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COMBINING MONTE CARLO SIMULATION WITH HEURISTICS TO SOLVE A RICH AND REAL-LIFE MULTI-DEPOT VEHICLE ROUTING PROBLEM

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ABSTRACT

This paper presents an optimization approach which integrates Monte Carlo simulation (MCS) within a heuristic algorithm in order to deal with a rich and real-life vehicle routing problem. A set of customers' orders must be delivered from different depots and using a heterogeneous fleet of vehicles. Also, since the capacity of the firm's depots is limited, some vehicles might need to be replenished using external tanks. The MCS component, which is based on the use of a skewed probability distribution, allows to transform a deterministic heuristic into a probabilistic procedure. The geometric distribution is used to guide the local search process during the generation of high-quality solutions. The efficiency of our approach is tested against a real-world instance. The results show that our algorithm is capable of providing noticeable savings in short computing times.

1 INTRODUCTION

Distributing goods to end customers, providing high levels of service at a reasonable cost, may be of paramount importance for road transportation firms. When products can be easily stored, employing a network of geographically-distributed facilities (depots) might be an appropriate strategy for reducing distribution costs. Typically, each depot uses a fleet of vehicles for delivering products to the end customers. When the capacity of these depots is not enough to cover the customers' demand, additional tanks (external to the firm) can be used to fill the vehicles. In addition, multi-tank vehicles are required when distributing different products that cannot be mixed (e.g., different types of fuels). Finally, due to the evolution of the automotive industry, vehicles of different capacity are acquired over time, thus leading to heterogeneous fleets. In such a rich environment, generating efficient routing plans on a daily basis is not trivial, and optimization algorithms might be required in order to obtain high-quality routing solutions. Being an extension of the well-known vehicle routing problem (VRP) (Prins 2004, Laporte 2007), these rich multi-depot VRPs are NP-hard in nature, which in practice means that heuristic-based approaches need to be used in order to obtain high-quality solutions in reasonable computing times, at least for medium- and large-scale instances.

In this paper, we introduce a probabilistic algorithm which combines Monte Carlo simulation (MCS) with a heuristic procedure in order to deal with the aforementioned optimization problem. The MCS component, which is based on the use of the geometric probability distribution, is employed to better guide the random-search process during the generation of high-quality solutions.

The rest of the paper is structured as follows. Section 2 describes in more detail the optimization problem considered in this paper. Section 3 reviews some related work. Section 4 explains our simulation-optimization solving approach. Then, Section 5 includes some numerical experiments that contribute to illustrate the utility of our methodology. Finally, Section 6 highlights the main conclusions of this work.

2 PROBLEM DEFINITION

The problem analyzed in this paper is based on a real-life case related to the distribution of fuel in the North of Spain, and it basically consists in a rich version of the multi-depot VRP with heterogeneous fleets. Each customer might order several products (different types of fuel in our case). Vehicles use compartments to avoid mixing products of different types. At each depot, there is a fleet of heterogeneous vehicles, i.e., these vehicles differ in their loading capacity as well as in the number of compartments they have. This number varies from two to four. Since products cannot be mixed in the same compartment, the maximum number of different products that a vehicle can transport is limited by its number of compartments. Moreover, the operational cost of each vehicle is given by a fixed cost (different for each type of vehicle) plus a distance-dependent variable cost. All in all, the goal is to minimize the total delivery cost while satisfying all the customers' demands and the additional constrains explained below.

For each depot, we know its stock level at the beginning of the day, as well as the number of vehicles of each type assigned to it. Each vehicle must return to its corresponding depot after completing its daily delivery plan. A vehicle can perform several routes in a single day, as far as the total amount of time employed does not exceed the maximum number of working hours (also given as an input). Each customer has to be fully served from a single depot. However, a customer requesting several products can be served by different vehicles (as far as all of them depart from the same depot).

