

DETERMINATION OF AN EMPIRICAL MODEL OF AVERAGE RANK FOR MULTI-DEEP AS/RS BASED ON SIMULATION

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ABSTRACT

We consider in this paper multi-deep automated storage/retrieval systems, where the cell capacity is strictly greater than one load. The main advantage of this class of AS/RS is a better use of space. Its main drawback is that, in order to retrieve a desired load, it is necessary to move all the loads in front of it. This is a common characteristic of all the multi-deep automated storage/retrieval systems. The number of loads to move is given by the rank of the load to retrieve. Our objective is to provide an empirical formula of the average retrieval rank in a multi-deep automated storage/retrieval system. With this formula, it is easy to deduce the mean retrieval time. This computation is based on multiple simulations of various AS/RS models and a regression on the obtained data. The particular case of the flow-rack automated storage/retrieval system will be considered to illustrate our contribution.

1 INTRODUCTION

Typically, an Automated Storage/Retrieval System (AS/RS) consists of storage racks, S/R machines and pickup/drop-off (P/D) stations. They are generally classified according to their physical configurations; namely:

1. The number and capacity of S/R machines,
2. The arrangement of racks and aisles in the system,
3. The position of P/D stations,
4. The depth of racks, i.e., the maximum number of loads that could be stored in a cell.

In this paper, we consider the class of multi-deep AS/RS, also called compact AS/RS, high-density AS/RS or 3D AS/RS. Although the application of multi-deep AS/RS is still limited, they have become increasingly popular. The main particularity of this class is that the capacity of a cell (the maximum number of loads that could be stored in the same cell) is strictly greater than one load. which significantly reduces the amount of floor space. Each storage cell is independently accessible, and so any load can be stored in any storage cell. Both sides of a cell are accessible either by the same Storage/Retrieval (S/R) machine or by different S/R machines. This allows to move: (1) loads to store from the pickup/deposit stations to cells, and (2) loads to retrieve from cells to the pickup/deposit stations.

In order to explain the contribution of this paper, it is important to point out the retrieval mechanism. When the load of rank m has to be retrieved, the $(m - 1)$ preceding loads must be retrieved first and stored again. The movement of these other loads here is called relocation. The operation of relocation is a part of the retrieval operation, which means that, in order to express the retrieval time of a given load, it is necessary to express the number of relocation operations to be carried out during the retrieval operation. Consequently, the rank in which the desired load is located (m) must be determined.

Obviously, a smart storage of an AS/RS improves its performance (like the mean access time). This is all the more true for multi-deep AS/RS, since the access time to a given load depends on the number and types of loads that precede it. Several works that propose storage heuristics for multi-deep AS/RS systematically compare the performances of the proposed heuristics with the performances of a random storage (Cardin et al. 2012; Le-Duc and De Koster 2007), as random storage is considered as the worst storage heuristic and therefore its performance as a classical lower bound. The purpose of this paper is to introduce a new way to evaluate this lower bound, using a calculation rather than a simulation study as commonly performed. This is meant to avoid the development of such model, which is generally time consuming. To achieve this objective, the last parameter to determine is the mean retrieval time, as expressed before. As a matter of fact, The main aim of this paper is to propose an empirical model, developed via simulation, of the average rank m_i of a load of type i , whose proportion in the system is α_i .

This paper is organized as follows: Section 2 gives a brief literature review on multi-deep AS/RS, section 3 presents a description of one kind of multi-deep-AS/RS, flow-rack AS/RS. Section 4 gives the analytic formula of the average retrieval time in the case of flow-rack AS/RS, that highlights the critical parameter considered here. Section 5 describes the simulation approach, while section 6 computes the average rank analytic formula using the simulation results. Finally, in section 7, we compare the results of the proposed empirical model to the performances obtained by simulation.

2 LITERATURE REVIEW

We will present in the following some of the works related to multi-deep AS/RS. Lerher et al. (2010) suggested analytic travel time models for the computation of single and dual cycle times for unit-load double-deep automated storage and retrieval systems. Le-Duc and De Koster (2007) developed an optimization method to determine the average throughput time for an order-batching problem in a 2-block rectangular warehouse; they applied the well-known method of S-shape heuristic. Yu and De Koster (2009) derived the expected single-command cycle time under the full turnover-based storage policy and developed a model to determine the optimal rack dimensions by minimizing the cycle time. They simplified the model, and analytically determined the optimal rack dimensions for any given rack capacity and any ABC curve skewness. They proved that the cycle time was significantly reduced as compared to a random storage strategy. Sari, Saygin, and Ghouali (2005) considered the flow-rack AS/RS, which is a special configuration of multi-deep AS/RS, to see whether it can be used for the storage of a large number of load types, by using intelligent heuristics for storage and retrieval processes.

