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AGENT-BASED EXPLORATION OF THE POLITICAL INFLUENCE OF COMMUNITY LEADERS ON POPULATION OPINION DYNAMICS

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ABSTRACT

Population consensus may lead to scenarios of positive feedback in which the momentum toward a consensus could result in outcomes that may not be in the best interests of society—the opinion dynamics that lead to support for a negotiated settlement or peace agreement might similarly lead to mass violence and riots. Given this, additional insight into how consensus might be influenced has broad implications for the betterment of society. We extend current literature in continuous opinion dynamics modeling under heterogeneous bounds of confidence by introducing population interactions with multi-track leadership. We theorize that the presence of multi-track political influence—particularly from non-formal, community-based authority—richly enhances the exploration of consensus formation and provides a new framework for understanding the opinion formation process. We present an agent-based approach to extend the Hegselmann-Krause opinion dynamics model to include multi-track leadership and show that community leaders can significantly contribute to consensus formation.

1 BACKGROUND

The formation of public opinion consensus and how it changes over time has significant real-world consequences. As public opinion evolves, it may lead to positive feedback within the system, with the momentum of increasing returns leading to substantial opinion change that may or may not be beneficial to society. Beneficial outcomes may include support for a political settlement or peace agreement, or voluntary adherence to community-focused norms that better manage common pool resources such as water. On the contrasting side, these positive feedback phenomena may result in an impetus for mob violence or riots. One facet of opinion dynamics and consensus formation is based on interactions of the public with multi-track leadership and authority. As such, greater insights into the role of leadership in the evolution of population opinion will be useful to better understand how consensus might be influenced so that opinion outcomes can be directed toward the betterment, safety, and well-being of society. In this research, we specifically focus on the context of the peacebuilding process.

1.1 Multi-Track Diplomacy

Multi-track diplomacy emerged from a distinction made by Joseph Montville in the 1980s between official governmental leaders (referred to as Track I actors) and unofficial non-governmental leaders (referred to as Track II actors) engaged in a conflict resolution process (Notter and Diamond 1996). Diamond introduced the term “multi-track diplomacy” as an extension of the concept in recognition of the variety of actors who could be engaged in this unofficial process (Diamond and McDonald 1996).

With respect to conflict resolution and peacebuilding, consideration of all actor populations is important, as each subgroup has nonlinear and interdependent influence on the others—individual opinion (and opinion shifts) may be strongly influenced by the opinion dynamics within and outside of an individual's actor or Track population group.

Figure 1 graphically represents a conceptual model developed by Lederach (1997) that describes the multi-track diplomacy concept as applied to the peacebuilding process. He describes three levels of actors—Level 1, or top leadership; Level 2, or middle-range leadership; and Level 3, or grassroots leadership (Lederach 1997). In this research, we operate on the assumption that individuals in Level 2 (middle-range leadership) are the authority figures (community leaders) who are mobilized for Track II diplomacy.

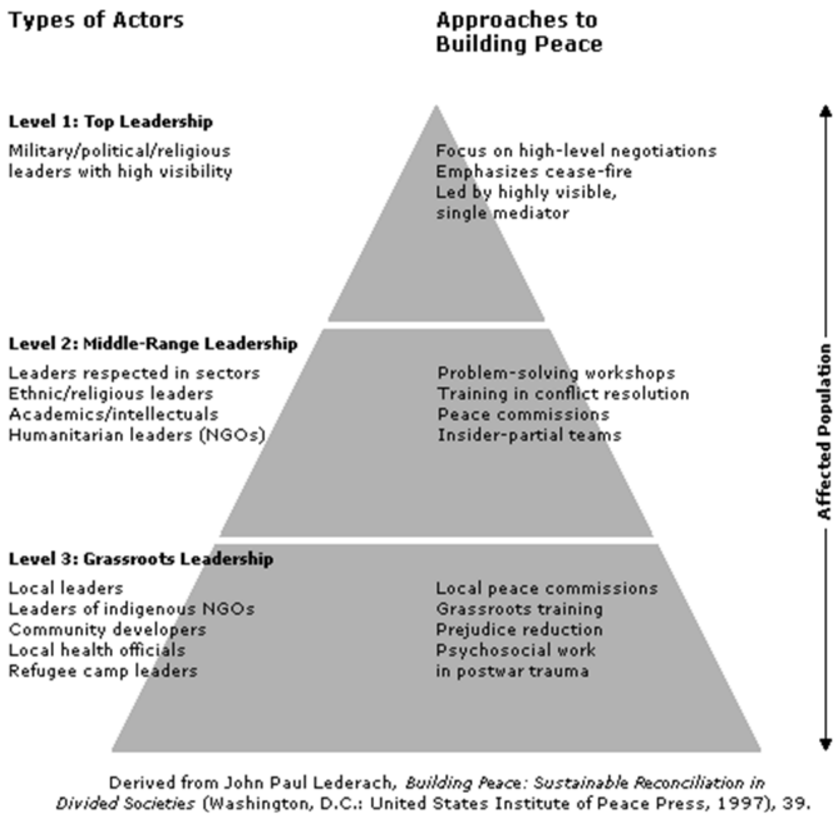

Figure 1: Lederach conceptual model for actors in a peacebuilding process.

