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ADD-MORE: AUTOMATED DYNAMIC DISPLAY OF MEASURES OF RISK AND ERROR

Ashkan Negahban

School of Graduate Professional Studies
Pennsylvania State University
30 E Swedesford Rd
Malvern, PA 19355, USA

Mohammadnaser Ansari
Jeffrey S. Smith

Department of Industrial and Systems Engineering
Auburn University
3301 Shelby Center
Auburn, AL 36849, USA

ABSTRACT

We develop an Excel® Add-In that automates the evaluation and visualization of measures of risk and error (MORE) for performance measures that change over time. We use an example with a non-stationary arrival process to demonstrate the applicability and importance of a such tool. The tool takes raw simulation output, automatically calculates the pertinent MORE values, and generates side-by-side MORE plots for different time ticks according to a user-specified interval to characterize how the mean and percentiles of a time-dependent statistic and their corresponding confidence intervals change over time. The ADD-MORE tool significantly reduces the burden on the simulation analyst and can potentially have a high impact on simulation practice as there is no easy way to perform such analysis using existing tools. The tool is made available online for free and can potentially be integrated into existing simulation and/or statistical software packages to support output analysis and decision-making.

1 INTRODUCTION

Simulation is commonly used for decision-making on the design and operation of manufacturing (Negahban and Smith 2014), healthcare (Mielczarek and Uzialko-Mydlikowska 2012), and military (Naseer, Eldabi, and Jahangirian 2009) systems as well as in supply chain management (Terzi and Cavalieri 2004), marketing (Negahban and Yilmaz 2014), and social sciences (Axelrod 1997). The work-in-progress (WIP) in a production line, number of patients waiting for treatment at an emergency department (ED), space utilization of a distribution center, and future sales/demand for a new technology are examples of typical performance measures estimated/predicted through simulation. Due to the stochastic nature of the different components of such dynamic systems, many of the inputs of a simulation model are random (e.g., stochastic processing times on machines in a production line, patient arrivals into an emergency department, the number of SKUs in an order received by a warehouse, or consumers' purchasing behavior and word-of-mouth after the launch of a new product). As a result, the output (performance measures) are also random variables making the assessment of the level of *error* in the predictions of the simulation model and the level of *uncertainty* in the possible values (i.e., distribution) of the measure(s) of interest critical for effective decision-making.

In simulation, a confidence interval (CI) on the estimated statistic is generally used as a measure of error (Law 2014) as it attempts to provide, based on the simulation output, an interval that is expected to cover the actual (unknown) value of the performance measure with high probability. Let Y_1, Y_2, ..., Y_n be a sample of independently and identically distributed (i.i.d.) simulation output on performance measure Y from n replications where Y has a distribution with mean μ_Y and variance $\sigma_Y^2 > 0$. Given the sample mean $\overline{Y}(n)$, sample variance $S^2(n)$, and $0 < \alpha < 1$, the approximate $100(1-\alpha)$ percent confidence interval for μ_Y is given by $\overline{Y}(n) \pm t_{n-1,1-\alpha/2}\sqrt{S^2(n)/n}$, where $t_{n-1,1-\alpha/2}$ is the upper $1-\alpha/2$ critical point for the t distribution with $n-1$ degrees of freedom. The interpretation of the CI is that if many of such CIs is constructed, a proportion of $1-\alpha$ of these CIs are expected to cover μ_Y. The width of the confidence interval is a measure of how precisely μ_Y is estimated. A wider CI indicates less accurate estimations suggesting that more replications are necessary to reduce the sampling error. Therefore, the CI helps the decision maker answer the question of whether enough replications are performed to be confident about the correctness of the decisions made based on the simulation results.

On the other hand, risk (commonly perceived as the variability of the distribution of the outcome) has been recognized as one of the most important factors for managers when making any financial investment since the conception of classical decision theory (March and Shapira 1987). Percentiles are generally used as measures of risk as they provide some insight about the likelihood of possible outcomes for the performance measure under investigation - the information we often need when making a decision. As discussed by Nelson (2008), the mean is essentially an expected long-run performance indicator that can provide a good single-point estimate of the performance measure while it does not provide any information about the uncertainty in the future outcome diminishing the power of simulation in characterizing risk. Based on the above, Nelson (2008) introduces the Measure of Risk & Error (MORE) plot that displays the sample mean, selected percentiles, and their confidence intervals all in one place (Figure 1). The reader is referred to Nelson (2008) for more details about the MORE plot such as the calculation of CI for percentiles.