Bessenouci, Sari, and Ghomri (2012) considered metaheuristics based control of flow rack AS/RS. They investigate the scheduling of retrieval request to minimize the system response. Sari and Bessenouci (2012) designed a new physical flow-rack AS/RS configuration and modeled the mean travel time of an S/R single machine. In the configuration they proposed, cells have a U form and are divided in segments; each segment is used for the storage of one unit. A single load is conveyed by a powered conveyor from the input side to the output one. For this same physical configuration, Hamzaoui and Sari (2015) determined the optimal dimensions, i.e., the dimensions that minimize the mean retrieval time. Traditionally, bins slope in the same direction in a flow-rack in order to make unit-loads slide from the storage face to the retrieval face driven by gravity. Unit-loads are then stored to the storage face and retrieved from the retrieval face. Chen, Li, and Gupta (2015) designed a new configuration of flow rack AS/RS in which bins are bidirectional, i.e. in adjacent columns slope to opposite directions. They developed a travel time model for the configuration they suggest. Liu et al. (2014) studied a new configuration of multi-deep AS/RS. This last is composed of several aisles, each aisle is served by one S/R machine. The cells are equipped by conveyors. When a given load needs to be retrieved, the S/R machine makes the horizontal and vertical movements and the conveyor of the cell makes the movement in depth. For this AS/RS, the authors develop an analytic model and present the minimum travel time for given dimensions. Fan et al. (2015) study the travel time of the S/R machine in multi-deep AS/RS based on simulation methods. A flexible model of multi-deep AS/RS with 3600 kinds of plans has been built

and simulated by the authors, aiming at the S/R machines travel time. They thus obtained a large number of simulation data, which were then used in the multiple regression analysis. The fitting formulas of the S/R machines travel time has been finally built.

However, we consider here the flow rack AS/RS, which is a special physical configuration of multi-deep AS/RS, the average rank model we develop here can be used to express the average retrieval rank of any physical configuration of multi-deep AS/RS.

3 FLOW RACK AS/RS DESCRIPTION

As shown in Figure 1, flow-rack AS/RS is composed of only one multi-deep rack, consisting of a set sloping cells. Each cell can contain several loads placed one after the other. Each cell is equipped with a gravity conveyor based on rollers or freewheels inclined in such a way as to allow the loads to slide from one end of the cell to the other, and therefore from one side of the rack to the other. The items are loaded on one end of the rack, i.e. the storage face. They slide on the gravity conveyors to the other end of the rack, i.e. the retrieval face. The retrieval machine withdraws the item from the retrieval face during the retrieval process. The storage and retrieval machines travel on parallel x-y planes to reach any cell on the rack. A drop-off station and a pickup station are located at the storage face and retrieval face of the rack, respectively. A restoring conveyor links the retrieval and storage machines, as shown in Figure 1.

