among others to estimate the parameters. In most of the applications, the estimation is applied in discrete-time.

The ROMs being described in continuous time, performing the estimation of the system parameters also in continuous-time offers several advantages over more conventional approaches: a better interpretation of the physical behavior of the system, models being derived from physical principles are represented usually by ordinary differential equations which preserve the prior knowledge, allow usage of variable sampling rates, the estimates converge to the real values as the sampling time approaches 0 (Unbehauen and Rao 1997). However, one known drawback of continuous-time identification techniques is that it can require the use of time-derivatives whose implicit derivation should be avoided. This can be handled by applying a set of signal processing techniques called R_{LD} (Unbehauen and Rao 1997). An earlier such technique is the so-called modulating function (MF) method described by Shinbrot (1957).

The main idea of the MF method is to multiply the measured signals and inputs of the system with a known function with specific properties and then proceed to time-integration (sza), thus by-passing the need for time differentiation of noisy signals. In literature, several types of MFs were introduced: Hermite functions (Takaya 1968), trigonometric functions (Co and Ungarala 1997), Hartley functions (Patra and Unbehauen 1995) (see for example Jouffroy and Reger (2015) for a brief summary). In the present paper, there is no a priori choice of a specific MF. The MF is determined based on simple constraints, as well as using the measured signals of the system. One of the consequences of such an approach is that the MF can be totally different depending on where it is applied on a time-interval.

In this paper, we look at the potential of applying the MF approach to the estimation of energy building dynamics. Because of its ability to tackle continuous-time directly, as well as its finite-time property and the fact that it does not require a big amount of data, this method could be a good candidate to estimate on-line the parameters of simple building energy models, which could then be used further for control, energy predictions and optimizations.

This introduction is followed by Section 2, where the mathematical building model is described starting from basic heat transfer considerations and a state-space representation is given. Afterwards, in Section 3, the MF method is briefly presented for linear-time invariant (LTI) systems put in the state-space form, together with specific aspects of how the modulating functions are obtained. Simulation results of actual scenarios for a ROM can be found in Section 4, where actual weather data inputs are used. Brief concluding remarks are finally given in Section 5.

2 MATHEMATICAL MODEL

The nodal modeling approach is currently the most used when dealing with ROM for buildings dynamics. It assumes that each building zone has a homogeneous volume characterized by uniform state variables. In the same time, this technique reduces considerably the complexity of the physical problem by linearization of the equations and making use of lumped parameters. As a consequence, this implies an optimization in terms of model development, implementation as well as computation time (Foucquier et al. 2013).

In the present paper, the building is modeled as a single zone with homogeneous temperature distribution. As it is usually done in the literature, we consider only linear conduction through the building elements and neglect the nonlinear heat transfer through radiation and convection. These effects on the overall dynamics of the building are taken into account by a direct input as heat gain from radiation and heating system.

Temperature change due to air exchange between the building and the outdoor environment is neglected as well. This factor becomes nonlinear for high wind speed (Goyal and Barooah 2012). One way to cope with these nonlinearities is to use on-line adaptive parameter estimation on the linear system as it is exemplified in Martinčević et al. (2014), and which also fits well with the estimation approach used in the present paper.

Note that the above simplifications are quite standard and extensively used in behavior predictions (Harb et al. 2016) as well as model predictive control (Fonti et al. 2017) in buildings.

We use a 2nd order-linear model to represent the building dynamics, similarly to what is done in Bacher and Madsen (2011). In the nodal approach, the dynamics are schematically represented by using the analogy of an electrical circuit composed of resistances and capacitors, as shown in Figure 1.

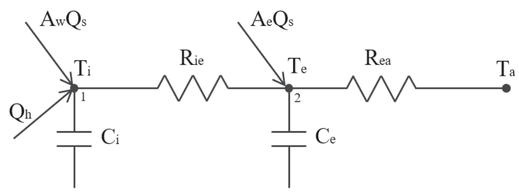

Figure 1: RC analogy of the considered ROM - 2R2C

Hence, in the present model, we have 2 nodes representing the indoor and envelope temperatures, respectively, 2 (thermal) resistances, 2 heat capacities (2R2C), and additional loads from heating system and solar radiation.