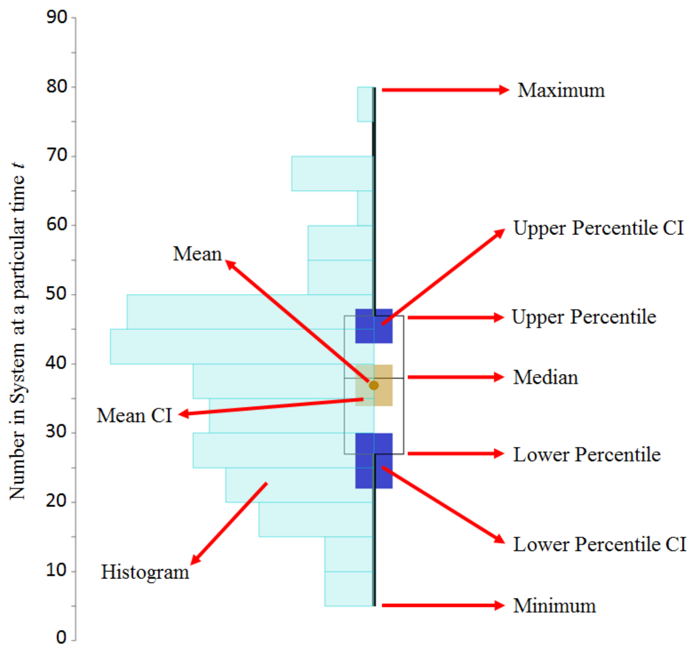


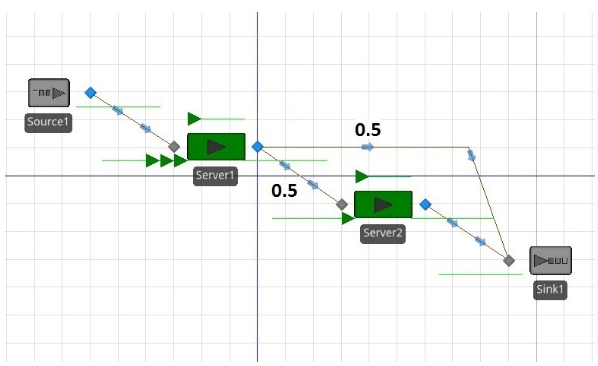
Figure 1: The original MORE plot and its different components.

Now consider a non-steady-state performance measure that varies over time (e.g., WIP in a production line, number of patients waiting at an ED, or monthly demand for an innovation which are not constant and vary throughout the operating hours or planning horizon). For any simulation time t, let Y_1^t, Y_2^t, ..., Y_n^t be a sample of i.i.d. simulation output from n replications where $Y(t)$ has a distribution with mean μ_{Y^t} and variance $\sigma^2_{Y^t} > 0$, i.e., the distribution of the performance measure and thus the level of risk and error change over simulated time. In order to make decisions based on such time-dependent metrics, we need multiple MORE plots, each representing the performance measure at a different time tick, to be able to assess how risk and error change over time.

