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TOWARDS A MODEL OF THE U.S. STOCK MARKET: HOW IMPORTANT IS THE SECURITIES INFORMATION PROCESSOR?

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ABSTRACT

Both the scientific community and the popular press have paid much attention to the speed of the Securities Information Processor – the data feed consolidating all trades and quotes across the US stock market. Rather than the speed of the Securities Information Processor, or SIP, we focus here on its importance to efficient, price discovery. Via extensions to a previous market model, we experiment with four different coupling mechanisms which operate across the US stock market. Of the four, we find that the SIP contributes most to efficient price discovery.

1 INTRODUCTION

While the scientific community and the popular press have paid much attention to the speed of the Securities Information Processor (SIP) – the data feed consolidating all trades and quotes across the US stock market, here we address its importance. Rather than the speed of the SIP relative to other data services, we focus on its importance to a key measure of market quality, namely efficient price discovery. Via extensions to a previous market model (Bookstaber, Foley, and Tivnan 2016), we experiment with four different coupling mechanisms which operate across the US stock market. Of the four mechanisms to overcome market fragmentation, we find the SIP contributes most to efficient price discovery.

In the sections which follow, we provide an overview of the National Market System and the SIP while adding some clarity to the debates surrounding the SIP. We summarize the relevant literature of previous computational models of financial markets and our subsequent contribution to this literature. We describe our four classes of experiments and present our findings. We conclude with a brief discussion of the implications of our results, both for policy and for future research – to include considerations for a related "Hilbert Problem" (Hilbert 1902) for the modeling of financial markets.

1.1 Overview of the National Market System

The National Market System (colloquially known as the "stock market") includes all market centers where investors can buy and sell shares of publicly traded companies. To facilitate the efficient exchange of capital and shares in the National Market System (NMS), each market center is required to publish both the *Best Bid* (i.e., the highest price at which an investor is willing to pay for a single share of a given stock) as well as how many shares the investor is willing to purchase at that price. In addition, each market center is also required to publish the *Best Offer* (i.e., the lowest price at which an investor is willing to sell a single share of a given stock) as well as how many shares the investor is willing to sell at that price. Across the entirety of the NMS, the highest bid and the lowest offer comprise what is known as the *National Best Bid and Offer* (NBBO) and the difference between the Best Bid and Best Offer is known as the spread.

Recall that the SIP distributes the best quoted prices across all exchanges. But what happens if the Best Bids and Best Offers cross; for instance, if the highest buy order for shares of Apple at BATS exceeds the lowest sell order for Apple at Nasdaq? Such an event is called a *crossed book*; a crossed book occurs whenever the best quote is not less than the best offer. While crossed books are prohibited by SEC Regulation National Market System (RegNMS), crossed books certainly happen and occasionally they occur with a high degree of dependence. Figure 1 depicts the time series of price for one of the most heavily traded securities in the market, SPY, on a day of particularly high volatility. Notice that though spreads tightened throughout the day, the SPY resulted in a cross book throughout the day. To date, no one has been able to model such events.