In addition to the depots owned by the firm itself, which have a limited capacity, there are also external facilities of unlimited capacity (Figure 1). These external depots can also be used to load empty vehicles (that is, due to regulations only empty vehicles can use these external facilities). As a result, vehicles might only visit the external facilities for a full-reload when they start a new route. Distances and estimated traveling times between any two locations (customers, firms' depots, or external depots) are known. Likewise, loading and unloading times are assumed to be known, and they are proportional to the quantity being delivered. Finally, given a daily set of customers' orders, a solution to the described problem will consist of: *(i)* a selection of vehicles to be used at each depot (from the different types available); and *(ii)* a routing plan to delivery the requested demands, which might include visits to the external facilities.

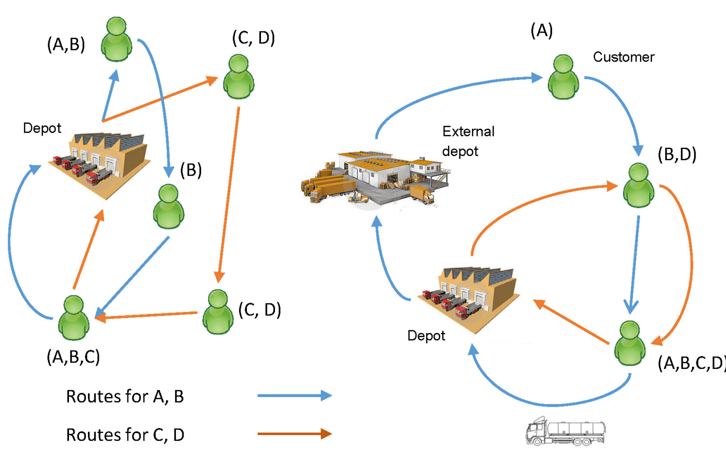

Figure 1: System configuration.

3 RELATED WORK

The savings heuristic (Clarke and Wright 1964) is the most popular constructive procedure in the VRP literature. Roughly speaking, it starts out by considering an initial dummy solution in which each customer is served by a dedicated vehicle –i.e., this initial solution assumes as many vehicles as customers. Next, an iterative process is started in which routes are merged according to a savings-based criterion. Given a pair of nodes to be served, a savings value can be assigned to the edge connecting these two nodes. This savings value is given by the reduction in the total cost function due to serving both nodes with the same vehicle instead of using a dedicated vehicle to serve each node. Accordingly, at each iteration of the merging process, the edge with the largest possible savings is selected as far as the following conditions are satisfied: *(i)* the nodes defining the edge are adjacent to the depot; and *(ii)* the two corresponding routes can be feasibly merged –e.g., the vehicle capacity is not exceeded after the merging, etc.

In Juan et al. (2010), the authors propose the use of Monte Carlo simulation as a way to transform the previous deterministic heuristic into a much more efficient probabilistic algorithm. In particular, they use a skewed theoretical probability distribution in order to randomly select the next edge to merge two routes. By using a skewed probability distribution, this selection can be random but, at the same time, it can assign more probabilities of being selected to those edges showing higher savings values. An extension of their approach is proposed by Juan et al. (2015) for solving the multi-depot VRP. In this case, the MCS component is employed not only to introduce an oriented random selection during the routes-merging stage, but also to generate a similar effect during the assignment of customers to depots –since these depots offer a limited capacity, assigning each customer to its closest depot is not a feasible option.

Regarding other works dealing with similar problems, Avella et al. (2004) propose a branch-and-price algorithm for solving a multi-product heterogeneous VRP related to the distribution of oil-derivatives. Similarly, Cornillier et al. (2008) design a heuristic-based approach for solving a multi-period petrol station replenishment problem. Other similar studies are related to selective garbage collection, in particular those considering trucks with multiple compartments for different types of waste. In this sense, Henke et al. (2015) present a problem addressing the collection of waste glass, where glasses of different colors must be stored in separated compartments.

Heterogeneous versions of the VRP have also studied, among others, by Taillard (1999), who propose a column generation heuristic. Likewise, Li et al. (2007) introduce a method for the heterogeneous VRP based on a variant of their algorithm "record-to-record travel" for the homogeneous version. Finally, Wang et al. (2014) make use of a tabu search metaheuristic for solving the heterogeneous VRP.