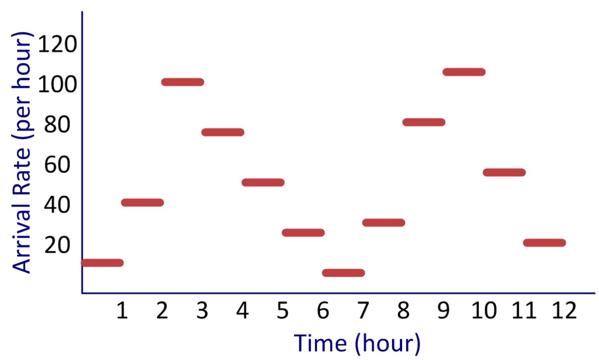
In this paper, we first demonstrate the shortcomings of existing simulation software in providing this type of information (Section 2). Next, through an example, we illustrate the importance of using side-by-side MORE plots to support decision-making under time-dependent performance measures (Section 3). Since there is no easy way of generating side-by-side MORE plots using any existing simulation or statistical software packages (that we are aware of), we develop a new graphical tool that automates the generation of side-by-side MORE plots from the raw simulation output (Section 4). We call the tool *Automated Dynamic Display of Measures of Risk & Error* (ADD-MORE) as it automatically calculates measures of risk and error and generates side-by-side MORE plots for the performance measure at different time steps as specified by the user. More specifically, the user can collect data from the simulation model at a fine resolution and use the tool to generate side-by-side MORE plots at any desired level of resolution without having to run additional replications. As a result, the proposed ADD-MORE tool significantly simplifies the evaluation of time-dependent statistics. The tool is implemented as an Excel® Add-In and made available online to simulation users and thus the main contribution of this paper is based on practical rather than theoretical grounds.

2 EXISTING APPROACHES

Consider a system of two stations in series as shown in Figure 2(a) where half of the entities skip the second server and exit the system after being processed on the first server. The system operates for twelve hours daily and the arrival process is non-stationary with arrival rates as shown in Figure 2(b). All inter-arrival times are exponentially distributed and processing times on both servers are uniformly distributed between 0.5 and 1 minute. Currently, there is only one server at each station (i.e., the processing capacity of each station is one).



(a) The Simio® model of the two-station system with passing.



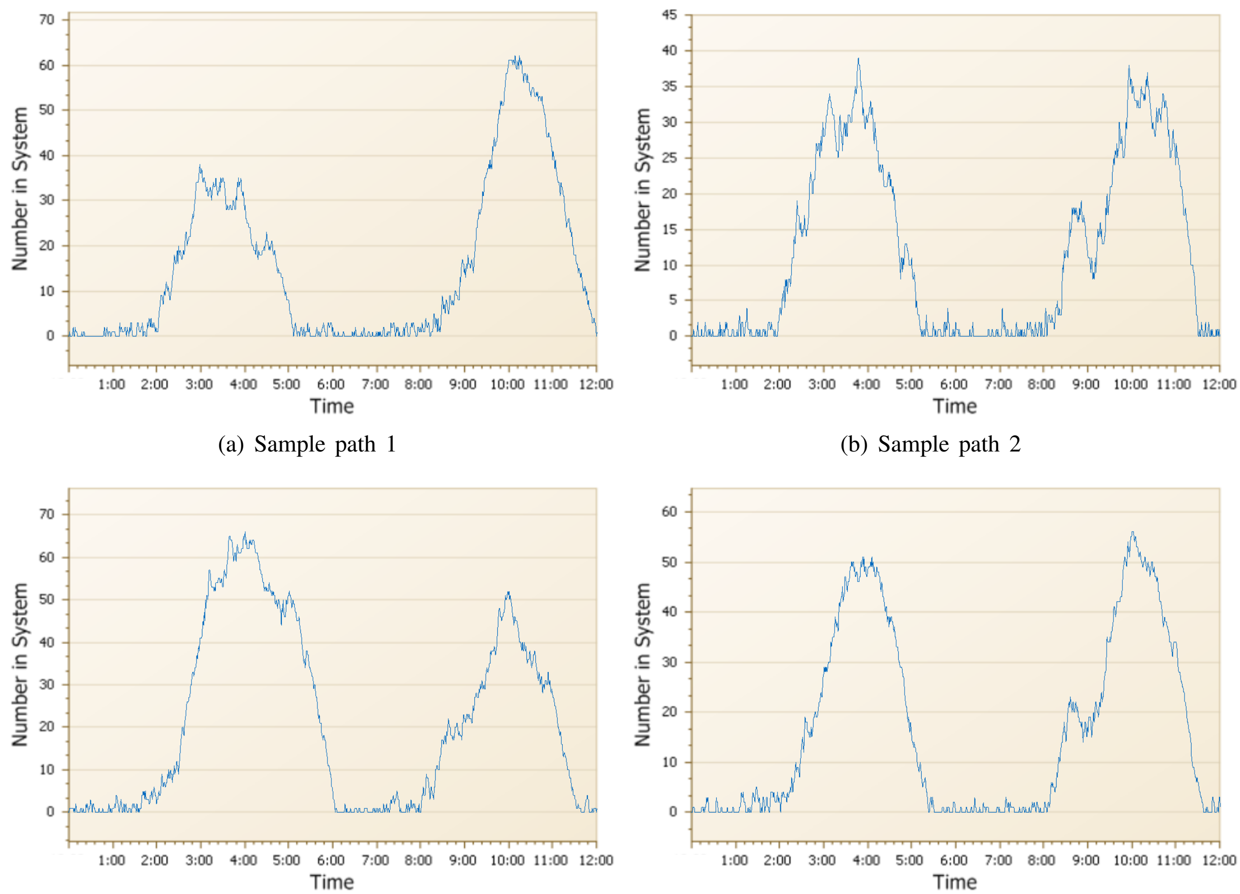
(b) Rate of the non-stationary arrival process.