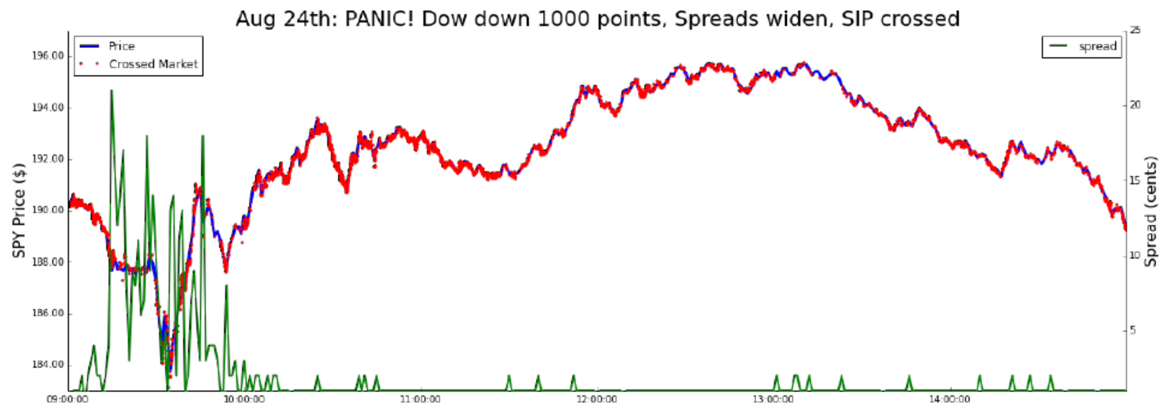

Figure 1. Crosses and Spreads in the SPY on August 24th, 2015 – "Manic Monday."

More context on the NMS might prove beneficial before delving into the challenge of modeling such events as those depicted in Figure 1. The NBBO reflects a distillation of the order flow across all the stock exchanges comprising the NMS. Each stock exchange has a proprietary data product commonly referred to as a *Direct Feed.* The Direct Feeds contain data on the entire order flow between the exchanges across the NMS. The Securities Information Processor, or SIP, consolidates and distills all the order flow from the Direct Feeds to determine and disseminate the NBBO as well as report all completed trades.

1.2 Debate Related to Relative Latencies between the Direct Feeds and the SIP

A swirling debate rages around the fairness of the relative latencies between the direct feeds and the SIP (e.g., see Arnuk and Saluzzi (2012), Patterson (2012), Lewis (2014), and Nanex (2014) among many others). As this debate can be nuanced and occasionally filled with arcane subtleties, elsewhere we lay bare the implications of these relative latencies via a sports analogy (Tivnan and Tivnan 2016). In this paper, we attempt to refine the related discourse by addressing two related question:

1. Does a sufficiently rich model of the NMS exist to inform the regulatory community of essential elements of a stable market?
2. If not, what would constitute a sufficiently rich model of the NMS to inform regulation?

In the ensuing sections, we begin to address these questions by developing a model with explicit representations of the four mechanisms which yield the interdependent coupling of the various components of the NMS. In the conclusion, we revisit both the importance of and challenges with modeling the NMS.

2 LITERATURE REVIEW OF MARKET MODELING

While there has been much discussion of the potential for agent-based models to advance the study of financial markets (Bookstaber 2012), it is important to note the evolution from agent-based models of an individual exchange to models of a financial market like the NMS.

2.1 Models of an Individual Exchange

Consistent with two of the prevailing design issues (LeBaron 2001a), the evolution of the set of agent-based models of an individual exchange follows along two distinct paths, one focused on the representation of the agents and the other focused on the representation of the trading mechanism (i.e., market microstructure). The former trajectory began in the early 1990s with the Santa Fe Institute (SFI) market model (Palmer et al. 1994). Largely predicated on Holland's (1977) genetic algorithms, the SFI market model was well received as a novel departure from equilibrium models, largely based on its qualitative depiction of agreement with empirical observations of market dynamics such as bubbles and crashes. Following the SFI market model, Lux and Marchesi (1999) introduced a model with a single trade type (i.e., market orders) that was the first to demonstrate clustered volatility, one of the stylized facts common to many markets. Similar to the Lux market with market orders, LeBaron (2001b) introduced a market model with agents that learn based on a neural network. In the LeBaron model, agents decided how much of their total wealth to invest; therefore, successful agents can have a large impact in the market. Cont and colleagues (Ghoulmie et al. 2005) developed a model that qualitatively reproduces the three prevailing stylized facts described above. Of note, the traders in the Cont models have heterogeneous trading thresholds, and the traders, many of whom trade rather infrequently, adapt their thresholds based upon performance feedback. More recently, Johnson et al. (2013) developed a model that quantitatively reproduces so called "Flash Crashes" in the market (i.e., dramatic price movements which are immediately reversed).