1.2 Role of Track II Leadership

Figure 1 highlights that the processes for conflict resolution or negotiation are not always—nor even in the usual case—limited to official governmental leaders (Track I). Much of the current conflict resolution international mediation process emphasizes the use of Track II leaders as a mechanism to promote resolution. There are two primary motivations for emphasizing Track II leaders. First, because Track II leaders are not official governmental leaders, they may have situational and political flexibility that formal leadership does not have and can participate more creatively in problem-solving processes. Second, these leaders have direct influence on Track I leadership and grassroots membership, potentially enabling them with more influence on the population than Track I leaders.

For our research, we also hypothesize that Track II leadership has a significant influence on public opinion. In the context of conflict resolution, the scope of our study is focused on the effect of Track II leaders on public opinion and consensus building. The current state of knowledge regarding Track II leadership is primarily anecdotal or based on case study descriptions (Diamond and McDonald 1996). The field does not currently have an empirical method to compare Track II–based interventions or diplomacy to other forms or bases of political influence. The authors are not aware of any robust theories or heuristic mechanisms for how Track II influence diffuses within a polity or the conditions under which political influence may be effective. This research provides an opportunity to explore a potential computational framework for representing how Track II leadership works in practice.

2 APPROACH

2.1 Prior Research and Literature

Opinion dynamics is the research field that examines the diffusion of opinions and how they might adapt and change over time. The field employs mathematical and computational tools that place an emphasis on the dynamics of the system, recognizing that the context of opinion formation is characterized by properties of complex adaptive systems (CASs), or systems that have many individuals (agents) that interact with each other, influencing system-level changes as the agents learn from and adapt to interaction outcomes as they occur (Holland 2006). Given this CAS characterization, opinions are an emergent outcome in the system that arise from individual opinions that are continuously adapting to the changing political environment (while taking into consideration their own heterogeneous perspectives). The transmission and adaptation of opinions is founded on nonlinear, spatiotemporal interactions between individuals and other aggregated levels of organizational hierarchy. This makes the study of opinion dynamics from a CAS perspective particularly challenging. To address these challenges, our research builds upon the literature for continuous opinion dynamics under heterogeneous bounds of confidence. The focus on continuous opinion dynamics (instead of discrete) and use of bounds of confidence (in contrast to equal treatment of all agents) was selected because of our bias that in conflict scenarios within a population, individual opinions are better represented on a spectrum and that agents are influenced by other agents only if they have some level of confidence in them.

With respect to the concept of continuous opinion dynamics, decisions are not binary and exist on a continuum in which small shifts of position are possible. As Lorenz (2007) observes, "Thus opinions in continuous opinion dynamics should be expressable [sic] in real numbers where compromising in the middle is always possible. Example issues are prices, tax rates or predictions about macroeconomic variables." Continuous opinion dynamics allows for gradual shifting of individual, small-group, and polity opinion without any one agent having to "flip-flop"—our focus is in contrast to binary opinion dynamics models in which agents hold one of two possible opinions (Holley and Liggett 1975; Clifford and Sudbury 1973; Latané and Nowak 1997).

The measurement of an acceptable distance around the individual's own opinion for which they would consider the opinions of others is referred to as the bounded confidence interval (Lorenz 2007), which allows an agent to evaluate and consider only other agents with sufficiently similar opinions. We argue that it is important to consider that individuals prefer to listen—and thus consider an opinion shift—only to others with whom they agree in some respect. Conceptually, this means that any one individual will only consider an influence set of other individuals whose opinion is similar to their own and would discount (or potentially ignore entirely) the opinions of those who are very different. We also argue that it is important that these confidence bounds be heterogeneous, meaning the number of others that an individual would consider, or the range of nearby opinions an individual will listen to, may be different from the others.

In the literature, there are two extensively studied models that explore continuous opinion dynamics under bounded confidence, the Deffuant-Weisbuch (DW) model (Deffuant et al. 2000) and the Hegselmann-Krause (HK) model (Hegselmann and Krause 2002). The DW and HK models are very similar in their use of bounded confidence on a continuous opinion scale. Differences arise in how an agent aggregates the opinions around them for consideration in their own opinion shift. With the DW model, agents aggregate opinions through pairwise evaluations, slightly shifting an agent's opinion after looping through all possible pairings within the population. In the HK model, an agent aggregates all other agent opinions within their confidence bound and considers the group average for shifting their opinion (Lorenz 2007).