Figure 2: The system configuration for Example 1.

Suppose the decision-making problem is to find the optimal capacity strategy, i.e., how to adjust the number of parallel servers in the stations over the 12-hour daily operation to avoid large number of entities in the system. Consider a restaurant where a long line is undesirable, or a call center where we do not want to have more than a certain number of calls on hold at any time, or a manufacturing process where WIP is costly and needs to be controlled by adjusting the production capacity. In order to make such decisions, given the upper bound for an acceptable number of parts/entities in system, the analyst requires statistically reliable information about how the number in system (NIS) and its distribution change over time. Here, we discuss the difficulties in approaching this problem using the existing methods and illustrate the need for a new tool that facilitates the analysis process.

Most existing simulation packages (including Simio®, Arena®, AnyLogic®, ExtendSim®, FlexSim®, SIMUL8®, etc.) provide dynamic graphical tools for tracing different variables as the simulation is running. We can run the model in the interactive mode and observe how the number in system changes over the current simulation run by looking at the dynamic plot of NIS. Of course, one sample path of NIS is not sufficient to draw any (statistically valid) conclusions and multiple replications using different random seeds are required.

We develop a simulation model of the system described above in the Simio® simulation package. Figure 3 presents four NIS plots (from four different interactive runs) generated by the simulation software. In Simio®, the analyst needs to manually change the random seed number before each interactive run; otherwise, the same random seed number will be used and thus the exact same replication will be repeated every time the model is run in the interactive mode. Although we can get some idea about the variation in the NIS over time by running the model multiple times in the interactive mode and looking at the resulting dynamic plot, the main problem associated with this approach is that we will only be able to see one sample path of the NIS at a time that pertains to the current run. Therefore, similar to what we did in Figure 3, we need to juxtapose the resulting plots from multiple interactive runs outside of the simulation software. Regardless of the simulation package, this approach is tedious and prohibits quantification and a meaningful evaluation of risk and the sampling error and thus any kind of decision-making could be flawed due to the lack of statistically valid information. To the best of our knowledge, none of the existing commercial simulation packages provide a built-in tool that allows for simultaneous comparison and interpretation of multiple time series data (in this case NIS) from different replications.



(a) Sample path 1 (b) Sample path 2

(c) Sample path 3 (d) Sample path 4

Figure 3: Four representative sample paths of the number in system generated by the simulation software.

As an alternative approach, simulation output from multiple replications can be written to a file to generate a line chart of a all sample paths in one plot. While most simulation packages by default report summary statistics such as the *average* NIS, we are not aware of any simulation software that automatically provides the time series data. As a result, additional *coding* in the simulation model may be required to monitor NIS and write the instantaneous/time-tick values to the file when it changes (along with the corresponding simulation time). In Figure 4, we used Excel® to generate the NIS plot for 25 replications of the simulation model of our two-station system. While this figure can better illustrate the variability in the process and the range for the data *across replications* (compared to the first approach), it fails to provide meaningful quantitative statistical information about the level of uncertainty and sampling error (i.e., the mean, percentiles, and their corresponding confidence intervals) that are necessary for effective decision-making. The figure is also visually unappealing and its illustrative qualities diminishes as the number of replications increases. With more replications, it actually becomes more difficult to distinguish individual sample paths and assess *within replication* variability and patterns over time. For example, it is hard to see whether or not the highest values during the two peak hours occur in the same run.