4 SOLVING APPROACH

To address such a complex optimization problem as the one described here, we used the three-stage approach depicted in Figure 2. First, each customer is assigned to one of the available base depots. This generates a customer-depot assignment map. Each base depot together with its assigned customers constitutes a submap. Then, each of these submaps is modeled as a VRP and solved, i.e., a routing plan is generated for each submap. Finally, a 2-opt local search (Croes 1958) is applied to each route to try improving it. Monte Carlo simulation is used in the first and second stages in order to introduce some 'oriented' randomness into the heuristics that are in charge of generating the assignment map (so different but promising maps are considered at each algorithm iteration) and constructing the subsequent routing plans (so different but promising routing plans are considered at each algorithm iteration).

As explained before, we make use of a skewed theoretical probability distribution as a way to introduce a random but oriented behavior into the assignment and routing heuristics. Thus, instead of following the greedy criterion specified by each heuristic during the map or routing plan construction process, a geometric distribution with parameter β ($0 < \beta < 1$) is used to randomly choose the next step during these constructive processes. In other words, while constructing an assignment map or a routing plan, we iteratively perform random sampling among the most promising movements identified by the respective heuristic. The use of

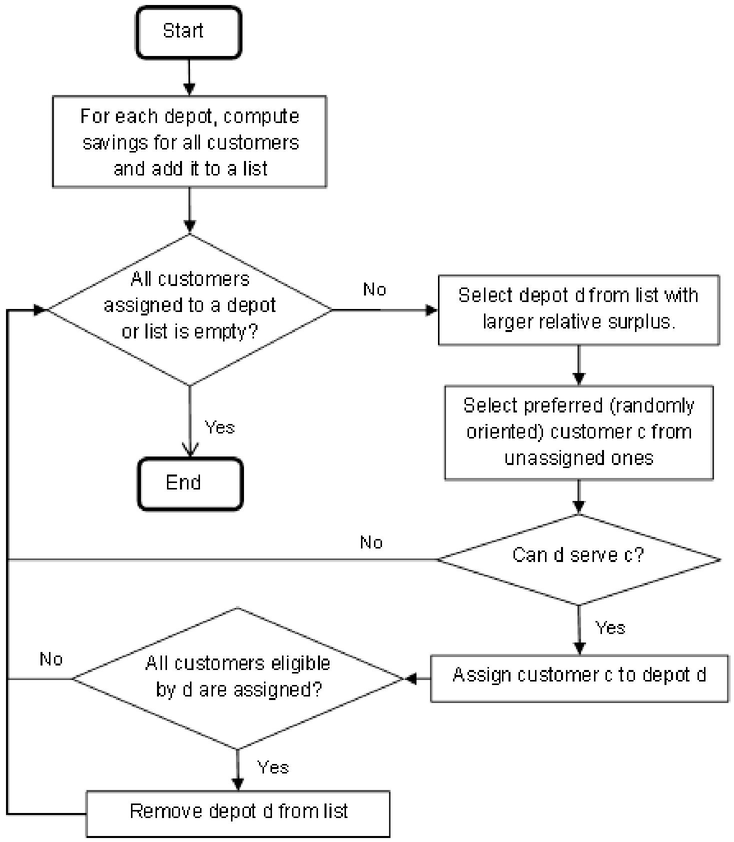

Figure 2: Submap generation chart.

the geometric distribution for introducing oriented random search during the construction of routing plans is discussed in Juan et al. (2009). Here, we have extended these ideas to the multi-depot case in which assignment maps need to be generated too.

In the following subsections, we provide some details on each of the different stages of the algorithm.

4.1 Map Generation: Assigning Customers to Depots

For each depot in the system, all the firm's customers are sorted according to their 'marginal assignment savings', which are computed as the difference between the cost of serving a customer from the closest depot and the cost of serving it from the best alternative depot.