The trajectory of models focused on trading mechanisms began with Maslov (2000) nearly a decade after the SFI market model. Maslov's model was quickly extended by Darley et al. (2001) and Farmer et al. (2005). The Farmer model (subsequently referred to as the ZIM for Zero-Intelligence Model) built a model of zero-intelligence traders active within the structure of a continuous, double auction placing two types of orders: market orders and limit orders. Market orders are orders that enter the market with an intent to buy or sell a certain number of shares and do not specify a particular price. Limit orders, on the other hand, enter the market with both a specified price and quantity of shares. Since market orders do not specify a price, they are executed immediately upon entering the market at the best available price. Limit orders, however, will accumulate in the market until their specified price is met or they are cancelled. The accumulation takes place in a prioritized queue by price and arrival time. This accumulation of limit orders is called the limit order book. It is this accumulation of limit orders that creates liquidity (i.e., the ability for market orders to be executed) in the market. In the ZIM, both market and limit orders arrive and are cancelled per a Poisson process. Although not as rich as the Johnson et al. (2013) model which can replicate the dynamics of market events such as the 2010 Flash Crash, the ZIM is useful in exploring structural aspects of market dynamics.

Adhering to the cumulative approach espoused by Axtell et al. (1996), Preis et al. (2006, 2007) first replicated and then extended the ZIM. Their model reproduced a leptokurtic distribution of returns, which the ZIM did not. Because of this extension, while also adhering to the Axtell et al. (1996) cumulative approach, Bookstaber, Foley and Tivnan (2016; referred to as BFT) used the Preis model as a building block for their extensions to explore market-relevant aspects of heterogeneity that are not contained within the Preis model. While demonstrating the impact of heterogeneous decision cycles on market resilience and the stochastic properties of market prices, BFT present the first model to depict recent, empirical findings relating to order flow toxicity (Easley et al. 2011).

2.2 Market Models with Multiple Exchanges

Unlike the evolving set of models of an individual exchange, there are very few market models with multiple exchanges, thereby even beginning to represent the complexity of the National Market System. Wah and Wellman (2013) were the first to model a simplified, fragmented market with two exchanges and a frictionless arbitrageur. Wah and Wellman find that market fragmentation and the latency arbitrageur reduce market liquidity, so they argue for modifications to the market microstructure (i.e., the replacement of continuous-time markets with periodic batch auctions as advocated by Budish et al. (2015)).

3 METHODS

To evaluate the importance of the SIP in the context of the various mechanisms which couple the fragmented market, we extended the simplified model of the National Market System (Wah and Wellman, 2013) and integrated the BFT model of the individual exchanges.

3.1 Our Model of the National Market System

Following the cumulative approach advocated by Axtell et al (1996), we build on the generally accepted approach of modeling financial markets established by BFT and their predecessors, to replicate and extend the BFT single market, continuous double auction order book models. The extensions to the previous models are the following:

- Representation of multiple markets
- Configurable communication links and message delay times
- A SIP/consolidated tape like facility for multi-exchange best bid and offer distribution to traders
- Cross-exchange latency arbitrage traders who can trade when prices across exchanges diverge
- *Circuit breakers* that halt trading if the percent change in price exceeds a limit over any interval less than a grace period
- Rejection of *stub* quotes that exceed a limit above or below the last trade price

The objective of the initial experiments is to analyze the effect of the SIP, cross-exchange traders, and latency arbitrageurs on price divergence when measured at different exchanges and the SIP. *Price divergence* is defined as the difference in price of a stock when measured at the same time at two different locations.

3.2 Simulation Methods

Four experiments were conducted to explore price divergence due to the coupling of finance and communications. The experiments are specified with the following:

- **Patient traders** who place buy or sell limit orders per their perception of the value of the stock being traded. Their perception is set by a trending variable that moves up or down at a fixed rate. A single trending variable is used for all markets to represent shared perception of the value of a stock. They place orders at rate $\alpha \in [0,0.5]$ and cancel them at rate $\delta \in [0,0.2]$ on their single primary exchange. 250 patient traders connect to each exchange.
- **Impatient traders** who place market orders at rate $\mu \in [0,0.2]$ on their single primary exchange. 250 impatient traders connect to each exchange. Impatient traders do not trade before timestep 500 to allow for orders to fill the order book from the patient traders and prevent order book collapse (where one side of the order book empties).