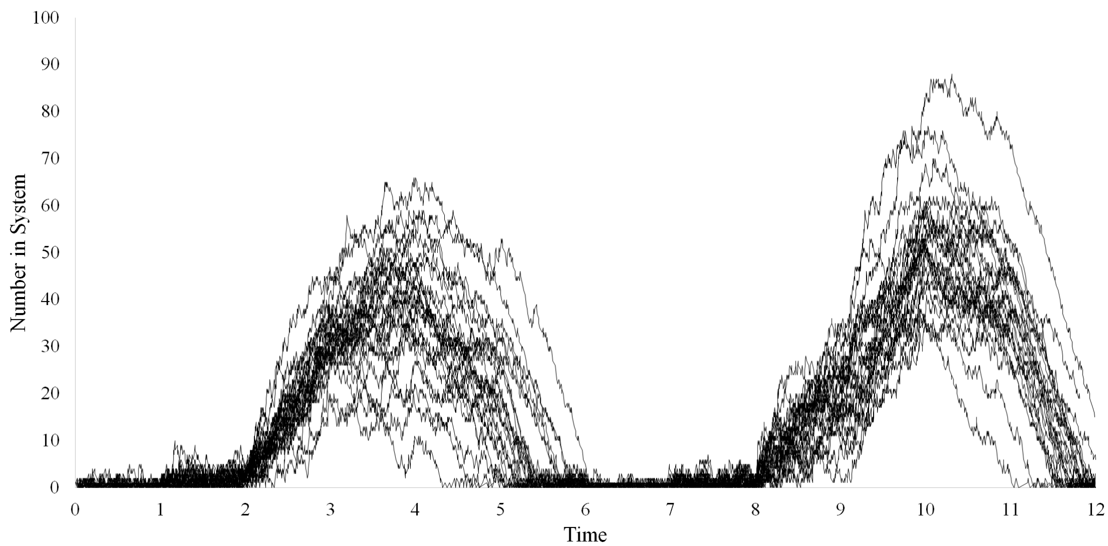


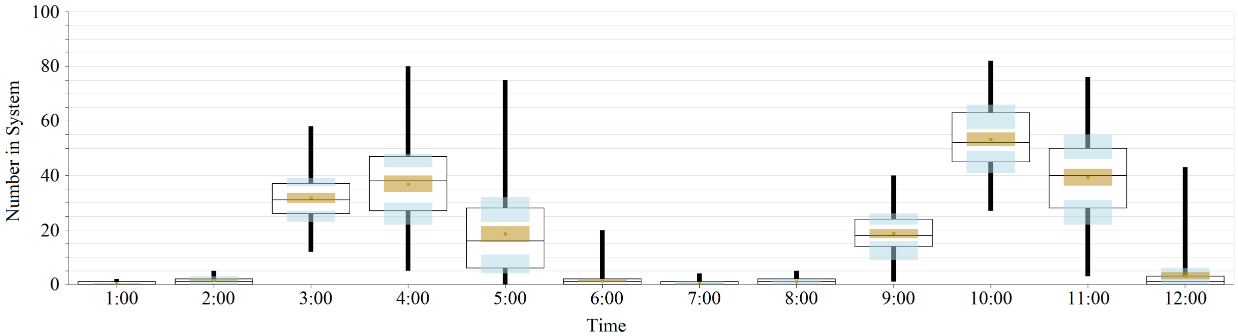
Figure 4: Sample paths of the number in system from 25 replications. Note that we do not deal with the entities that are still in the system when the run is finished after twelve hours as terminating conditions are not relevant to the purpose of this paper and will not affect the validity of the plots or the proposed tool.

Clearly, there is a need for an alternative type of plot that provides the necessary (statistical) information and quantifies measures of risk and error for different time ticks to support decision-making.