Then, a first-order differential equation is given to represent the heat balance for each node as follows:

- node 1 - interior air of the building/zone

$$C_i \frac{dT_i}{dt} = \frac{1}{R_{ie}}(T_e - T_i) + Q_h + A_w Q_s \tag{1}$$

- node 2 - envelope of the building/zone

$$C_e \frac{dT_e}{dt} = \frac{1}{R_{ie}}(T_i - T_e) + \frac{1}{R_{ea}}(T_a - T_e) + A_e Q_s \tag{2}$$

where, in equations (1) and (2), we have

- a set of state variables:
  - T_e temperature of the envelope of the building
  - T_i temperature of the indoor air
- the input signals:
  - Q_h heat load added from the heating system
  - Q_s heat load from solar radiation
  - T_a outdoor ambient temperature
- constant parameters:
  - A_e effective wall area (fraction of solar gain that is affecting the envelope)
  - A_w effective window area (fraction of solar gain that is added directly to indoor air due to windows)
  - C_e heat capacity of the envelope
  - C_i heat capacity of the indoor environment
  - R_{ie} thermal resistance between the indoor air and the envelope
  - R_{ea} thermal resistance between the envelope and the outdoor environment.

2.1 State-Space Representation

The equations for each node are then usually group into a state-space form, so that we have

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{3}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \tag{4}$$

where the state vector is given by

$$\mathbf{x} = \begin{bmatrix} T_i & T_e \end{bmatrix}^T,$$

while the input vector is

$$\mathbf{u} = \begin{bmatrix} Q_h & Q_s & T_a \end{bmatrix}^T.$$

Matrices **A**, **B** and **C** of state-space representation (3) - (4) are thus written as

$$\mathbf{A} = \begin{bmatrix} -\frac{1}{C_iR_{ie}} & \frac{1}{C_iR_{ie}} \\ \frac{1}{C_eR_{ie}} & -(\frac{1}{C_iR_{ie}} + \frac{1}{C_eR_{ea}}) \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} \frac{1}{C_i} & \frac{A_w}{C_i} & 0 \\ 0 & \frac{A_e}{C_e} & \frac{1}{C_eR_{ea}} \end{bmatrix},$$

and

$$\mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

3 PARAMETER ESTIMATION

In this section, we briefly recall the steps of the MF method, for the sake of clarity. For more details on the MF method and more recent developments, see for example Jouffroy and Reger (2015). Since our energy building dynamics are described under a state-space representation, we first need to convert it to input-output form. To do so, start with the multi-input single-output linear system description

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$

$$y(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \tag{5}$$

where $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{u} \in \mathbb{R}^m$ and $y \in \mathbb{R}$.

First, differentiate output equation (5) $n-1$ times to obtain the expression

$$\bar{\mathbf{y}} = \mathbb{O}\mathbf{x} + \mathbf{T}\bar{\mathbf{u}} \tag{6}$$

where $\bar{\mathbf{y}} = [y, \dot{y}, \ldots, y^{(n-1)}]^T$ contains the measured output and its derivatives, and, similarly for input signals, $\bar{\mathbf{u}} = [\mathbf{u}^T, \dot{\mathbf{u}}^T, \ldots, \mathbf{u}^{(n-1)^T}]^T$. Matrix $\mathbb{O}$ is the well-known observability matrix of the Kalman criterion, while matrix **T** is the system specific Toeplitz matrix given by

$$\mathbf{T} = \begin{bmatrix} \mathbf{D} & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{CB} & \mathbf{D} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \mathbf{0} \\ \mathbf{CA}^{n-2}\mathbf{B} & \mathbf{CA}^{n-3}\mathbf{B} & \ldots & \mathbf{D} \end{bmatrix}.$$

The system must be observable so that $\mathbb{O}^{-1}$ to exist. This can be verified by computing the determinant, $det(\mathbb{O})$, which must be different from 0.