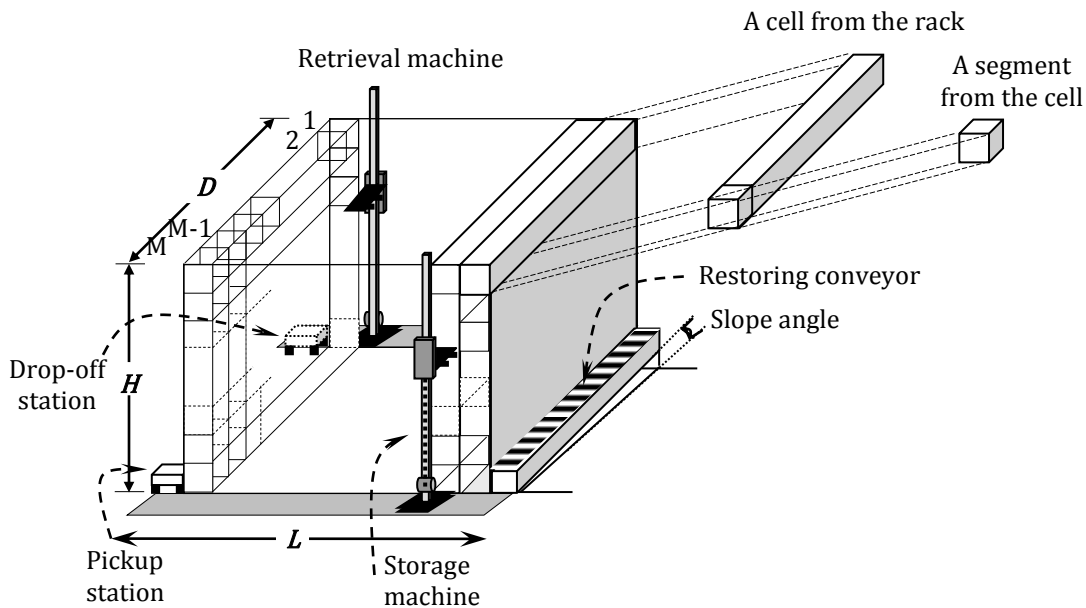


Figure 1: Configuration components and dimensions of a typical Flow-rack AS/RS.

In the case of unit-deep AS/RS, modelling average retrieval time is a rather easy problem, since all items are on the front side of the system. In multi-deep AS/RS, as flow rack AS/RS, the retrieval process is more complex to compute. The same cell is used for the storage of different types of loads with different proportions. The retrieval of an item often requires the retrieval of all the items that precede it.

To retrieve an item, the retrieval machine starts by transporting, one by one, all the items that precede it in the cell, to the front end of the restoring conveyor. This last carries the items from the retrieval face to the storage face, so that the storage machine stores them again in the rack. The storage and retrieval machines move simultaneously in the horizontal and vertical directions. These movements known as Tchebychev moves allow a faster machine operation.

4 AVERAGE RETRIEVAL TIME IN A FLOW RACK AS/RS

In this section, we detailed the computation process of the average retrieval time. This will show the importance of the rank parameter.

Consider the retrieval time of the item stored in the rank number m of the cell whose time coordinates (X, Y) . Figure 2.a shows the flow-rack AS/RS retrieval face. This face has 3 important stations, which are:

1. The dwell position of the retrieval machine located at the center of the retrieval face.
2. The AS/RS drop-off station located at the lower left-hand corner of the face.
3. The front end of the restoring conveyor located at the lower right-hand corner of the face.

Figure 2.b shows a column (a set of cells arranged one on the other) from the rack of a flow-rack AS/RS. Retrieving the item stored in the segment number m , requires from the retrieval machine to remove, one by one, all the items stored before it, i.e. the items stored in the segments from 0 to $m - 1$, until the desired item reaches the first segment (number 0) on the retrieval face of the rack. After this, the retrieval machine retrieves it.

The retrieval machine performs three elementary travels when retrieving an item (Figure 2.a):

- E_1^R is the travel time from the dwell point of the retrieval machine to the backside of the cell containing the desired item; during a retrieval operation the machine makes the travel E_1^R once.
- E_2^R is the travel time from the front end of the restoring conveyor to the cell containing the desired item; during a retrieval operation the machine makes this travel once.
- E_3^R is the travel time from the drop-off station to the cell containing the desired item; during a retrieval operation the machine makes this travel $2m$ times; where m is the segment containing the desired item.

The time required to retrieve the item stored in the cell (X, Y) and in segment m can be expressed as follows:

$$E^R = E_1^R + 2mE_2^R + E_3^R \quad (1)$$

Since the variables m and E_2^R are independent, we can write:

$$\overline{mE_2^R} = \bar{m} \overline{E_2^R} \quad (2)$$

The average value of E^R is calculated as follows:

$$\overline{E^R} = \overline{E_1^R} + 2\bar{m} \overline{E_2^R} + \overline{E_3^R} \quad (3)$$

In equation (3) E_1^R , E_2^R and E_3^R are time variables that depend on the physical dimensions of the flow rack AS/RS, whereas m is an integer variable that depends on rate of existence of the item in the rack. It also depends on the number of loads in the system.

The analytic computation of the average movement times $\overline{E_1^R}$, $\overline{E_2^R}$ and $\overline{E_3^R}$ is well solved in the literature. The retrieval face of the rack is approximated by a continuous face. It was used by Bozer and White (1984) to model the average travel time for unit load AS/RS and by Ghomri et al. (2008) to model the average travel time in multi aisle AS/RS. Sari, Saygin, and Ghouali (2005) model the average travel time in the case of flow rack AS/RS under the assumption that the rack is used for the storage of a very large number of different types of loads. However, the parameter m remains to be computed.