The original DW and HK models did have some limitations for our specific research. Both models assumed that all agents in the population have equal influence on the others (for example, no accounting for distinct agent classes, such as community leaders having a different degree of influential power than other citizens) and that any sub-populations within the larger group would engage in opinion formation in the same way (related to the previous point, no accounting for distinct agent classes potentially having different mechanistic rules for considering influences; for example, community leaders listening to their respective constituency but not to leaders from other communities). The models also did not differentiate between how much an agent might consider their own opinion over the opinion of others.

The contribution of this research builds upon the HK model, which we felt was a more plausible representation (as opposed to the DW model) of how citizens in a conflict resolution context might become aware of opinions (e.g., media campaigns, access to social media). The conceptual process of a DW model might be a better representation if opinion shifts were primarily driven by repeated individual interactions.

2.2 Hegselmann-Krause (HK) Model

To describe the HK model, consider a system of n agents, whose opinions can be expressed by a real number that is located in one-dimensional Euclidean space. The agent set is defined as $V = \{1, 2, \cdots, n\}$, where the opinion of agent $i \in V$ at time t is represented by $x_i(t)$. We use ε as the bounded confidence threshold—agent i will only consider neighbors whose opinions are within its own confidence threshold ε_i. The HK model is described in Fu, Zhang, and Li (2015) as

$$x_i(t+1) = \begin{cases} x_i(t) + \frac{1}{|N_i(t)|} \sum_{j \in N_i(t)} \left(x_j(t) - x_i(t) \right), & N_i(t) \neq 0 \\ x_i(t), & otherwise \end{cases},$$

where $N_i(t) = \{j \in V \mid \left| \left(x_j(t) - x_i(t) \right) \right| \leq \varepsilon_i\}$ is the set of "influential" neighbors for agent i at time t and $|N_i(t)|$ is the cardinality of $N_i(t)$. This assumes that agent i considers itself as part of the set. The agent updates its opinion by averaging selected opinions (along with itself) with a uniform weight, $\frac{1}{|N_i(t)|}$.

The agent then adjusts its opinion based upon the distance between their opinion and that of the average opinion of those within the confidence interval. If no other agents exist within that interval, the agent maintains its opinion and does not change position.

2.3 Modified Hegselmann-Krause (MHK) Model

An expansion of the HK model by Fu, Zhang, and Li (2015) addressed some of the limitations in the original Hegselmann-Krause model; these limitations include no consideration for self-opinion and universal treatment for all agents. The modified HK (MHK) model features an implementation of heterogeneous bounded confidence for all agents but introduces sub-populations of open-, moderate-, and closed-minded agents (Fu, Zhang, and Li 2015), each category of which used different ranges of ε_i. Most significant to our research, however, they introduced a self-weight term α that could be used to represent a measure of resistance to change or stubbornness. This allowed an agent to consider their own opinion independently from others. The MHK model presented by Fu, Zhang, and Li (2015) may be described as

$$x_i(t+1) = \begin{cases} \alpha_i x_i(t) + (1 - \alpha_i) \frac{1}{|\bar{N}_i(t)|} \sum_{j \in \bar{N}_i(t)} x_j(t), & \bar{N}_i(t) \neq 0 \\ x_i(t), & otherwise \end{cases},$$

where $\overline{N}_i(t) = \{j \in V \cap\ j \neq i| \ \left|\left(x_j(t) - x_i(t)\right)\right| \leq \varepsilon_i\}$ is the redefined set of "influential" neighbors for agent i at time t and $|\overline{N_i}(t)|$ is the cardinality of $\overline{N_i}(t)$. Using this new term for self-weighting, if $\alpha = 1$, then the agent is completely stubborn and will only consider their own opinion, $x_i(t)$, effectively ignoring all of their surrounding neighbors.

Our model builds upon this earlier work by adding an element of multi-track diplomacy to enable the exploration for how leaders at different levels may influence the opinion dynamics and consensus building process.

2.4 Multi-Track Diplomacy Hegselmann-Krause (MDHK) Model

By extending the MHK model to include multi-track leadership influences, we present the multi-track diplomacy Hegselmann-Krause (MDHK) model. Our MDHK model uses three agent classes: citizen (C) agents, Track II ($T2$) agents, and Track I ($T1$) agents.