We continue by differentiating equation (5) one more time (i.e. to order n) and isolating $\mathbf{x}$ in expression (6), so that we get

$$y^{(n)} = \mathbf{C}\mathbf{A}^n \mathbb{O}^{-1}(\bar{\mathbf{y}} - \mathbf{T}\bar{\mathbf{u}}) + \mathbf{C}\mathbb{C}^R\bar{\mathbf{u}} + \mathbf{D}\mathbf{u}^{(n)} \tag{7}$$

where the reversed controllability matrix $\mathbb{C}^R$ is written as

$$\mathbb{C}^R = [\mathbf{A}^{(n-1)}\mathbf{B}, \mathbf{A}^{(n-2)}\mathbf{B}, ..., \mathbf{A}\mathbf{B}, \mathbf{B}].$$

Define now the new quantity $\mathbf{Y}$ as

$$\mathbf{Y} = [\bar{\mathbf{y}}^T, \bar{\mathbf{u}}^T, \mathbf{u}^{(n)^T}]^T$$

and let the unknown parameter vector $\theta \in \mathbb{R}^{n+nm+m}$ be given by

$$\theta = [\mathbf{C}\mathbf{A}^n\mathbb{O}^{-1}, \mathbf{C}\mathbf{A}^n\mathbb{O}^{-1}\mathbf{T} + \mathbf{C}\mathbb{C}^R, \mathbf{D}]^T$$

so that expression (7) can finally be re-written as

$$y^{(n)} = \mathbf{Y}^T\theta. \tag{8}$$

Next, pre-multiply each side of system (8) with a sufficiently smooth function $\phi : [0,T] \to \mathbb{R}$ and whose boundary conditions are given as

$$\phi^{(i)}(0) = \phi^{(i)}(T) = 0 \qquad , i = \overline{0, n-1}. \tag{9}$$

The function $\phi(t)$ is referred to as modulating function. The idea behind its use is that it allows to avoid the differentiation of noisy measurement signals. To see this, notice that, thanks to integration by parts, pre-multiplication of $\dot{y}(t)$ and integration gives:

$$\int_0^T \phi(\tau)\dot{y}(\tau)\,d\tau = \phi(T)y(T) - \phi(0)y(0) - \int_0^T \phi'(\tau)y(\tau)d\tau.$$

meaning that $y(t)$ is not explicitly differentiated since the differentiation is transferred to $\phi(t)$. Also, with this expression, we avoid needing the initial conditions (Co and Ydstie 1990). More generally, and for higher-order derivatives, we have:

$$\int_0^T \phi(\tau)y^{(n)}(\tau)\,d\tau = \begin{cases} \int_0^T \phi(\tau)y^{(i-1)}(\tau)\,d\tau & , i = \overline{1,n} \\ \int_0^T \phi(\tau)u^{(i-n-1)}(\tau)\,d\tau & , i = \overline{n+1, 2n+1}. \end{cases}$$

Using the boundaries properties (9) and integration by parts on all terms of (8) we obtain

$$z = \mathbf{w}^T\theta$$

where

$$z = \int_0^T (-1)^n\phi^{(n)}(\tau)y(\tau)\,d\tau \tag{10}$$

and w_i, the i-th component of $\mathbf{w}$, is given by

$$w_i = \begin{cases} \int_0^T (-1)^{i-1}\phi^{(i-1)}(\tau)y(\tau)\,d\tau & , i = \overline{1,n} \\ \int_0^T (-1)^{i-n-1}\phi^{(i-n-1)}(\tau)u(\tau)\,d\tau & , i = \overline{n+1, 2n+1}. \end{cases} \tag{11}$$

Using $m_\phi \geq 2n+1$ such modulating functions, we get the set of linear equations

$$\mathbf{z} = \mathbf{W}^T\theta$$

where $\mathbf{z} = [z_1, z_2, ..., z_{m_\phi}]^T$ and $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, ..., \mathbf{w}_{m_\phi}]$. Finally, an estimate of parameter vector θ is obtained by computing simply

$$\hat{\theta} = \left(\mathbf{W}\mathbf{W}^T\right)^{-1}\mathbf{W}\mathbf{z}. \tag{12}$$