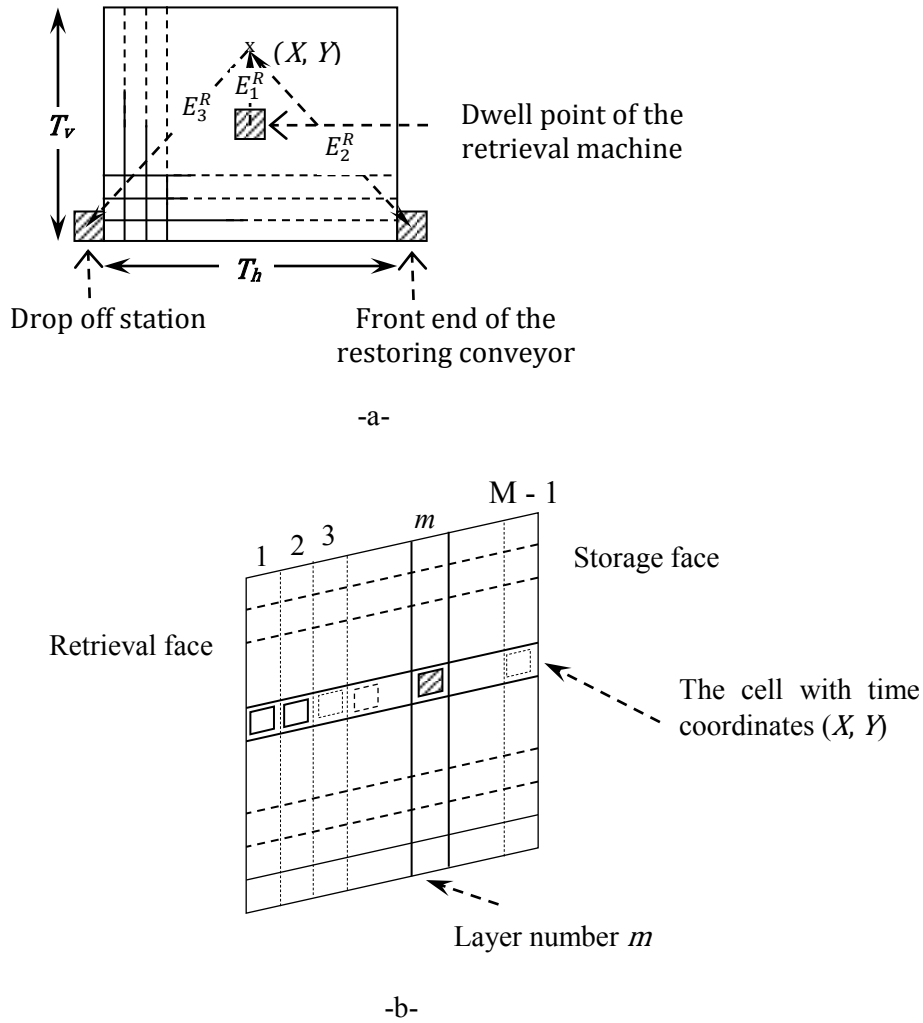


Figure 2: a. Flow-rack AS/RS retrieval face; b. A column from the rack in a flow-rack AS/RS.

5 SIMULATION APPROACH

The objective of this section is to determine a set of data corresponding to the average rank of a load type i : \bar{m}_i (equation 3). Two important parameters have to be considered:

- ρ : The load rate of the flow rack AS/RS, It is given by the proportion of the loads in the rack to the capacity of the rack;
- α_i : the proportion of the load of type i in the AS/RS, it is given by the ratio of the number of loads of type i present in the system out of the total number of loads present in the system.

As shown in Figure 1, M represents the total number of segments in a cell. Then ρM corresponds to the mean number of items in cells. It is the first parameter in the computation of the average rank. The second parameter is α_i , it identified the product i . Performing simulations, these two parameters were found as the basic terms for the determination of \bar{m}_i . It is why our set of simulations is structured with two inputs $(\rho M, \alpha_i)$ and on output the average rank \bar{m}_i .

The input ρM varies from 1000 to 19000 every 1000, while α_i varies from 5% to 100% each 5%. This set of data generates 340 experiments, with various physical configurations of the rack. To cope with this set, a Rockwell Arena simulation model was created, able to generate automatically the number of cells given in a configuration file. A Microsoft Excel file was used in order to handle the set of inputs, generate the configuration file, run the model and retrieve the simulation results. This process is detailed in the sequence diagram of Figure 3. The queues creation is made in the “Runbegin” phase, i.e. before the compilation of the model. The assignment of the parameters of the simulation run is made in the “RunBeginReplication” phase, i.e. just after the compilation.