To implement our MDHK model, we must first redefine the influence neighbor set $N_i(t)$ from how it was generated in the HK and MHK models for each agent i. In the MDHK model, every agent considers three different subgroups for influential neighbors, one for each of the different agent types. Thus, we define $\overline{N}_i^C(t)$ and $\overline{N}_i^{T2}(t)$ as the influence neighbor set for other citizens and $T2$ leaders within an agent i's confidence bounds, respectively. With respect to Track I, we assume that for all of our model run scenarios in this research that there is only one representative $T1$ agent; therefore, $\overline{N}_i^{T1}(t)$ represents one agent and not a set of multiple agents. We formally define each agent subgroup, as given by $\overline{N}_i^C(t)$, $\overline{N}_i^{T2}(t)$, and $\overline{N}_i^{T1}(t)$—in the following formalizations.

For the consideration of other citizens, $\overline{N}_i^C(t)$ is described as

$$\overline{N}_i^C(t) = \begin{cases} \{j \in V^C \mid \left|\left(x_j(t) - x_i(t)\right) \cap j \neq i\right| \leq \varepsilon_i\} & if\ i \in V^C \\ \{j \in V^C \mid \left|\left(x_j(t) - x_i(t)\right)\right| \leq \varepsilon_i\} & if\ i \in V^{T2}. \\ \{j \in V^C\} & if\ i \in V^{T1} \end{cases}$$

$\overline{N}_i^C(t)$ specifies that if agent i is of the population set of citizens, $i \in V^C$, then the agent has the same influence neighbor set as specified by the MHK model, in which the agent considers all agents, other than themselves, provided they are within their bounded confidence—citizens only listen to those with whom they are comfortable.

If agent i is of the population set of $T2$s, $i \in V^{T2}$, then the agent will consider all citizens within their bounded confidence interval—Track II leadership are only swayed by their local group of agents, such as communities, organizations, and constituencies.

If agent i is a $T1$, then they consider all citizen opinions regardless—Track I leaders are concerned with the opinions of the entire general population. Our assumption for $T1$ agents is easily changed depending on the political context for a given analysis and may be adapted to a different weighting scheme. For example, we can have $T1$ be more self-interested and therefore they will largely ignore the populist opinion, or we can identify influential citizens through use of an inverse square law that would make far away citizen opinions less influential to the $T1$ agent.

For the consideration of community leadership opinion, $\overline{N}_i^{T2}(t)$ is described as

$$\overline{N}_i^{T2}(t) = \begin{cases} \{j \in V^{T2} \mid \left|\left(x_j(t) - x_i(t)\right)\right| \leq \varepsilon_i\} & if\ i \in V^C \\ \{j \in V^{T2} \mid \left|\left(x_j(t) - x_i(t)\right) \cap j \neq i\right| \leq \varepsilon_i\} & if\ i \in V^{T2}. \\ \{j \in V^{T2} \mid \left|\left(x_j(t) - x_i(t)\right)\right| \leq \varepsilon_i\} & if\ i \in V^{T1} \end{cases}$$

$\overline{N}_i^{T2}(t)$ specifies that if agent i is of the population set of citizens, $i \in V^C$ or a $T1$ agent, then that individual considers all $T2$s that are within their confidence bound interval—citizens (and the Track I authority) only listen to community leaders with whom they are comfortable.

If agent i is of the population set of $T2$s, $i \in V^{T2}$, then the agent will consider all $T2$s other than themselves—community leaders only listen to other community leaders with whom they share similar opinions.

Finally, for the consideration of central authority leadership opinion, $\overline{N}_i^{T1}(t)$ is described as

$$\overline{N}_i^{T1}(t) = \begin{cases} \{j \in V^{T1} \mid x_j(t)\} & if\ i \in V^C, V^{T2} \\ 0 & if\ i \in V^{T1} \end{cases}.$$

$\overline{N}_i^{T1}(t)$ specifies that if agent i is a citizen or a $T2$, they will always consider the $T1$ opinion at some level. If agent i is a $T1$, then $\overline{N}_i^{T1}(t) = 0$.

Once all of the influence neighbor sets have been identified, we introduce multi-track influences into the second term of the MHK formulation described earlier and incorporate these agent sets. This requires some additional agent parameters that allow coefficient weights, which adjust the degree to which other influences will be considered in the final opinion shift. These agent parameters are owned by all agent classes and are shown in Table 1. The parameters were designed to facilitate future calibration of this model to real-world scenarios—we assume that these parameters could be approximated in some way by available public data sets or through population surveys.

Table 1: Agent parameters for specifying the MDHK model.