- Experiments 3 and 4 represent an additional trader class, **cross-exchange, latency arbitrage traders.** These traders are connected to both exchanges directly with communication links that allow them to receive market information faster than the other types of traders. When prices diverge across exchanges, cross-exchange traders place buy orders at the exchange with a lower price and sell orders at the exchange with a higher price. Experiments 3 and 4 had 10 cross-exchange traders that placed orders at a fixed rate of 1000.
- **Two exchanges** with continuous double auction order books. Market orders execute immediately while limit orders are placed in a queue that executes with price, then time priority. Orders that happen at the same time are executed per the following rules. If the sell order was placed after the buy order the sell order is executed. If the sell order was placed before the buy order, then the buy order is executed. If they were placed at the same time, then a random draw chooses which order to execute.
- All traders buy and sell a single stock of share size 1 and initial price of $1,000.

Each replicate is run for 5,000 timesteps.

3.3 Experimental Design

The four experiments are defined as follows:

1. **Experiment 1, Figure 2** – Two exchanges with no communication and all traders only communicate with their primary exchange.
2. **Experiment 2, Figure 3 –** Two exchanges with patient traders using the SIP to determine the current best price at both exchanges rather than just their primary exchange.
3. **Experiment 3, Figure 4** – Two exchanges with cross-exchange latency arbitrage traders using direct exchange communication links. When prices diverge across exchanges cross-exchange traders place buy orders at the exchange with a lower price and sell orders at the exchange with a higher price. All other traders use only their primary exchange feed to determine the last trade price.
4. **Experiment 4, Figure 5** – Two exchanges with cross-exchange latency arbitrage traders using direct exchange communication links and patient traders using the SIP to determine the current best price at both exchanges.

Experiments were executed in a Design of Experiments across the μ, α, δ parameter space based on an Orthogonal, Latin Hypercube (Kleijnen et al. 2005) sampling of those parameters. Each μ, α, δ combination represents a design point in the experiment. 25 replications of the simulation were performed at each design point where each replication used a different random number seed. At each replicate within each design point a set of so-called stylized facts were calculated to determine how well the statistical characteristics of the represent the characteristics of real-world markets. The stylized facts calculated for this experiment were the kurtosis, clustered volatility, and Student's T-test of the autocorrelation function. An initial 500 design points were pared down to 78 that exhibited at least one replicate that had kurtosis > 5, clustered volatility > 5 and Student's T-test of the autocorrelation function > 0.05. These 78 design points were run for the remaining three experiments for a total of 18,350 realizations of the model (i.e., 25 replications at each of the initial 500 design points in Experiment 1, then 25 replicates for each of the 78 design points that passed the stylized facts for the remaining three experiments, (25 x 500) + (25 x 78 x 3) = 18,350).

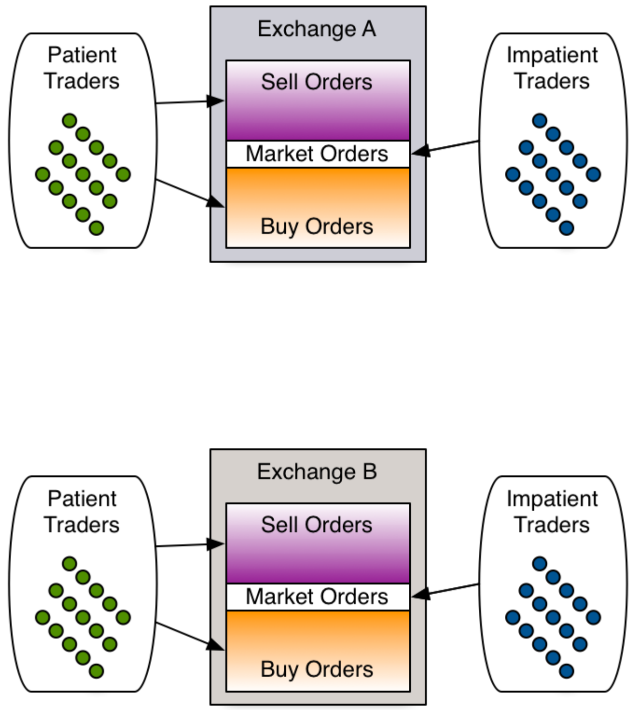

Figure 2. Experiment 1 - two Exchanges, no communication between them.