3.1 Choosing the Modulating Functions

A great deal of studies consider the definition of different types of modulating functions (i.e. splines, trigonometric functions, Hartley, etc, see Jouffroy and Reger (2015) for a short summary). In the present paper, the modulating functions are deduced on-line from the problem formulation and the data itself. Note that this can be related to the interesting work presented in Schmid and Roppenecker (2011) where the parameters of the considered system are estimated independently.

Thus, we begin by re-writing equations (9) - (11) by focusing on the n-th derivative of a MF $\phi(t)$, i.e. we define the new function $\alpha : [0,T] \rightarrow \mathbb{R}$ as

$$\alpha(\tau) := \phi^{(n)}(\tau).$$

Then, using the anti-derivative notation

$$f^{(-i)}(t) := \int_0^t \int_0^{\tau_i} ... \int_0^{\tau_2} f(\tau_1) d\tau_1 \, d\tau_2 \dots d\tau_i,$$

the boundary conditions at $t = T$ in (9) can be written as

$$\alpha^{(-i)}(T) = 0 \quad , i = \overline{1,n} \tag{13}$$

while the boundary conditions at $t = 0$ are directly fulfilled by integration of α. Expression (10) can be simply re-written as

$$z = \int_0^T (-1)^n \alpha(\tau) y(\tau) \, d\tau,$$

while the w_i terms in (11) are now expressed as

$$w_i = \begin{cases} \int_0^T (-1)^{i-1} \alpha^{(-n+i-1)}(\tau) y(\tau) d\tau & , i = \overline{1,n} \\ \int_0^T (-1)^{i-n-1} \alpha^{(-2n+i-1)}(\tau) u(\tau) \, d\tau & , i = \overline{n+1, 2n+1}. \end{cases}$$

As a reference to the energy building model of Section 2, if we follow the presented steps, the $\mathbf{w}$ vector is:

$$\mathbf{w} = \begin{bmatrix} \int_0^T \alpha^{(-1)}(\tau) y(\tau) \, d\tau \\ \int_0^T \alpha^{(-2)}(\tau) y(\tau) \, d\tau \\ \int_0^T \alpha^{(-1)}(\tau) u_1(\tau) \, d\tau \\ \int_0^T \alpha^{(-2)}(\tau) u_1(\tau) \, d\tau \\ \int_0^T \alpha^{(-1)}(\tau) u_2(\tau) \, d\tau \\ \int_0^T \alpha^{(-2)}(\tau) u_2(\tau) \, d\tau \\ \int_0^T \alpha^{(-2)}(\tau) u_3(\tau) \, d\tau \end{bmatrix} \tag{14}$$

because, for our specific model, in vector $\hat{\theta}$ from equation (12) $\mathbf{D} = \mathbf{0}$ and we have no $\dot{u}_3$ in the model.

In order to obtain the modulating functions or the corresponding functions $\alpha(t)$, we set first the normalizing constraint

$$\mathbf{W} = \mathbf{I}_{m_\phi}, \tag{15}$$

To be able to tackle the problem from this angle, we have to make sure that $\mathbf{W}$ is square. Next, we rewrite together the right hand-side boundary conditions (13) for each MF as

$$\Gamma = \mathbf{0}_{n \times m_\phi}, \tag{16}$$

where $\Gamma = [\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, ..., \boldsymbol{\alpha}_{m_\psi}]$, with each vector $\boldsymbol{\alpha}_j$ containing all boundary conditions (13) for this particular MF, i.e. $\boldsymbol{\alpha} = [\alpha^{(-n)}, \alpha^{(-n+1)}, ..., \alpha^{(-1)}]^T$. Expressions (15) - (16) represent together a system of integral equations with the functions $\alpha(t)$ as unknowns. There are different ways to solve (15) - (16). In the present work, we simply obtain the m_ϕ $\alpha(t)$'s numerically (see the appendix A at the end of the paper for more details focused on the application of the MF method on the model from Section 2). Once this is done, we simply use (15) and obtain the estimate of θ as