Variable	Variable Name	Description
$x_i(t)$	Opinion	The opinion of agent i at time t, on the interval [0, 1]
ε_i	Bounded confidence	The size of the boundary that determines whose opinion is considered by the agent and whose is different enough not to be considered
α	Self-weight	The proportion from 0 to 1 that the agent considers their own opinion or the opinion of others
β_i^C	Citizen influence	The extent to which an agent considers the opinion of citizens (influence variables add up to 1)
β_i^{T2}	$T2$ influence	The extent to which an agent considers the opinion of $T2$ leaders (influence variables add up to 1)
β_i^{T1}	$T1$ influence	The extent to which an agent considers the opinion of $T1$ leaders (influence variables add up to 1)

Given our definition of agent parameters in Table 1 and the influential neighbor sets $\overline{N}_i^C(t)$, $\overline{N}_i^{T2}(t)$, and $\overline{N}_i^{T1}(t)$, we extend the MHK model of Fu, Zhang, and Li (2015) and formalize our MDHK model as

$$x_i(t+1) = \begin{cases} \alpha_i x_i(t) + (1-\alpha_i) \begin{bmatrix} \beta_i^C \left(\frac{1}{|\overline{N}_i^C(t)|} \sum_{j \in \overline{N}_i^C(t)} x_j(t)\right) \\ + \beta_i^{T2} \left(\frac{1}{|\overline{N}_i^{T2}(t)|} \sum_{j \in \overline{N}_i^{T2}(t)} x_j(t)\right) \\ + \beta_i^{T1} \left(x_j(t)\right) \end{bmatrix}, & \overline{N}_i^C(t), \overline{N}_i^{T2}(t) \neq 0 \\ x_i(t), & otherwise \end{cases},$$

where $\beta_i^C + \beta_i^{T2} + \beta_i^{T1} = 1$. This model of opinion $x_i(t+1)$ shows that each of the multi-track leadership influences are linearly combined within the "non-self" component of the formulation, and each agent subgroup term is weighted with its own βweight.

3 IMPLEMENTATION

3.1 Model Description

We implemented the MDHK model in NetLogo (Wilensky 1999), an open-source, agent-based modeling platform. Our implementation is based on a model developed by Lorenz (2007) in which he used NetLogo to demonstrate the DW and HK models. We redesigned the interface and leveraged some aspects of Lorenz's approach to include our MDHK model, as well as the MHK model by Fu, Zhang, and Li (2015).

Following our earlier description of the MDHK model, we instantiate in the NetLogo implementation three types of agents—C, $T2$, and $T1$—representing citizens, Track II leaders, and Track I leaders, respectively. Each agent has the six instance variables described in Table 1. At each time step, every agent evaluates their surroundings and, using our MDHK formulation, updates their opinion appropriately. Agent updates are executed by default in NetLogo by using asynchronous random activation order.

3.2 Verification

Figure 2 shows a screenshot of our model interface, which features user controls for adjusting agent population sizes and MDHK model parameters. The center plot visualizes opinion formation by plotting opinion position on the y-axis and time on the x-axis. As the model progresses, lines draw from left to right. Blue lines represent citizen opinion, orange lines represent Track II opinion, and the red line represents Track I opinion. Converging lines represent the formation of opinion clusters. The histogram and line plot on the right-hand side of the interface may be used to indicate the magnitude of the opinion consensus clusters, where we define an opinion cluster as any distinct grouping on the histogram of greater than zero.