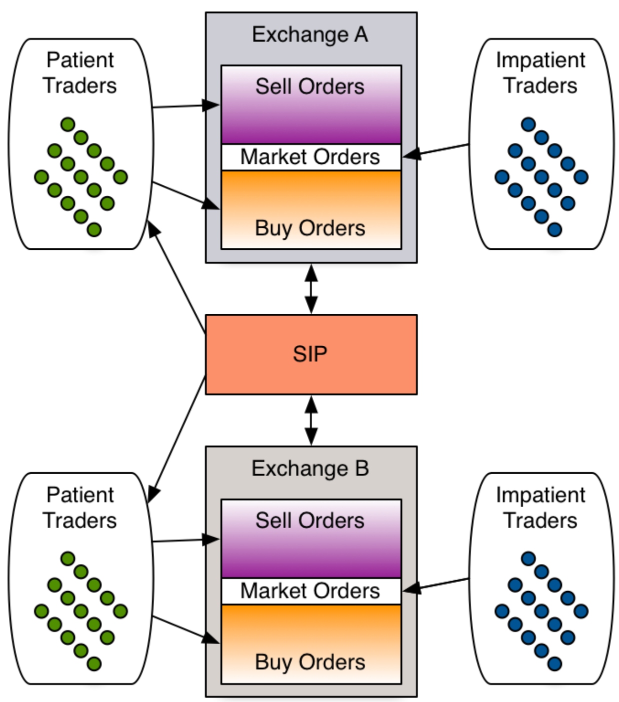

Figure 3. Experiment 2 - two Exchanges, SIP communicates best bid/offer to patient traders.

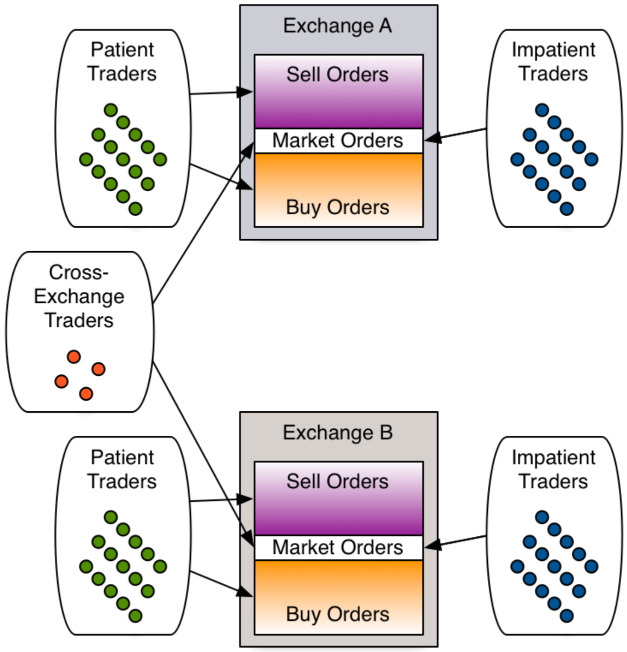

Figure 4. Experiment 3 - two Exchanges, cross-exchange traders place orders at the market with the best price.