Then, a round robin process starts in which, at each iteration, the depot with the highest surplus (in terms of delivery capacity) chooses its next customer from the sorted list. However, instead of applying a greedy selection criterion, the selection is driven by a Monte Carlo simulation: a random position in the list is chosen according to a geometric probability distribution with parameter β_{MAP}. This round robin process is repeated until all customers have been assigned to a depot. In some extreme instances with a very tight depot capacity (i.e., when the total demand is extremely difficult to satisfy due to the limited number of available vehicles), it might occur that the randomly generated map does not cover all customers. In that case, and assuming that the problem is feasible, the round robin process is re-started until one of the randomly generated maps includes all customers.

Table 1: Properties of each base depot.

Base	Vehicles	Initial Stocks	Security Stocks
1	1 small, 2 medium, 1 big	12000, 17000, 18000, 14000	15000, 0, 0, 0
2	1 small, 1 big	0, 8000, 1000, 0	0, 0, 0, 0
3	1 medium	25000, 7000, 22000, 0	0, 0, 0, 0

4.2 Generating Routing Plans

Once every customer has been assigned to a depot, the next step consists in generating the routing plan for this submap (i.e., to solve the associated vehicle routing problem). In case that a customer requires more than one product, it is considered by the algorithm as several customers with the same geographic location (one per product). A 'route-merging savings' for every pair of nodes i and j is computed as $s_{ij} = c_{ij} - c_{0j} - c_{0i}$, where c_{ij} is the cost corresponding to the trip from i to j, and 0 corresponds to the base depot. Then, an iterative process starts until the termination criterion (usually available time) is met. At each iteration, a vehicle type is randomly selected and the problem is solved as a homogeneous VRP as explained in Juan et al. (2010), i.e., Monte Carlo simulation is combined with the savings heuristic in order to generate promising routing plans. Similar as with the map generation process, a geometric distribution of parameter β_{ROUTE} is employed to introduce an oriented random behavior into the savings heuristic. Loading and unloading times are computed according to the customers being visited.

As a result of the previous steps, a routing plan is generated for a particular type of vehicle. Those routes for which it is possible to find available vehicles of the right capacity are kept aside. They will be used as potential routes for the global routing plan. Accordingly, the customers in those routes are deleted from the initial set of inputs. The remaining (and reduced) problem (including the customers who have not been covered so far as well as the non-used vehicles) is re-started and solved again using another type of vehicle. This process is repeated, each time using a different type of vehicle, until all customers have been covered. Then, as a final step, a construction is built by merging the different feasible routes found in the iterative process.

5 COMPUTATIONAL RESULTS

To assess the performance of the proposed algorithm, several computational experiments were performed. The proposed algorithm has been implemented as a Java® 7SE application. All tests have been executed on a standard desktop computer with an Intel® Core™ 2 Duo at 2.2 GHz and 4 GB RAM running on Windows 10.

After performing a series of quick preliminary tests, the β_{MAP} parameter for the geometric distribution was fixed to 0.2 in the customers-depot assignment stage, and β_{ROUTE} to 0.5 in the routing-generation stage. A real case study, proposed by a firm in the transportation sector, has been used to check the performance of the algorithm. The data employed in our experiments relates to three base depots and a total of thirty-one customers. There are four kinds of fuel products (A, B, C, C+), one external facility, and three kinds of vehicles (big, medium, and small). The big vehicles have five compartments, with capacities 10000, 8000, 8000, 4000, and 2000 liters, respectively; the medium vehicles have two compartments, with capacities 6000 and 5000 liters, respectively; and the small vehicles have two compartments, with capacities 2000 and 2000 liters, respectively. Finally, each base depot has a different number of available vehicles and stocks, as depicted in Table1.