$$\hat{\theta} = \mathbf{z}. \tag{17}$$

4 CASE STUDY

The proposed ROM for buildings together with the parameter estimation method were implemented and simulated in Matlab/Simulink. For testing the estimation algorithm we are considering a scenario where the indoor temperature T_i is generated from a 4th-order building model, so that we increase a bit the complexity of the problem. We are going to refer further to this temperature as the reference indoor temperature. We estimate the parameters for the 2nd order model using as measured output the reference indoor temperature simulated previously with applied noise.

Following the procedure described above for the 2nd order model considered we have the parameter vector which we estimate $\hat{\theta} = [a_1 \ \ a_0 \ \ b_{11} \ \ b_{10} \ \ b_{21} \ \ b_{20} \ \ b_{30}]^T$, with: $a_1 = \frac{1}{C_e R_{ie}} + \frac{1}{C_e R_{ea}} + \frac{1}{C_i R_{ie}}, a_0 = \frac{1}{C_i R_{ie} C_e R_{ea}}, b_{11} = \frac{1}{C_i}, b_{10} = \frac{1}{C_i C_e R_{ie}} + \frac{1}{C_i C_e R_{ea}}, b_{21} = \frac{A_w}{C_i}, b_{20} = \frac{A_w}{C_i C_e R_{ie}} + \frac{A_w}{C_i C_e R_{ea}} + \frac{A_e}{C_i C_e R_{ie}}, b_{30} = \frac{1}{C_i R_{ie} C_e R_{ea}}$.

4.1 Persistence of Excitation

In order to generate a reliable model, with valuable estimates, a key point is to make sure that persistent inputs are used. This implies that the input signal has sufficient variations such that the observed response of the system contains the required information to perform the parameter estimation. The input is said to be persistent if a specific matrix related to this input signal is non-singular (Ljung 1999).

The measurements from a building, in a normal operation period, almost always do not fulfill the persistence of excitation criteria presented above, as described also in Bitmead (1984). This is why, for the identification purpose, usually some control inputs are generated using a series of specific signals as multi-sine signals, Pseudo-Random Binary signal (PRBS) or Pseudo-Random Sequences with very well defined frequencies (Braun et al. 2002). Unfortunately, there are not enough studies which consider the importance of persistent inputs for building models as discussed by Li and Wen (2014).

Since the data related to external ambient temperature (T_a) and solar radiation (Q_s) cannot be controlled, we are using data taken from an actual Test Reference Year-type (TRY) weather data set (Crawley and Huang 1997) which resembles to real measurements sampled at 1 hour, as shown in Figure 2.

To ensure persistence of excitation, we are looking at the heating power input which can be chosen by the modeler for the purpose of estimation. The heating system is operated as on/off processes based on a PRBS in order to excite the system enough and obtain relevant (Q_h) input. A similar approach is described in Royer et al. (2014), where the heating set-point is set based on a PRBS to assist the estimation process of a real building. This is a common way of setting up an experiment to gather reliable data for estimation purposes in a real case application scenario regardless of the estimation method chosen.

4.2 Simulation Results

For the considered case study, the integration time for estimation is $T = 48$ hours with a sampling time of $T_s = 6$ minutes. This implies a set of $n = 480$ sampled data used for the estimation window.