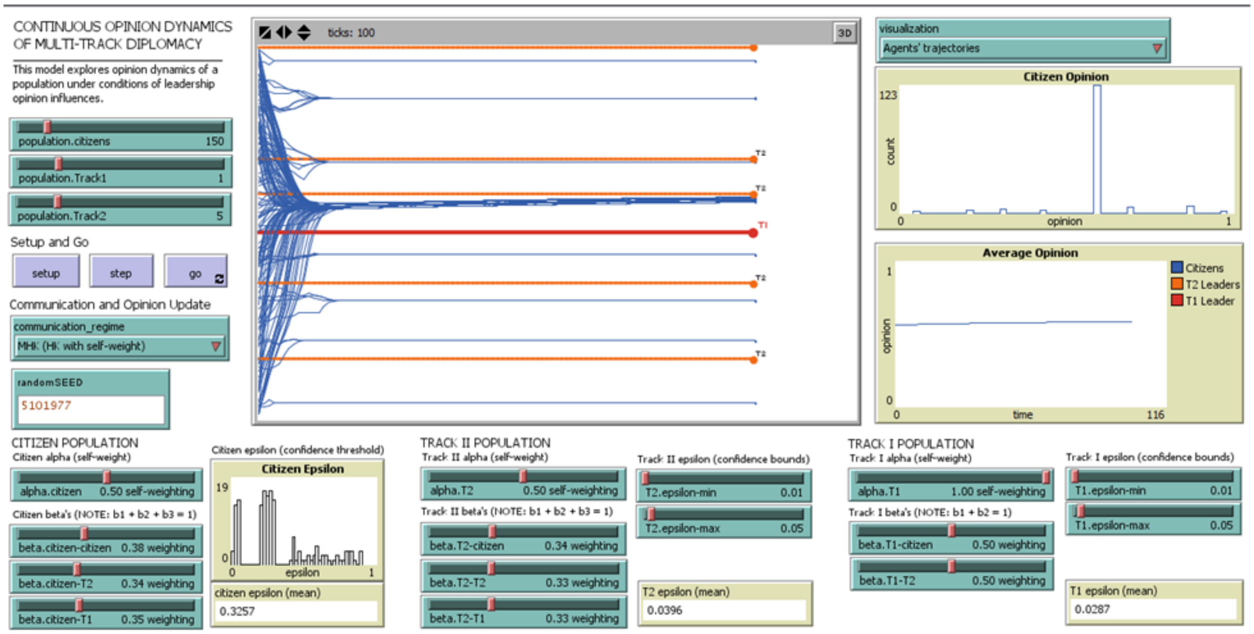

Figure 2: Screenshot of the model interface.

Model verification was a continuous process throughout model development. Our primary means for verifying that the presence of Track I and Track II leadership resulted in opinion outcomes different than with MHK was to comparatively evaluate a model run by using each of these two models when using the same parameters and random-seed initialization.

As an example, Figure 3 shows comparative results for one realization of MHK and MDHK, with both models using the same random seed. The random seed used in Figure 3 resulted in $T2$ agent instantiation in a relatively symmetric pattern on either side of the $T1$ agent.

Figure 3.a. shows the results for an MHK model run where opinion consensus is clustered near the $T1$ agent at the middle point of 0.5 on the opinion scale. There are 10 distinct smaller opinion clusters that are dispersed over the opinion range. While $T2$ and $T1$ agents are shown in the model, they are not being recognized by the citizen agents and thus have no influence on the final opinion clusters. They were only instantiated to ensure repeatable initial conditions for the MDHK model run as the $T1$ and $T2$ agents also draw from the random number stream.

Figure 3.b. shows the MDHK model results when the $T2$ and $T1$ influences have been activated with some default parameter set that notionally represents moderate-minded multi-track leadership, where moderate-minded confidence is defined by Fu, Zhang, and Li (2015) with ε uniformly drawn from the interval [0.2, 0.3]. Results shown in the opinion visualization indicate that the MDHK model results are significantly different than the MHK model results. The histogram and line plot also confirm that results are concentrated into two distinct opinion clusters; thus, multi-track leadership acted in a way that facilitated opinion convergence.

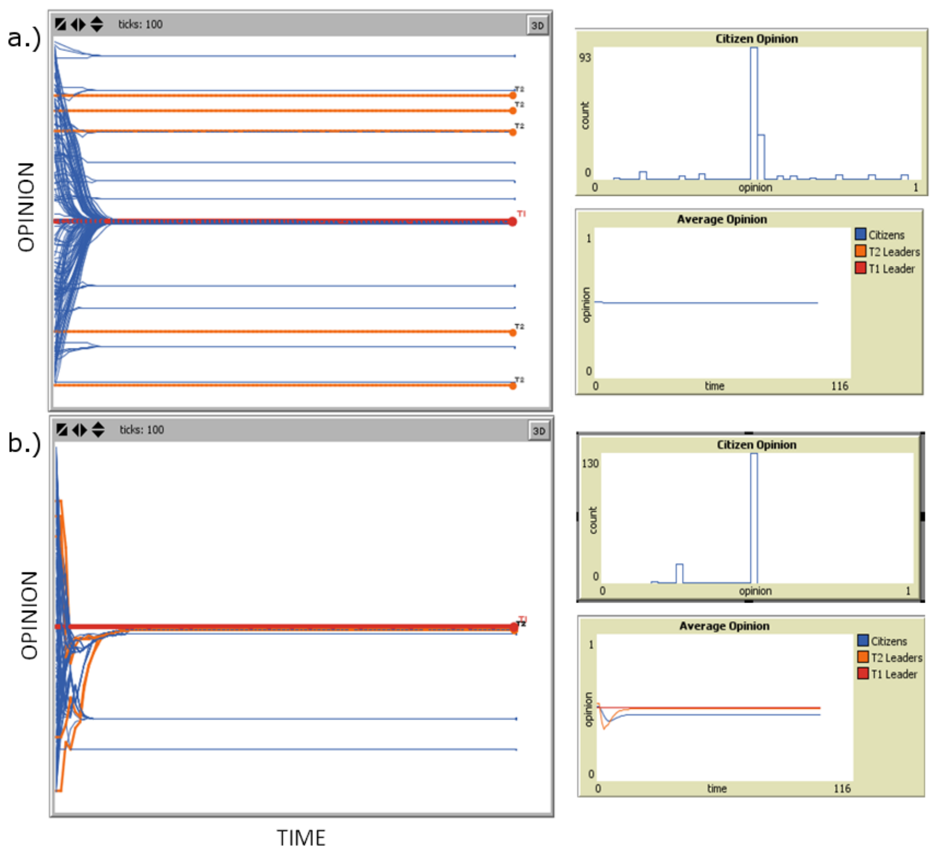

Figure 3: Model screenshot comparing the MHK and MDHK model results for one realization when using the same random seed.