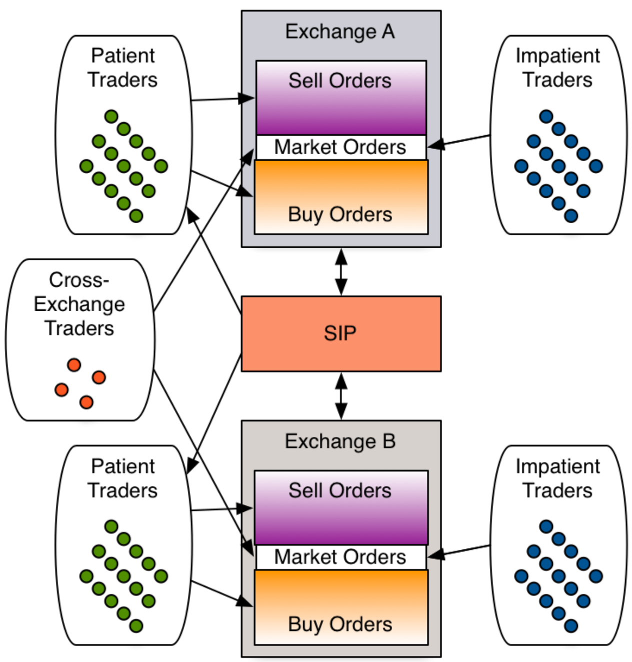

Figure 5. Experiment 4, SIP communicates best bid/offer to patient traders and cross-exchange traders place orders at the market with the best price.

4 PRELIMINARY SIMULATION ANALYSIS

The preliminary results from these experiments show clear differences in market behavior because of the changes in configuration. For each of the experiments the exchanges are labeled BATS and NYSE for identification purposes only. This labeling is not meant to imply that the exchanges in the model represent any Bats exchange or the New York Stock Exchange but rather the differing connections between the exchanges as shown in Figure 2 through 5.

With no connection between the exchanges, Experiment 1, we do not expect to see any relationship in price between them other than what is driven by the trending variable. In the plot of Experiment 1 in Figure 6 we see, for example, a minimum price of approximately $400 at BATS and at the same time a minimum price of approximately $200 at NYSE. The same phenomenon is seen in the other direction for Experiment 1 where BATS can have a lower minimum than NYSE. When a SIP is added in Experiment 2 we see in Figure 6 that the minimum at BATS is always equal to or greater than the minimum at NYSE. In Figure 7 the maximum price can be above or below the other exchange's maximum for Experiment 1 while in Experiment 2 the maximum price at BATS is always equal to or less than the maximum price at NYSE.

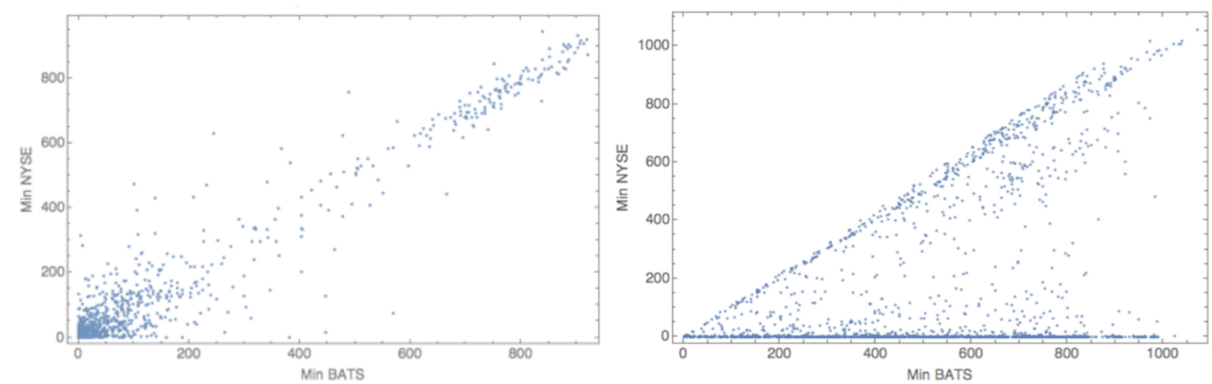

Figure 6. – Experiment 1 (left), Experiment 2 (right) – Minimum price seen at the exchanges.