This sustains the fact that only a relatively limited and low amount of data is necessary in order to obtain relevant estimates (see Figure 3) for a ROM compared with the case of full data-driven models (Foucquier et al. 2013) when sometimes one year of recorded data is necessary. On the same aspect of training data intervals, note that the MF estimation method presented in this paper compares favorably

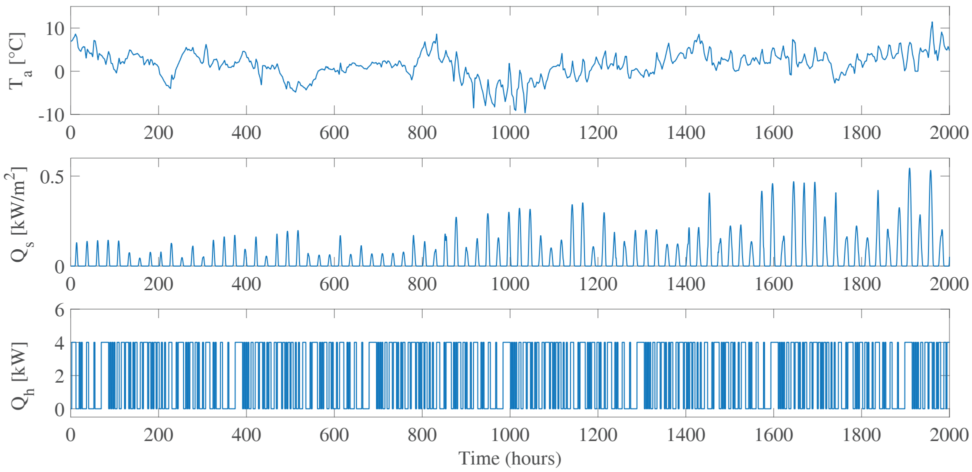

Figure 2: Input signals for the ROM.

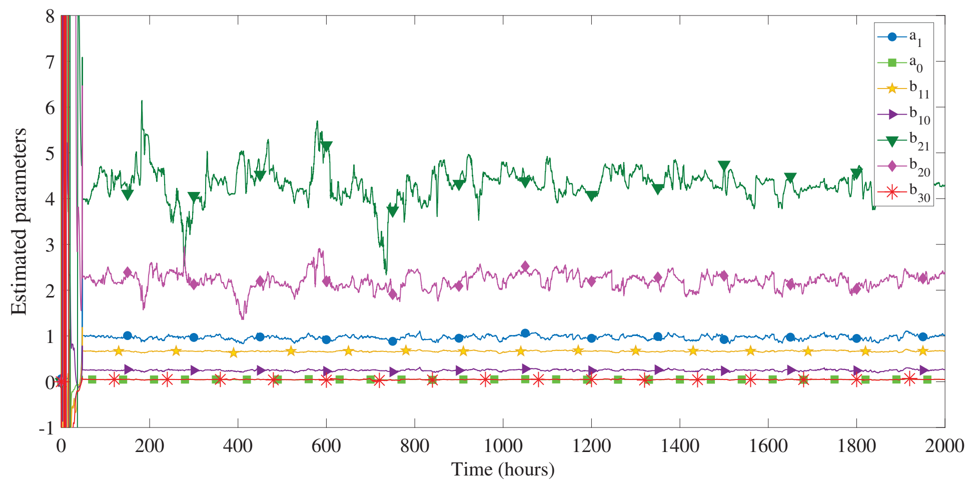

Figure 3: Estimated parameters.

with other approaches using Unscented Kalman Filtering (Radecki and Hencey 2012) or statistical methods based on maximum likelihood estimation (Bacher and Madsen 2011) where the estimates are typically obtained after a few days (from 4 days to 60 days in some models, see Berthou et al. (2014) for the summary).

We do not have any particular knowledge of the actual values of the parameters to be estimated. In this case, we can still evaluate the results by comparing the estimated output with the reference measurement $y(t)$. Looking at Figure 4, where such comparison is done, one can also see a reasonably good match, thereby illustrating the potential of the method.