We ran other verification scenarios not shown here, but all had similar outcomes in which the MDHK model was proven to produce different results than from the MHK model under the same initial conditions. We assume from these findings that the MDHK model is functioning in an expected way.

4 SIMULATION ANALYSIS AND EXPERIMENT RESULTS

4.1 Research Question and Hypothesis

One of the research conclusions by Fu, Zhang, and Li (2015) through the use of their MHK model was that closed-minded agents dominated the formation of opinion consensus. This was supported by simulation results that indicated as the proportion of closed-minded agents in the population increased (with the difference being open-minded), so did the number of final opinion clusters after reaching a period of steady state. We hypothesized that the introduction of $T2$ leadership influence would dampen this effect, reducing the influence of closed-minded citizens on the overall population opinion.

To test this theory, we sought to first replicate this finding by using our implementation of the MHK model and the same experiment parameter settings used by Fu, Zhang, and Li (2015) with respect to proportions of closed- and open-minded citizens and the respective ε values for these mindedness categories. We then ran our MDHK model in a sensitivity analysis that incremented the proportion of closed-minded citizens while also changing how much influence $T2$ leadership had on all citizens by incrementally increasing β_i^{T2}.

4.2 Experiment Initialization

To initialize the experiment runs, we used the following parameters in Table 2.

Table 2: Experiment parameter settings.

Model Parameter	Experiment Value(s)
Population	150 citizens, 5 $T2$s, and 1 $T1$.
Proportion of closed-minded agents	To be changed for citizens over a sensitivity range [0%, 30%], incrementing by 10%. The remaining proportion was considered as open-minded.
α	0.5 for citizens, 0.75 for $T2$s, and 1.0 for $T1$.
ε_i	For closed-minded agents, the value is drawn uniformly from the confidence interval range [0.01, 0.05]. For open-minded agents, the value is drawn uniformly from the confidence interval range [0.2, 0.3].
β_i^C	For a given experiment value of β_i^{T2}, the value is $\left(1-\beta_i^{T2}\right)/2$.
β_i^{T2}	To be tested at values of 0.5, 0.75, and 0.9.
β_i^{T1}	For a given experiment value of β_i^{T2}, the value is $\left(1-\beta_i^{T2}\right)/2$.

4.3 Experiment Process Flow and Scheduling

The process flow of our model for the experiment followed these steps:

1. Populations are instantiated for citizens, $T2$, and $T1$ on the basis of the scenario conditions that are being studied.
    a. For a time step of the model, in the sequence of citizens, $T2$, then $T1$.
        i. Each agent determines some influence set of neighbors that are within their bounded confidence, excluding themselves. Three influence sets are generated;

based on their current opinion, we generate a set for influential citizens, $T2$s, and $T1$s.

ii. Each agent shifts their opinion as appropriate, using the MDHK model logic.

2. If the time steps are less than 100, go to process flow step 2, else terminate the model run.

We ran 10 replications for each sensitivity combination in Table 2 for a total of 120 runs to complete the experiment.

4.4 Results

Figure 4 shows the experiment results and plots the average final opinion clusters from over the 10 replication runs for each scenario of a sensitivity parameter combination. Parameter combinations tested the sensitivity of increasing the proportion of closed-minded citizens from 0% to 30%, incrementing by 10%, where the remaining agents in each scenario were instantiated as open-minded. The lines represent the control case of only using the MHK model (no consideration of multi-track leadership influence) and also for a range of citizen β_i^{T2} values for how much the closed-minded agents weigh $T2$ opinion by using values of 0.5, 0.75, and 0.9.