Figures 3 and 4 show realistic examples of alternative routes corresponding to different assignment plans. Each of these routes represent a single piece of a global solution to the problem, which is composed of several routes. In this case, the routes refer to a region located in the NW of Spain. Notice that different assignment plans can lead to completely divergent routes (and, eventually, to completely different solutions). The high sensitivity of the final distribution plan with respect to the customer-depot assignment

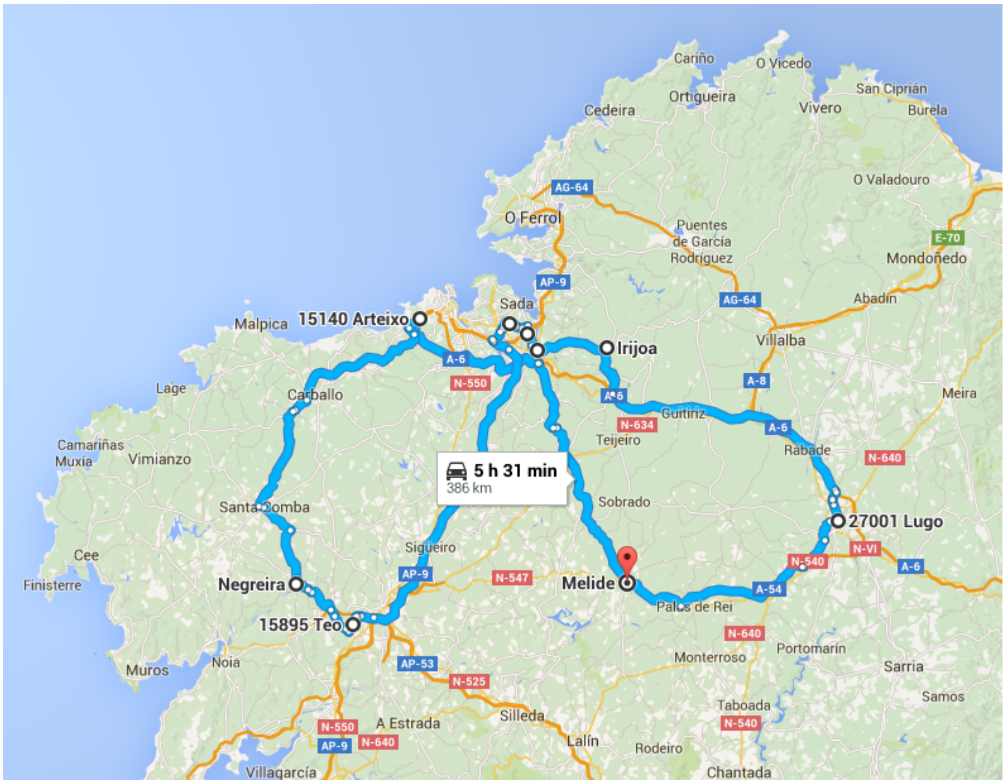

Figure 3: A first example of a possible route for a given assignment map.

map explains the importance of using an integrated solving approach –as the one presented in this paper– instead of a two-stage methodology.

The expert solution provided by the firm to this case study has been evaluated taking into account one monetary unit (m.u.) per kilometer. Fixed costs for each kind of vehicle have not been considered. The cost of this solution is 1016 m.u., and the maximum delivery time for any vehicle is 609 minutes. As a first experiment, the proposed algorithm has been run during 60 seconds, and then we have checked the best solution obtained. This solution was reached in about 32 seconds. Its cost is 827 m.u., which represents an improvement of about 19% with respect to the firm's solution, and its maximum delivery time for any vehicle is 603 minutes. Being a stochastic algorithm, we run a second test in order to analyze the variability in the results as different seeds are used for the pseudo-random number generator. Thus, in this new experiment a total of 30 runs have been performed. The best solution obtained after these runs has an associated cost of 739 m.u., i.e., an improvement of about 27% with respect to the firm's solution.

The third experiment performed consists in calculating the different cost obtained as the maximum-allowed delivery time varies. Figure 5 shows the results of this study. The x axis corresponds to the maximum delivery time and the y axis corresponds to the cost. Notice that the algorithm is able to find solutions with a maximum-allowed delivery time of 530 minutes, which is 13% lower than the one initially proposed by the firm and about 17% cheaper in terms of monetary cost. Additionally, using the same maximum-allowed delivery time as the firm's solution, the algorithm provides a solution which is about 25% cheaper.

