# A NOTE ON SIMULATION FOR ESTIMATING THE VARIANCE OF THROUGHPUT IN FLOW LINES WITH FINITE BUFFERS

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## ABSTRACT

Although analytical approaches and exact solution methods are best for manufacturing system design, it is not easy when the systems become complex. Thus, approximation methods are required and the accuracies of the methods are analyzed by simulation. However, unlike the first order measures such as the mean of throughput, the second order measures such as the variance rate of throughput is difficult to be converged using simulation study. This paper introduces various phenomena empirically when the variance rate of throughput is estimated by simulation in flow lines.

## **1 INTRODUCTION**

There have been extensive literatures for analyzing the manufacturing systems with unreliable machines servers finite buffers and. Most of the works related to the performance evaluation of manufacturing systems have been focused on analyzing the first order measures such as average production rates (or throughput) and average buffer levels in steady-state (Shin and Moon, 2017). The first order measures have been interested because we can estimate the stable capabilities of a manufacturing system. However, there may be tremendous variability such as the variance of throughput in a given time period, the interdeparture time distributions and the covariance between consecutive inter-departure times, which are also very useful to design and operating manufacturing systems effectively. For example, the variance of throughput is closely related to the delay of due dates and the late penalties for the delays.

Despite the importance of second order measures, there have not been many studies dealing the variabilities in manufacturing systems. Hendricks (1992) developed an analytical approach for analyzing the inter-departure time distribution and correlation structures of serial line with exponential processing time distributions, reliable machines and finite buffers. However, his approach is difficult to apply to the long lines or big buffer capacities. He et al. (2007) considered two measures in the same flow lines considered in Hendricks (1992) and suggested approximation methods for analyzing long lines. In their paper, the variance of the number of parts produced (we call it as throughput) in a given time period and the variance of the time to produce a given number of parts were selected as the measures. However, they did not addressed the accuracy of the approximation method. Recently, Wu et al. (2016) explained various properties of variability in flow lines, and Tan and Lagershausen (2017) presented an analytical method to determine the auto-correlation of inter-departure times in both open and closed queueing networks. However, their approach also has a limitation in applying to the long lines.

Analytical approaches and exact solution methods are highly required for manufacturing system design, but it is not easy when the systems become complex. Thus, many researchers have developed various approximation methods and showed the accuracy of their methods by simulation. However, unlike the mean of throughput, the convergence of variance is difficult to be obtained by simulation study. In this study we will discuss the various things to be careful when estimating the variance rate using simulation. The variance rate is defined as  $VR_t = Var(N_t)/t$  where  $N_t$  is the variance of throughput during time period *t*.

# 2 SIMULATION RESULTS AND CONCLUSIONS

The system considered in this study is a flow line with k machines, k-1 buffers, exponential process time with mean one, reliable machines and Blocking after service (BAS) rule. Simulation models are developed with ARENA<sup>TM</sup>. Because of the limited space, the only a few observations about  $VR_t$  are discussed.

Figure 1 shows the behaviors of  $VR_t$  obtained from simulation using default random number stream (RNS), when the time length t is changed. It tends to converge when t is greater than 200,000, but it shows different patterns when t is relatively small such as 1,000 or 10,000. In practice small value of t is our usual concern, not big t. Figure 2 shows the behavior of  $VR_t$  when the number of replications increases. Usually we have insight that  $VR_t$  is monotonously decreasing as the increase of replications for the given t. However, it is not always true when t=1,000 and the number of replications is 300. Note that we rarely set the number of replications greater than 100 in simulation study.

Another concern of simulation is RNS. We compared the effects of RNS on  $VR_t$  in flow lines when number of machines are 2~16 and buffer capacities are 0, 3, 5 and 7. Five RNSs are selected randomly in ARENA<sup>TM</sup> and obtained the range and mean of  $VR_t$  for five RNSs. Then, the relative ratio of the range over the mean were calculated, and found that the maximum value of the ratio is 20%. Thus, we should design the simulation experiments, such as number of replications and RNS, very carefully when t is relatively small and estimating the second order measures.



Figure 1.  $VR_t$  with changing t



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