5 CONCLUSIONS

In this paper, we presented results on using the Modulating Function method for parameter estimation of a ROM for buildings. The advantages of this method make it suitable for this purpose, especially because of the reduced amount of data that is needed for estimation as well as the fact that it eliminates the need for

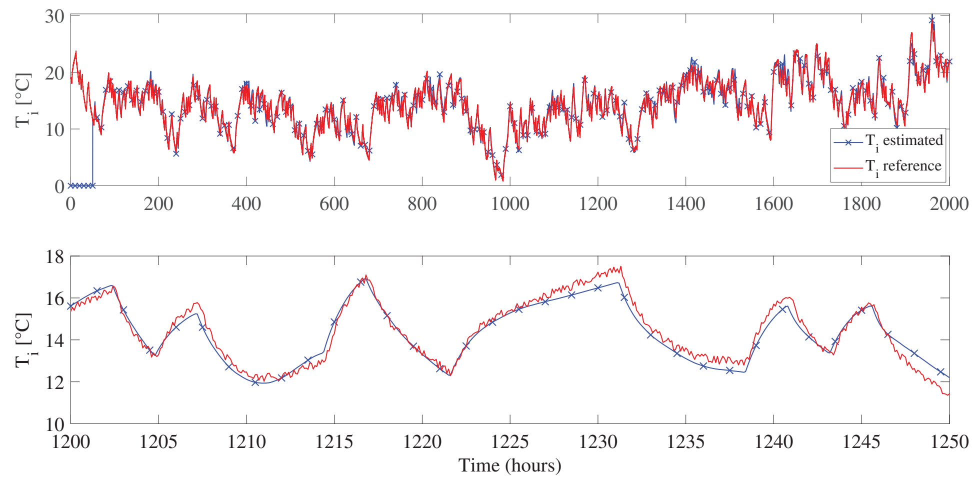

Figure 4: Estimated vs. reference indoor temperature.

knowing the initial conditions. Its simplicity makes it relatively easily implementable by simple application of least-square-based techniques.

The results show a good match between the actual and estimated parameters as well as between the reference and estimated system output. Following the steps described in Section 3, the method can be successfully applied also to higher order building models. This will generate a higher dimension of the modulating functions used and parameters that are estimated.

From a building simulation perspective, this method is quite promising in the sense that it can be used on-line to automatically correct the estimated parameters and in a receding-horizon fashion according to sudden changes in the building conditions. This, in turn, could lead to significant improvements regarding the choice of operation settings of heating systems, contributing to a better performance in terms of energy efficiency and thermal comfort. Further work should consider the persistence of inputs and how to deal with this in a real case set-up according to available measurements. Simulation tests using real measurements from a building could assist this process. Another interesting aspect to investigate is the order of the model, model selection and extension to state estimation and on-line control.

ACKNOWLEDGMENTS

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A SYSTEM DISCRETIZATION

Since the signals $\mathbf{y}(t)$ and $\mathbf{u}(t)$ are in practice sampled at regular intervals, we used a simple Riemann sum for implementation purposes. Hence, for the simple boundary condition (13), where $i = 1$, we have the approximation

$$\alpha^{(-1)}(T) = \int_0^T \alpha(\tau)\, d\tau \approx \sum_{k=1}^{N} T_s \underline{\alpha}(k) = T_s \mathbf{1}^T \underline{\boldsymbol{\alpha}}$$

where N is the number of samples over interval, T_s is the sampling period, and $\underline{\alpha}(k)$ is the sampled value of function α at iteration k, which gives the vector $\underline{\boldsymbol{\alpha}}^T = [\underline{\alpha}(1), \underline{\alpha}(2), ..., \underline{\alpha}(N)]^T$. Vector $\mathbf{1}$ is a vector of dimension N containing only ones. Using a similar reasoning, $\alpha^{(-1)}(t)$ can be approximated as

$$\alpha^{(-1)}(t) = \int_0^t \alpha(\tau) d\tau \approx \sum_{l=1}^{k} T_s \underline{\alpha}(l) =: \underline{\alpha}^{(-1)}(k)$$

so that we have

$$\underline{\boldsymbol{\alpha}}^{(-1)} = T_s \mathbf{Q} \underline{\boldsymbol{\alpha}}$$

where $\underline{\boldsymbol{\alpha}}^{(-1)T} = [\underline{\alpha}^{(-1)}(1), \underline{\alpha}^{(-1)}(2), \ldots, \underline{\alpha}^{(-1)}(N)]^T$, while the matrix $\mathbf{Q}$ is a lower triangular matrix of ones given by