6 CONCLUSIONS AND FUTURE WORK

A methodology for solving a rich and real-life multi-depot vehicle routing problem has been introduced. This methodology makes use of Monte Carlo simulation to better guide the random search of a heuristic-based

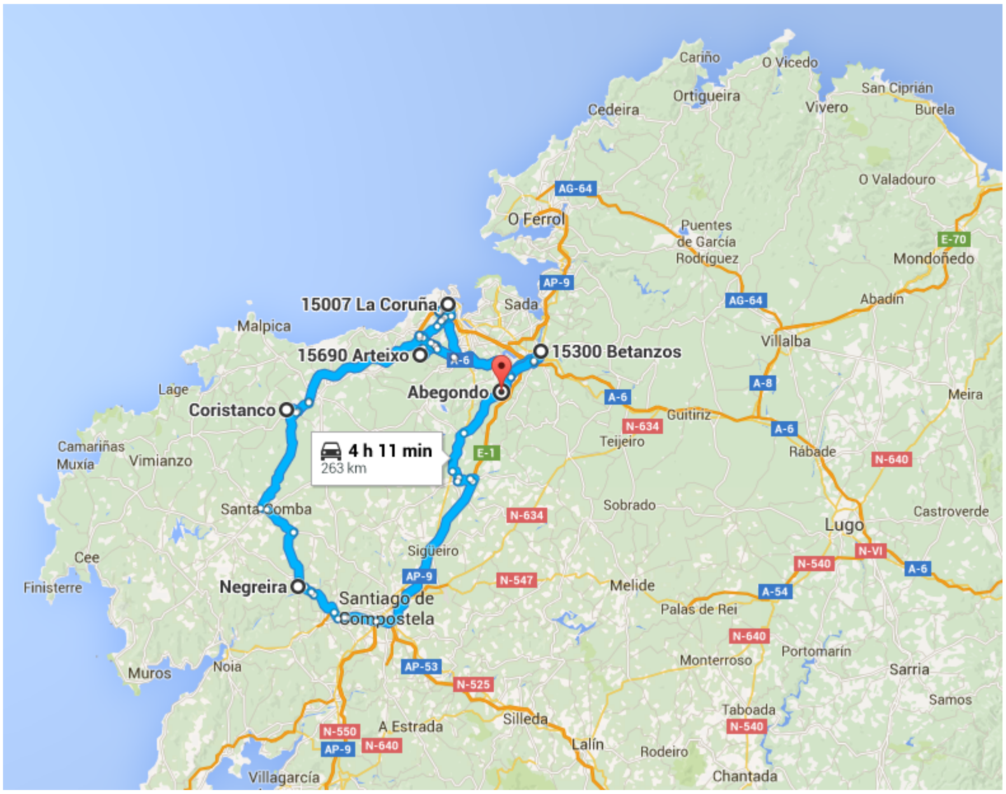

Figure 4: A second example of a possible route for a given assignment map.

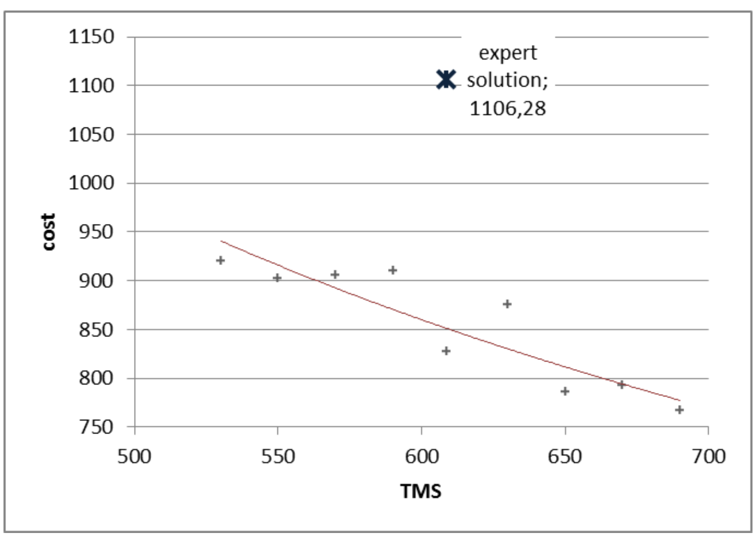

Figure 5: Costs comparison.