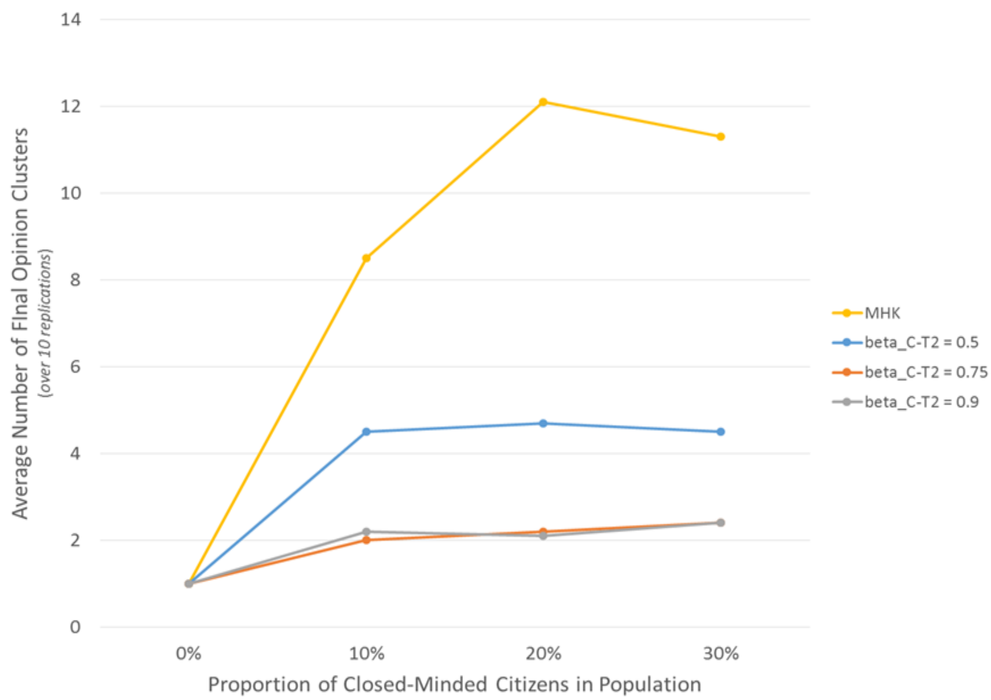

Figure 4: Average number of final opinion clusters with respect to sensitivity to β^{T2}.

Figure 4 also shows that we were able to replicate findings from the MHK model research demonstrating that as closed-minded agents increased within the population, the final opinion clusters also increased. This makes plausible sense, as stubborn individuals are likely to hold onto their viewpoints and maintain separate and isolated opinions. The other lines in the plot prove our experiment hypothesis correct—as we increase the influence weight for T2 opinion, β_i^{T2}, we see that the number of final opinion clusters decreases and that closed-minded agents have less of an impact on consensus formation.

It is also interesting that for the scenarios using the three β values tested, when at greater than a 10% proportion of closed-minded citizens, there is a negligible change in final opinion clusters when $T2$ influences are considered.

5 CONCLUSIONS

Our model was developed to aid in the understanding of population opinion dynamics and consensus formation. Public opinion consensus and how it changes over time has significant real-world consequences. We were interested in the influence of Track II leaders on decisions and consensus in conflict resolution projects, so we built upon previous work in opinion dynamics, specifically the HK and MHK models, to develop our multi-track diplomacy variant of the MHK model (called the MDHK model). By introducing Track 1 and Track II leaders into our models and allowing for different levels and weighting of influence, we were able to create a model that may potentially be calibrated to different kinds of political cultures, expanding opportunities for where our model might be applied in practice.

This is a potentially feasible proposition as we have verified that the MDHK model did in fact change the outcomes from MHK model-based results without multi-track leadership influences. We also conducted an experiment that confirmed results found by the MHK model in which the number of final opinion clusters are dominated by closed-minded agents—open-minded agents cannot contribute significantly to the formation of consensus. Our hypothesis (which was proved to be true) was that the introduction of Track II leadership will dampen this phenomenon in a way that closed-minded agents will have less influence.

6 FUTURE RESEARCH

The model was designed to allow for calibration to a real-world conflict scenario through adjustment of the parameters of population size, opinion initialization for citizens and $T2$ leaders, and the proportion of citizens with respect to open-, moderate-, and closed-mindedness through the setting of ε (bounded confidence), α (self-weighting), and β (weighting of other's opinions). Given this motivation of our model design, we are planning significant empirically based future research with this model, both to understand how the model can help us explain the consensus development process and also to apply it to real-world scenarios by using existing data sets and population survey results. As a demonstrative case study, we are currently using Northern Ireland Life and Times Survey data to parameterize our model and approximate the political climate in Northern Ireland immediately following the signing of the Good Friday Agreement. Replication of historical outcomes on some representative scale would help to verify the utility of this approach and of the MDHK logic.

There is a need to conduct more sensitivity analyses to fully explore the dynamics of the MDHK model implementation and gain better understanding of the mechanics of our model. We would like to repeat our proof-of-concept experiment with more replications and parameter combinations.

In addition, we would like to test additional hypotheses for how Track II leadership influences formation of opinion consensus. We hypothesize that diffuse Track II leadership will result in greater opinion clusters, a decreased convergence rate, and increased convergence time. Interactions with Track I leadership are also useful to investigate further—we hypothesize that when Track II leadership is generally aligned with Track I, it will improve the overall consensus convergence rate and decrease convergence time.

In accomplishing these additional explorations, the MDHK model will present a robust case for serving as a foundation for studying conflict in a novel way that may provide insights that are limited or challenging to produce when using current methods.

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