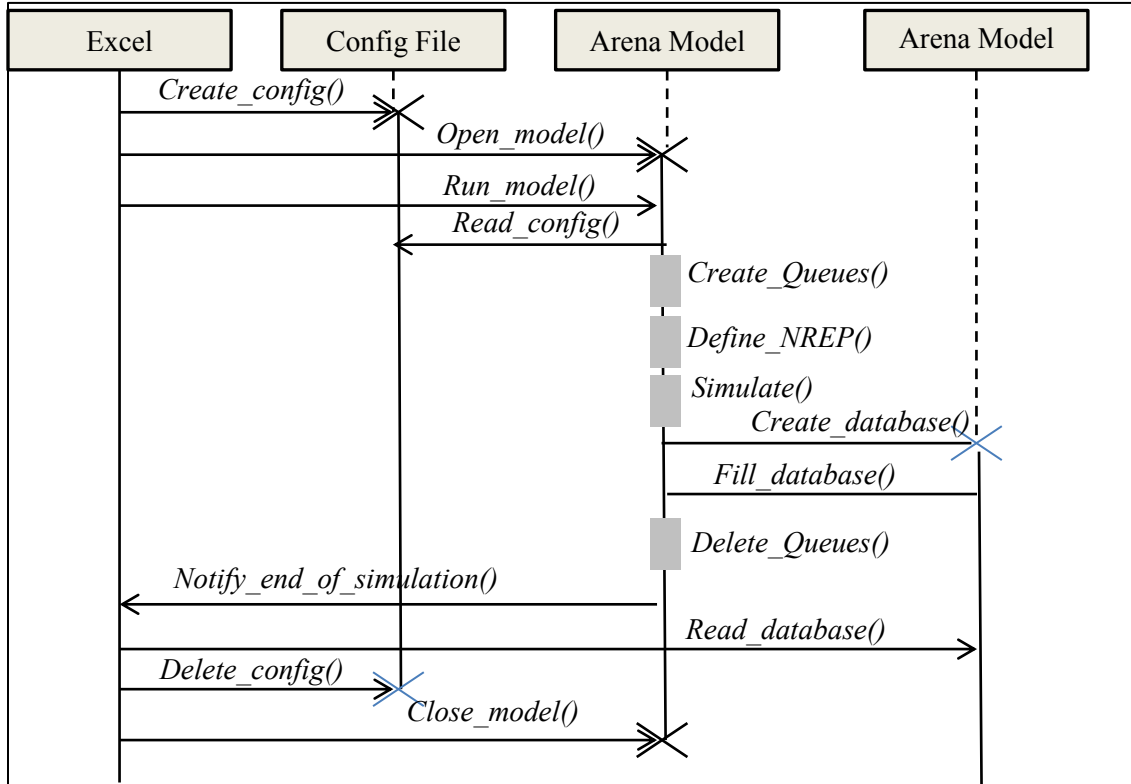


Figure 3: Sequence diagram of the simulation run.

The result (\overline{m}_l) is estimated via a VBA procedure, calculating, for each cell containing a load i , the average value of the first occurrence of such a load in the cells. With this procedure, the simulation model in itself is extremely simple, as it is only used to store the loads in the random cells. As a matter of fact, the simulation run ends when all the loads are stored.

Running with multiple replications was chosen in order to deal with the re-initialization of random distributions. The sequence diagram of Figure 3 exhibits the NREP parameter. This notation corresponds in Arena to the number of replications of the run. A preliminary study was run in order to estimate the correct number of simulations to define in the previous process. To do so, three physical configurations of racks were created: (1000 cells of 50 segments), (100 cells of 25 segments) and (25 cells of 10 segments) corresponding to a huge, an intermediate and a small rack. For each configuration low and high ρ (0.1 and 0.9) and α_i (0.1 and 0.5) were tested in a set of 6 configurations. For these 6 configurations, the same random heuristics were simulated on 5, 10, 20, 30, 40, 50, 60, 70, 80, 90 and 100 replications. Considering that the highest NREP provides more accurate results, we chose to express the obtained results in the form of the error relative to the average of the values obtained for NREP=80, 90 and 100. The results are shown in Table 1. Configurations providing less than 1% error are highlighted in green.

The table clearly shows that the reliability problems occur most often with small racks, which is quite logical as the number of loads is lower than in bigger configurations. In order to be secure the process, we chose to run systematically NREP=70 replications.

Table 1: NREP study results.