algorithm. A skewed theoretical probability distribution is used to induce a bias in the random selection of the next step during the constructive assignment and merging processes. The efficiency of our approach has been tested against a real-world instance provided by a firm. In short computing times (less than a minute) we obtain solutions that outperform by a 20% gap the solution proposed by the firm's experts. By running our algorithm several times (employing a different seed each time), we have been able to improve our solution ever further, which shows the potential of this simulation-optimization approach. As further work, we plan to test the proposed approach in a larger set of benchmarks. Likewise, we plan to introduce random demands to make the problem even more realistic and also to increase the contribution of the simulation component in this hybrid approach. Finally, despite the problem is probably too complex to be solved using exact methods –at least without relaxing some constraints and reducing the size of the real-life instances being considered in this work–, another research line would consist in finding lower-bounds for the optimal solutions. This will help to assess the quality of the solutions provided by our randomized algorithm.

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REFERENCES

Avella, P., M. Boccia, and A. Sforza. 2004. “Solving a Fuel Delivery Problem by Heuristic and Exact Approaches”. *European Journal of Operational Research* 152 (1): 170–179.

Clarke, G., and J. W. Wright. 1964. “Scheduling of Vehicles from a Central Depot to a Number of Delivery Points”. *Operations Research* 12 (4): 568–581.

Cornillier, F., F. F. Boctor, G. Laporte, and J. Renaud. 2008. “A Heuristic for the Multi-period Petrol Station Replenishment Problem”. *European Journal of Operational Research* 191 (2): 295 – 305.

Croes, G. A. 1958. “A Method for Solving Traveling-Salesman Problems”. *Operations Research* 6 (6): 791–812.

Henke, T., M. G. Speranza, and G. Wscher. 2015. “The Multi-compartment Vehicle Routing Problem with Flexible Compartment Sizes”. *European Journal of Operational Research* 246 (3): 730 – 743.

Juan, A. A., J. Faulin, J. Jorba, D. Riera, D. Masip, and B. Barrios. 2010. “On the Use of Monte Carlo Simulation, Cache and Splitting Techniques to Improve the Clarke and Wright Savings Heuristics”. *Journal of the Operational Research Society* 62 (6): 1085–1097.

Juan, A. A., J. Faulin, R. Ruiz, B. Barrios, M. Gilibert, and X. Vilajosana. 2009. “Using Oriented Random Search to Provide a Set of Alternative Solutions to the Capacitated Vehicle Routing Problem”. In *Operations Research and Cyber-Infrastructure*, 331–345. Springer.

Juan, A. A., I. Pascual, D. Guimarans, and B. Barrios. 2015. “Combining Biased Randomization with Iterated Local Search for Solving the Multidepot Vehicle Routing Problem”. *International Transactions in Operational Research* 22 (4): 647–667.

Laporte, G. 2007. “What You Should Know About the Vehicle Routing Problem”. *Naval Research Logistics* 54:811–819.

Li, F., B. Golden, and E. Wasil. 2007. “A Record-to-Record Travel Algorithm for Solving the Heterogeneous Fleet Vehicle Routing Problem”. *Computers & Operations Research* 34 (9): 2734 – 2742.

Prins, C. 2004. “A Simple and Effective Evolutionary Algorithm for the Vehicle Routing Problem”. *Computers & Operations Research* 31 (12): 1985 – 2002.

Taillard, E. D. 1999, 1. “A Heuristic Column Generation Method for the Heterogeneous Fleet VRP”. *RAIRO - Operations Research* 33:1–14.

Wang, Q., Q. Ji, and C.-H. Chiu. 2014. “Optimal Routing for Heterogeneous Fixed Fleets of Multicompartment Vehicles”. *Mathematical Problems in Engineering* 847630.

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