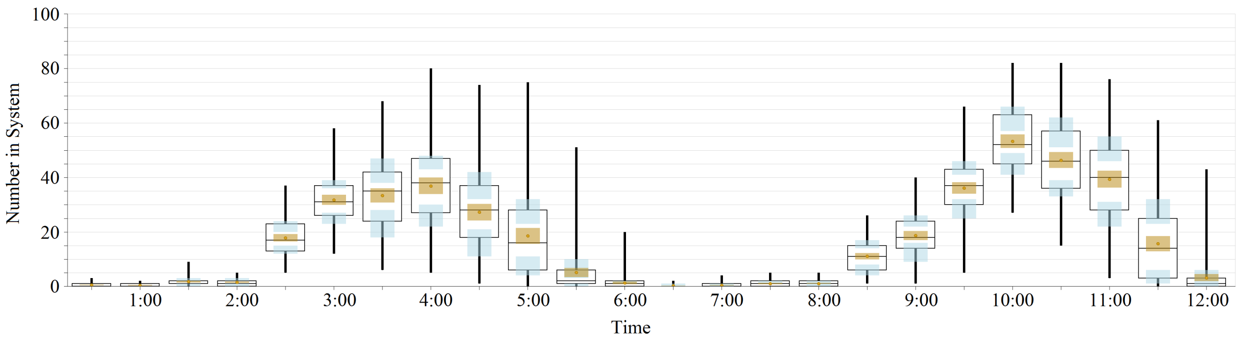
3 SIDE-BY-SIDE MORE PLOTS

We extend the idea of a MORE plot for a single output to a series of side-by-side MORE plots that provide measures of risk and error for the performance metric at different points in time as shown in Figure 5. In this figure, each individual MORE plot is generated from 100 observations of the instantaneous NIS at the respective simulation time (i.e., 100 replications). We use the 25$^{\text{th}}$ and 75$^{\text{th}}$ percentiles and 95% confidence intervals in the plots throughout this paper. Figure 5(a) provides the MORE plots for NIS at the end of every hour while, for the same set of 100 replications, Figures 5(b) through 5(d) present the side-by-side MORE plots of the number in system collected every half an hour, 15 minutes, and 5 minutes, respectively.

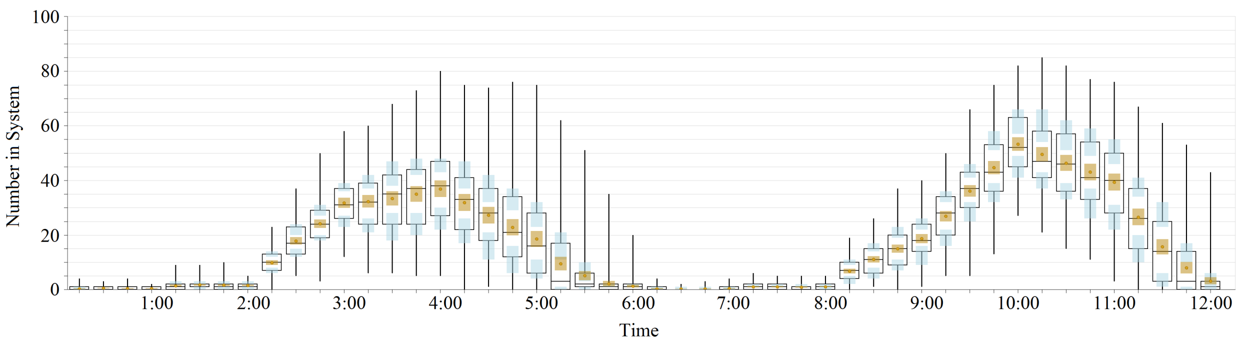
Suppose the objective is to keep the average NIS below 20 during the first peak hours. Based on Figure 5(a), additional capacity seems to be necessary from 2:00 to 6:00 since it is not clear at what point between 2:00 and 3:00 the average NIS exceeds 20 and at which point between 5:00 and 6:00 drops below 20. We are able to get more accurate information by evaluating NIS every 30 minutes as in Figure 5(b) where the decision becomes adding additional capacity from 2:30 to 5:30. Based on 15-minute elapsed times between consecutive observation collections, additional capacity seems to be needed between 2:30 and 5:15, and finally, from 2:30 to 5:05 based on 5-minute apart MORE plots in Figure 5(d). Considering the cost of an additional unit of capacity, the decision using the 5-minute apart plots saves the equivalent cost of 85