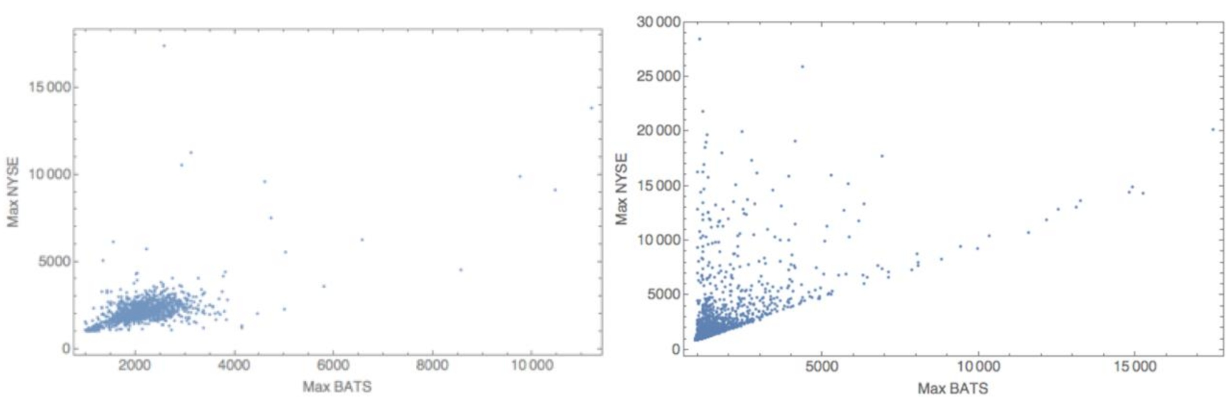

Figure 7. – Experiment 1 (left), Experiment 2 (right) – Maximum price seen at the exchanges.

Recall that the SIP distributes the best price across all exchanges. These results in Figures 6 and 7 show that the addition of a SIP keeps the price divergence at minimum and maximum price points to one exchange. In other words, a buy order at BATS in Experiment 1 could be higher than a sell order at NYSE – resulting in what is called a *crossed book*. If exchanges were connected as in Experiment 2, a crossed book occurs whenever the best quote is not less than the best offer. As discussed above, crossed books are prohibited by the SEC in RegNMS. These results provide preliminary indication that the SIP, as modeled for these experiments, limits crossings in order books between exchanges in accordance with RegNMS.

In these experiments, price divergence can be thought of as the difference in how a stock is valued at different places at the same time. The analysis of the experiments was done on the mean price divergence defined as the average difference in price at one exchange and the other at the times where every trade occurs. This adds up the difference in price every time a trade occurs and divides by the total number of trades.

The experiments show some clear differences in price divergence shown in Figure 8. In Experiment 1 with no communication between exchanges the divergence has two peaks at \$100 and \$220 and other divergences distributed across both sides of those peaks. In contrast the addition of the SIP in Experiment 2 leads to a single peak at \$100 and other divergences spread to one side of the peak. InTable 1 we find that compared to Experiment 1, Experiment 2 has nearly double the number of replicates have a mean price difference of \$100 or less. The difference in distributions indicates that a trader operating in the world of Experiment 2 would expect to see a smaller difference in price for a stock than they would in Experiment 1. This implies there is more stability in the stock price with the addition of the SIP in Experiment 2.

The addition of cross-exchange traders in Experiments 3 and 4 results in some notable differences with Experiments 1 and 2. Comparing Experiment 1 with Experiment 3 where neither has a SIP but cross-exchange traders operate in the latter we find that in Experiment 3 the second peak disappears appears to have a higher peak in the distribution of mean price divergence found in its replicates. Experiment 3 has just 58% of replicates that fall under \$100 mean price divergence compared to Experiment 1. Adding a SIP, Experiment 4, we find nearly two-and-a-half times the number of replicates with a mean price divergence of less than \$100 than without a SIP. While this is a significant improvement the total number of replicates with a small price divergence is less than what we saw for Experiment 2 where there were no cross-exchange traders. These preliminary results show that the presence of cross-exchange traders does not decrease price divergence in cases with and without a SIP.

