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PHANTOM PARETO SYSTEMS FOR MULTI-OBJECTIVE RANKING AND SELECTION

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ABSTRACT

Consider the context of the Multi-Objective Ranking and Selection (MORS) problem, which is a multi-objective simulation optimization problem on a finite feasible set of systems. Since the Pareto set can only be observed with error, MORS methods often are concerned with the possibility that a misclassification (MC) event occurs, in which a system is misclassified on its status as Pareto or non-Pareto. In two dimensions, phantom Pareto systems have been used to assist in analyzing the probability of an MC event. We construct phantom Pareto systems in d dimensions, describe an algorithm to efficiently locate the phantom Pareto systems in d dimensions, and describe how the phantom Pareto systems can be used in a SCORE framework.

1 INTRODUCTION

The Multi-Objective Ranking and Selection (MORS) problem is a multi-objective simulation optimization problem on a finite feasible set of systems. We define the MORS problem as

$$\text{Problem } M: \quad \text{Find: } \operatorname{argmin}_{s \in \mathcal{S}} \quad \mathbf{g}_s := (g_{s1}, \ldots, g_{sd})^\intercal,$$

where $\mathbf{g}_s \in \mathbb{R}^d$ is a vector representing the expected performance of system s on each of the d objectives, and $\mathcal{S} := \{1, \ldots, r\}$ is a set containing the system indices. Since each system is evaluated on multiple objectives, there may not be a single system that is best on all objectives. Thus the solution to Problem M is the set of non-dominated systems, which we call the *Pareto set*. We define the Pareto set as $\mathcal{P} := \{\text{systems } s \in \mathcal{S} : \nexists \text{ system } s' \in \mathcal{S} \text{ such that } \mathbf{g}_{s'} \preceq \mathbf{g}_s\}$, where $\mathbf{g}_{s'} \preceq \mathbf{g}_s$ if $g_{s'k} \leq g_{sk}$ for all $k \in \{1, \ldots, d\}$ and there exists $k' \in \{1, \ldots, d\}$ such that $g_{s'k} < g_{sk}$. Figure 1 shows the objective function space of an example Pareto set.

The true Pareto set, however, is unknown. Since we can only estimate the objective vectors with error, for example as the output from a Monte Carlo simulation, we can only observe an estimated Pareto set $\hat{\mathcal{P}}$. Some MORS methods attempt to control the probability that, after some total simulation budget has been expended, a *misclassification* (MC) event occurs. An MC event occurs whenever $\hat{\mathcal{P}} \neq \mathcal{P}$. MORS methods that consider MC events include Multi-objective Optimal Computing Budget Allocation (MOCBA) (Lee et al. 2010), Myopic Multi-Objective Budget Allocation (M-MOBA) (Branke and Zhang 2015), and bi-objective sampling allocations such as Hunter and McClosky (2016) and bi-objective Sampling Criteria for Optimization using Rate Estimators (SCORE) (Feldman and Hunter 2016).

Hunter and McClosky (2016) and Bi-objective SCORE exploit the special structure of the bi-objective MORS problem to create *phantom Pareto systems*, which assist in the analysis of MC events. In this poster, we construct phantom Pareto systems in d dimensions, describe an algorithm to efficiently locate the phantom Pareto systems in d dimensions, and describe their use in a SCORE framework.

2 FINDING THE PHANTOM PARETO SYSTEMS

To define the true phantom Pareto systems, consider a scenario in which the objective values of all the Pareto systems are known and fixed, but the non-Pareto systems are unknown and must be estimated. Then loosely speaking, phantom Pareto systems are points constructed from the objective values of the Pareto systems such that, if the objective values of the systems in the Pareto set were known and fixed, then (a) a non-Pareto system would be falsely included in the Pareto set if and only if it were estimated as dominating a phantom Pareto system, and (b) no phantom Pareto systems dominate other phantom Pareto systems. Geometrically, phantom Pareto systems in d objectives can be found at the vertices of the boundary of the union of the dominated cones of d unique Pareto systems, as shown in Figure 1.

A straightforward way of finding the phantom Pareto systems is to perform a brute force enumeration of all possible combinations of the objective vectors of the Pareto systems (Feldman 2017). Then, a "clean-up" procedure removes points that are not phantom Pareto systems. While this method works, it is computationally inefficient.

Instead of using brute force enumeration, we develop a "sweep" algorithm. The sweep moves through the set of Pareto points in one objective from higher objective values to lower objective values ("back" to "front"). At each Pareto point in a d-dimensional sweep, we project the Pareto points in front of the current sweep point into a plane to determine the $(d-1)$-dimensional phantom Pareto systems. Combining the $(d-1)$-dimensional phantom objective values with the fixed sweep objective value results in the d-dimensional phantom Pareto systems we desire. Our algorithm involves recursion; we use the sweep procedure within the sweep procedure until the dimensionality is such that the phantom Pareto systems are trivial to find. Our numerical experiments show that our sweep algorithm finds all of the same phantom Pareto systems that a brute force enumeration finds, and our algorithm runs in a much shorter wall-clock time.

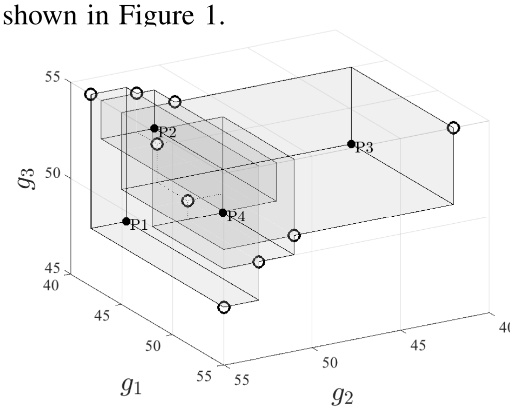

Figure 1: For $d = 3$ objectives, four Pareto systems and nine phantom Pareto systems are shown as solid and open circles, respectively. The cone of points dominated by each Pareto system is shaded in gray.

3 USING THE PHANTOM PARETO SYSTEMS

The d-dimensional objective vectors of the phantom Pareto systems can be used in a SCORE framework. We allocate to the non-Pareto systems j in inverse proportion to their score $\mathbb{S}_j := \min_{\ell \in \mathcal{P}^{\text{ph}}} \inf_{\mathbf{x}_j \leq \mathbf{g}_\ell^{\text{ph}}} I_j(\mathbf{x}_j)$, where $\mathcal{P}^{\text{ph}}$ is the set of indices of the phantom Pareto systems, $\mathbf{g}_\ell^{\text{ph}}$ is the objective vector of the ℓth phantom Pareto system, and $I_j(\mathbf{x}_j)$ is the large deviations rate function corresponding to the estimated objective vector, under appropriate regularity conditions. Under a normality assumption, I_j is quadratic.

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