NREP	Cells=1000	Cells =1000	Cells =100	Cells =100	Cells =25	Cells =25
	Segments=50	Segments =20	Segments =25	Segments =25	Segments =10	Segments =10
	$\rho=0.9$ $\alpha_i=0.1$	$\rho=0.1$ $\alpha_i=0.5$	$\rho=0.9$ $\alpha_i=0.1$	$\rho=0.1$ $\alpha_i=0.5$	$\rho=0.9$ $\alpha_i=0.1$	$\rho=0.1$ $\alpha_i=0.5$
5	0.35	0.77	0.68	0.09	3.33	-1.77
10	0.19	-0.03	0.34	0.12	-1.29	2.52
20	-0.21	-0.2	0.89	1.38	-4.7	2.57
30	-0.18	0.16	-0.21	1.12	-1.35	2.77
40	0.08	0.23	0.53	0.92	0.41	1.22
50	-0.11	0.24	0.3	0.57	0.17	1.12
60	0.01	0.25	-0.29	0.12	-0.13	0.88
70	0	0.08	-0.74	-0.04	0.2	0.51
80	-0.01	0.04	-0.27	-0.03	0.75	-0.38
90	0.06	0	0.46	0.1	-0.42	0.37
100	-0.05	-0.04	-0.19	-0.07	-0.33	0.01

6 ANALYTIC DETERMINATION OF THE AVERAGE RANK

From the set of data obtained by simulation as explained above, we used regression techniques in order to obtain an analytic formula of the average rank. The general solution is complex and not unique. It depends strongly on the parameter α_i , it is why we present a set of solutions function of the range of α_i . We have considered 3 intervals for this parameter.

1) α_i close to 0:

We represent hereafter, using the software Maple the set of points obtained by simulation. Each point gives m_i as a function of ρM and α_i . From Figure 4, we can observe that for α_i close to 0, which means that there are very few loads of type i in the system, m_i has an inclined straight line. Intuitively, in this situation, the load of type i will be retrieved from the middle of the full part of the cell.

$$\bar{m}_i = \frac{1}{2} \rho M \tag{4}$$

In Figure 5 hereafter we represent the straight line $m_i = \frac{1}{2} \rho M$ and the cloud of points obtained by simulation, by setting α_i to 0.05.

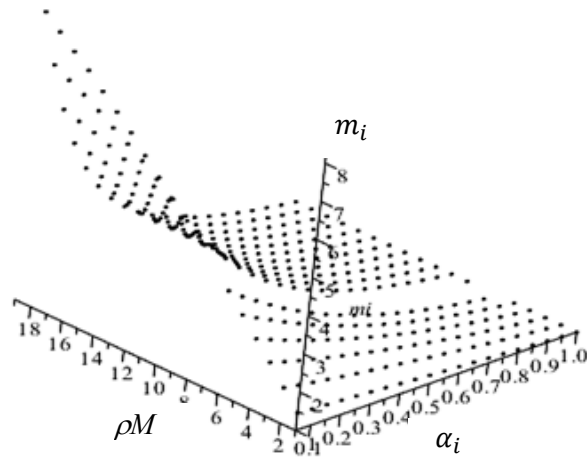


Figure 4: Curve representing the cloud of points obtained by simulation.