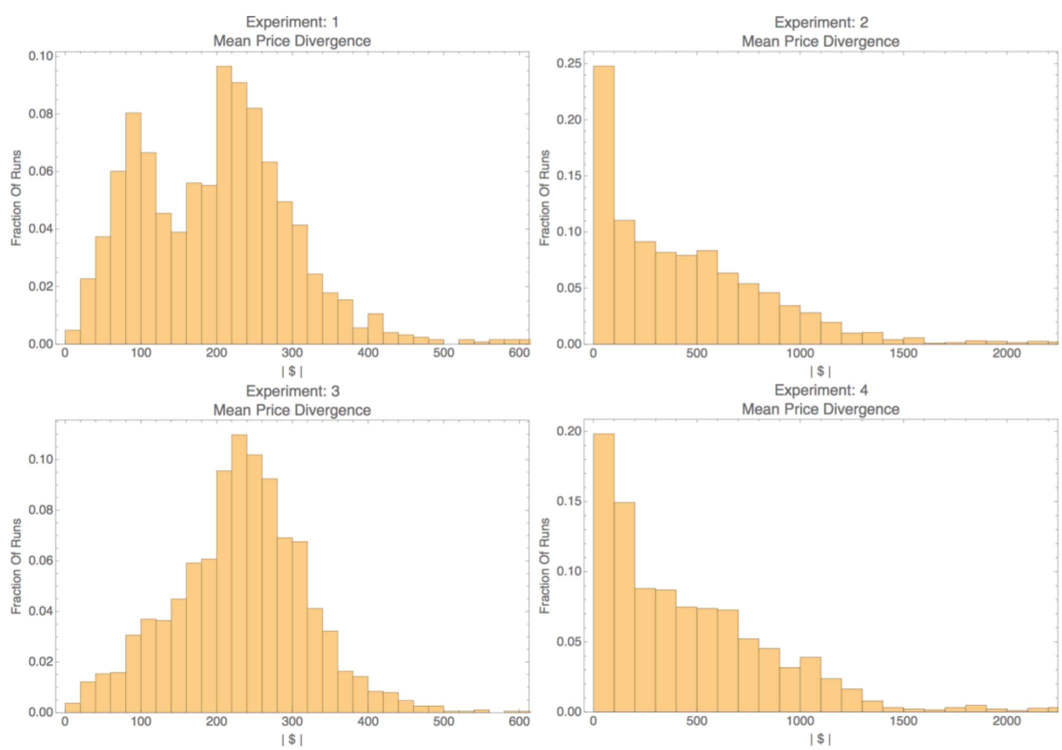

Figure 8. Mean price divergence.

Table 1. Mean price divergence of < $100 across runs.

Experiment	Replicate count	Percentage of replicates (%)
1	253	13
2	469	24.1
3	147	7.5
4	376	19.3

5 CONCLUSION

We conclude with a brief discussion of the implications of the results from our computational experiments as well as a brief discussion of a related "Hilbert Problem" for the modeling of financial markets. Our results give a preliminary indication that, with similar market conditions, the presence of a SIP lets traders operate in markets with low expected price divergence. This could mean that the SIP provides a beneficial coupling between exchanges via communication networks. The experiments conducted with the National Market System (NMS) Model are a first step at exploring the complex space where financial markets and communication networks interact. From here the model could be used to analyze many different questions of relevance to policy makers including (1) the cost of interruptions to data flow due to events ranging from weather to targeted attacks, (2) the impact of regulatory policies on market behavior, and (3) determining the circumstances where beneficial couplings between infrastructures emerge or collapse.

We conclude this discussion with a brief description of a possible "Hilbert Problem" for the modeling of financial markets. Consider that an essential component of the foundation of human civilization is exchange, from the exchange of ideas to goods even to the exchange of actual DNA. Building on advances in computational complexity, Axtell (2005) identifies the limitations of the conception that a Walrasian auctioneer can centrally determine prices – an essential aspect of the market's current state. Buss et al. (1991) show that simulation provides the best analytical approach to solve the state prediction problem. It stands to reason then that a possible "Hilbert Problem" should address the development of a broad class of models to solve the state prediction problem for general exchange. In the special case of financial exchange (e.g., National Market System), it has only recently become feasible to fully specify the state of the current set of agent preferences as manifested by their order flow. Can we build from our knowledge of the interaction rules governing exchange and our ability to specify current state to develop a class of models to solve the state prediction problem for exchange systems?

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