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 1 & \ldots & 1 & 0 \\ 1 & 1 & \ldots & 1 \end{bmatrix}.$$

Hence, for $i = 2$, condition (13) gives

$$\alpha^{(-2)}(T) = \int_0^T \alpha^{(-1)}(\tau)\, d\tau \approx T_s^2 \mathbf{1}^T \mathbf{Q} \underline{\boldsymbol{\alpha}}.$$

The terms of $\mathbf{w}$ in (14) can be similarly approximated, so that we have

$$\int_0^T \alpha^{(-1)}(\tau) y(\tau)\, d\tau \approx T_s^2 \underline{\mathbf{y}}^T \mathbf{Q} \underline{\boldsymbol{\alpha}} \tag{18}$$

for w_1, where $\underline{\mathbf{y}}$ is the vector of samples of continuous-time signal $y(t)$, while for w_2 we have

$$\int_0^T \alpha^{(-2)}(\tau) y(\tau)\, d\tau \approx T_s^3 \underline{\mathbf{y}}^T \mathbf{Q}^2 \underline{\boldsymbol{\alpha}} \tag{19}$$

and similarly for the remaining terms of $\mathbf{w}$ in (14) where vector $\underline{\mathbf{y}}$ in (18) or (19) is replaced by $\underline{\mathbf{u}}_1$, $\underline{\mathbf{u}}_2$, $\underline{\mathbf{u}}_3$. Taking now m_ϕ modulating functions, $\mathbf{W}$ in (15) can thus be approximated by

$$\mathbf{W} \approx \underline{\mathbf{W}} = \underline{\mathbf{K}}\, \underline{\underline{\alpha}}$$

where $\underline{\underline{\boldsymbol{\alpha}}} = [\underline{\boldsymbol{\alpha}}_1, \underline{\boldsymbol{\alpha}}_2, \ldots, \underline{\boldsymbol{\alpha}}_{m_\phi}]$ and the matrix $\underline{\mathbf{K}}$ is given by

$$\underline{\mathbf{K}} = \begin{bmatrix} T_s^2 \underline{\mathbf{y}}^T \mathbf{Q} \\ T_s^3 \underline{\mathbf{y}}^T \mathbf{Q}^2 \\ T_s^2 \underline{\mathbf{u}}_1^T \mathbf{Q} \\ T_s^3 \underline{\mathbf{u}}_1^T \mathbf{Q}^2 \\ T_s^2 \underline{\mathbf{u}}_2^T \mathbf{Q} \\ T_s^3 \underline{\mathbf{u}}_2^T \mathbf{Q}^2 \\ T_s^3 \underline{\mathbf{u}}_3^T \mathbf{Q}^2 \end{bmatrix}.$$

Proceeding similarly with the discrete approximation of Γ in (16), we have

$$\Gamma \approx \underline{\Gamma} = \underline{\mathbf{B}}\, \underline{\underline{\alpha}}$$

where

$$\underline{\mathbf{B}} = \begin{bmatrix} T_s^2 \mathbf{1}^T \mathbf{Q} \\ T_s \mathbf{1}^T \end{bmatrix}.$$

Then, matrix $\underline{\underline{\boldsymbol{\alpha}}}$ is obtained by simple pseudoinversion, i.e. we have

$$\underline{\underline{\boldsymbol{\alpha}}} = \begin{bmatrix} \underline{\mathbf{K}} \\ \underline{\mathbf{B}} \end{bmatrix}^+ \begin{bmatrix} \mathbf{I}_{m_\phi} \\ \mathbf{0}_{n \times m_\phi} \end{bmatrix}.$$

By virtue of (17), the parameter estimate vector is finally obtained through the discrete approximation of $\mathbf{z}$, which is given by

$$\mathbf{z} \approx \underline{\mathbf{z}} = (-1)^n T_s \underline{\mathbf{y}}^T \underline{\underline{\boldsymbol{\alpha}}}.$$

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