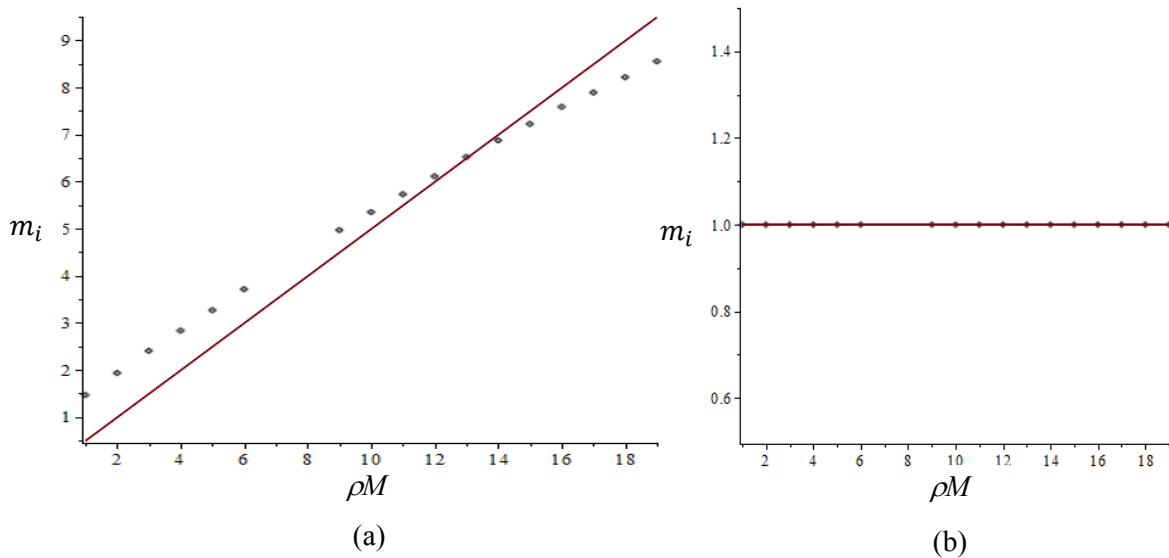


Figure 5: Curve representing the cloud of points obtained by simulation with: (a) $\alpha_i = 0.05$. (b) $\alpha_i = 1$.

2) α_i close to 1:

From Figure 4, we observe that for α_i close to one (all the loads or the majority of them are of type i), m_i has a straight line. Intuitively, in this situation, the load of type i will always be retrieved from the first segment of the cell. In Figure 6 hereafter we represent the straight line $m_i = 1$ and the cloud of points obtained by simulation, by setting α_i to 1.

3) α_i in comprised between 0 and 1:

In the general case, when α_i is between 0 and 1, the shape of the curve of the function m_i is as represented in Figure 6. We have chosen a polynomial solution for its simplicity. The first tests have shown that the output depends on the square of the input parameters. The powers greater than 2 are neglected. By applying a linear regression on the data obtained by simulation, we found that the polynomial function that best models the relation among the average rank m_i , the load rate ρ and the proportion of the load of type i in the AS/RS α_i , is as follows:

$$m_i = 6.10\alpha_i^2 - 10.37\alpha_i + 0.16\rho M + 4.38$$

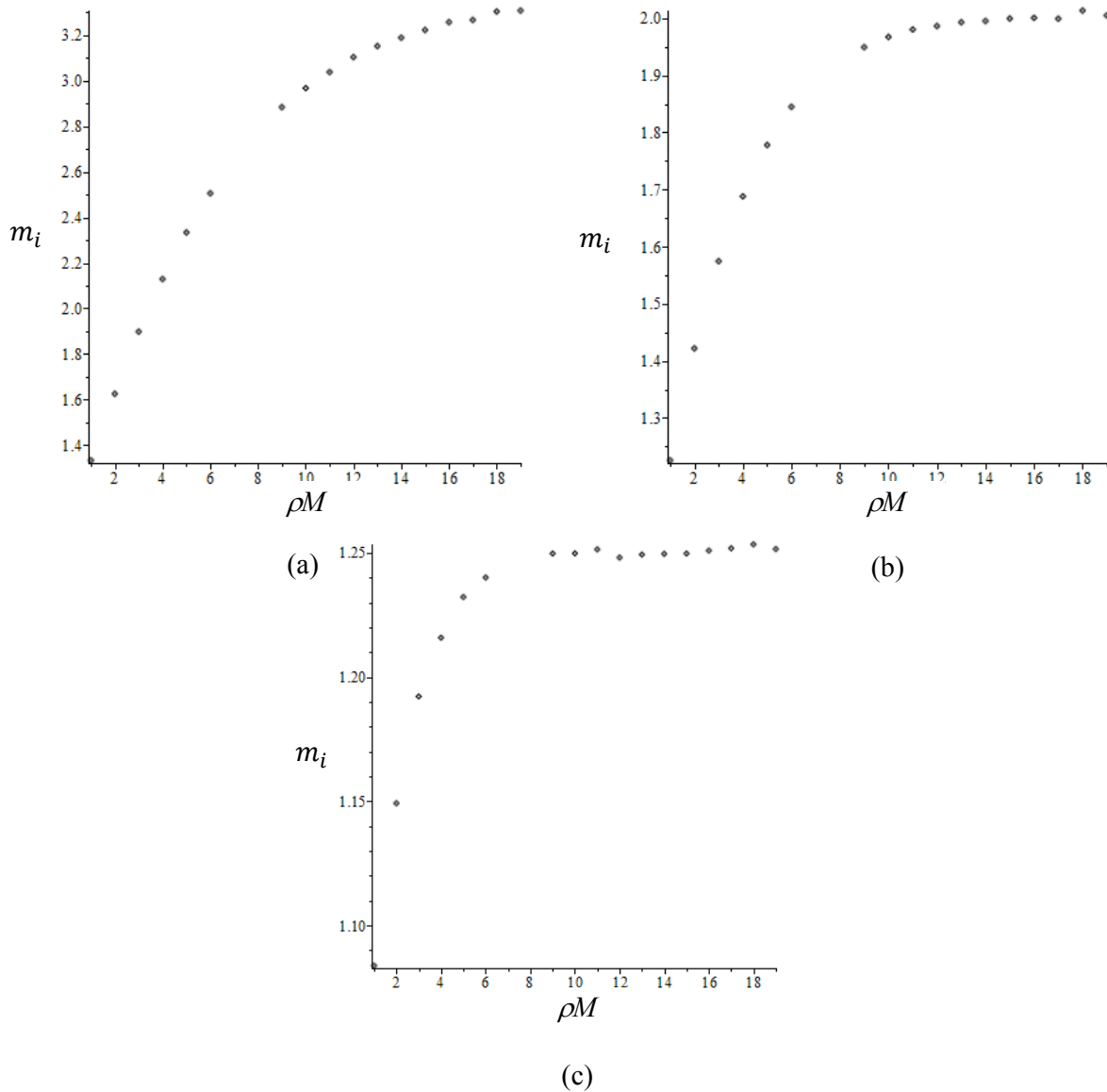


Figure 6: Curves representing the cloud of points obtained by simulation by setting: (a) α_i to 0.3, (b) α_i to 0.5 and (c) α_i to 0.8.

Figure 7 hereafter presents both the curve of the empirical proposed solution and the cloud of points obtained by simulation. These formulas can be used efficiently for the catalytic computation of the average retrieval time in any kind of AS/RS. This corresponds to the usefulness of our contribution.

7 VALIDATION OF THE EMPIRICAL MODEL

In this section we will compare the result of the empirical model with simulation results. To this purpose we have simulate several configuration, that were different from those used to the model development. We consider 1000 cells. Each cell has a capacity of 50 items. In this system we vary the load rate ρ between 10% and 90%, with a step of 20%. This means that ρM varies between 5 and 45 with a step of 10. We also vary the item existence rate α_i between 10% and 90% with a step of 20%. The results are summarized in Table 2 below. From table 2, we can notice that the error of the empirical model is relatively interesting. It is low when the parameter α_i is low and increases when α_i approaches 1. This can be improved by increasing the points considered during the fitting phase.

Table 2: Validation of the empirical model.

ρM	α_i	\bar{m}_i	\bar{m}_i	Error (%)
5	0.1	3.95	4.20	6.49
	0.3	2.47	2.62	5.95
	0.5	1.75	1.52	13.07
	0.7	1.34	0.91	32.19
	0.9	1.09	0.79	27.71
15	0.1	6.80	5.80	14.63
	0.3	3.06	4.22	37.96
	0.5	1.92	2.12	10.36
	0.7	2.40	2.51	4.77
	0.9	1.80	2.39	32.79
25	0.1	8.21	7.40	9.79
	0.3	5.23	5.82	11.34
	0.5	3.97	4.72	18.89
	0.7	3.41	4.11	20.38
	0.9	2.90	3.99	37.38
35	0.1	9.09	9.00	0.90
	0.3	6.30	7.42	17.73
	0.5	4.99	6.32	26.58
	0.7	4.42	5.71	29.06
	0.9	4.11	5.59	36.06
45	0.1	9.66	10.60	9.77
	0.3	8.33	9.02	8.23
	0.5	7.00	7.92	13.17
	0.7	6.73	7.31	8.64
	0.9	5.11	7.19	40.67

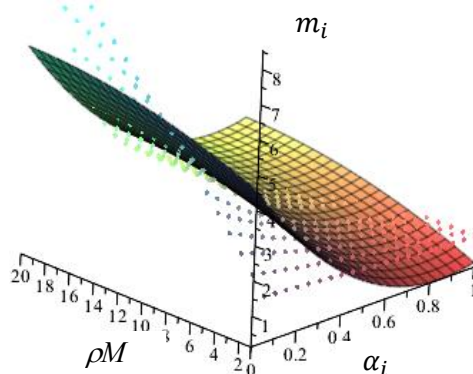


Figure 7: The curve of the empirical proposed solution and the cloud of points obtained by simulation.

8 CONCLUSION

In this paper, we have provided an analytic formula for the computation of the average retrieval rank in a multi-deep automated storage/retrieval system. There is no mathematical solution to the general problem, hard assumptions are considered to tackle this problem. Then we have proposed a parametrized simulation associated with a regression technique to answer this problem. It is then easy to deduce the mean retrieval time. Polynomial solutions have proved to be satisfactory approximations. The particular case of the flow-rack automated storage/retrieval system has been considered to illustrate our contribution. This formula can easily be used for other multi-deep systems.

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