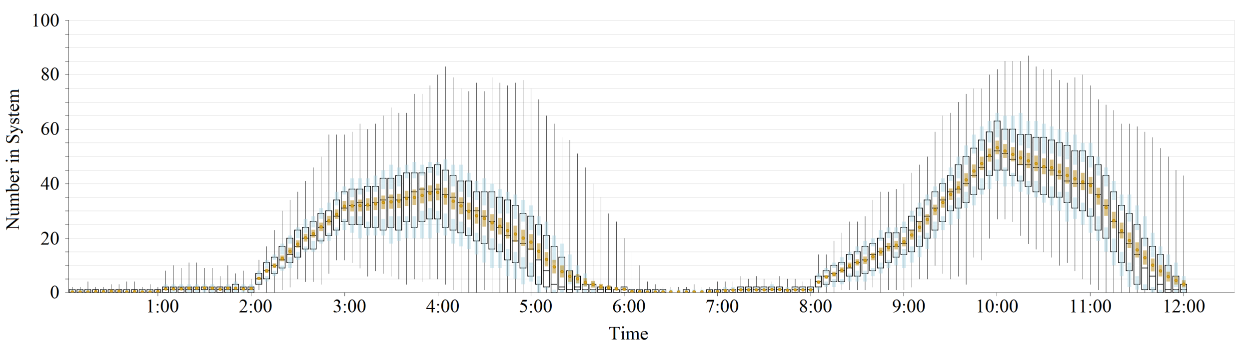
(a) 1-hour intervals between observations



(b) 30-minute intervals between observations



(c) 15-minute intervals between observations



(d) 5-minute intervals between observations

Figure 5: Side-by-side MORE plots of number in system over time.

resource-minutes compared to the decision under 1-hour elapsed times between observations. Of course, in many systems, there are restrictions on how frequently the number of resources can be adjusted and thus the *minimum practical* interval size becomes important. For instance, if the minimum time between capacity expansions/reductions in our system is 30 minutes, then the decision from figures 5(b)-5(d) would all be the same due to practical constraints (regardless of the higher accuracy from shorter intervals).

3.1 Issues Related to the Generation of Side-By-Side MORE Plots

The plots in Figure 5 are generated in Simio®. However, this is not automatically done by the simulation software and requires additional effort and coding. In Simio®, an *experiment* is used to define a set of *scenarios* to be executed (replicated) where each scenario can be thought of as a different parameter configuration of the model. Scenarios may be defined through a set of *control variables* that assign values to different properties of the model. The output are defined by a set of *responses* that are evaluated at the end of each replication of the scenarios in the experiment. For the model of our two-station system, consider an experiment consisting of a single scenario with 100 replications where the instantaneous number in system at the end of hourly intervals are defined as the responses (i.e., a total of 12 responses). While Simio® automatically generates the MORE plot for each response based on the 100 observations, the current version of the software does not generate side-by-side MORE plots for the twelve responses in the experiment and thus the MORE plots related to different time-ticks can only be viewed separately (one at a time). The reader is referred to Kelton, Smith, and Sturrock (2014) for more details about creating experiments, responses, and MORE plots in Simio®.

In order to generate side-by-side MORE plots, the experiment needs to be set up differently. What the software does provide is the side-by-side MORE plots of a selected response for different scenarios in the experiment (the feature is mainly designed to facilitate direct comparison of different scenarios, i.e., different system configurations). Therefore, in order to generate side-by-side MORE plots for NIS, multiple scenarios need to be defined such that in each scenario we record the NIS value at a different time tick. Figure 6 shows the experiment table used for generating hourly NIS MORE plots in Figure 5(a). The experiment contains 12 scenarios each replicated 100 times. The response for the first scenario is the NIS at time 1:00, the NIS at time 2:00 for the second scenario and we continue until the last scenario where the NIS at time 12:00 is recorded in each replication. In order to be able to set up the experiment this way, additional coding is required so that the instantaneous NIS at different time ticks is recorded in a *vector*. A control variable (named *ObservationOfInterestForScenario* in our experiment) is then used to specify the index of the observation of interest in the NIS vector to be reported at the end of each replication for each scenario where replication *r* of each scenario is run using the exact same random seed number. This leads to running the same 100 replications of the exact same model twelve times (total of 1,200 runs) to generate the twelve hourly side-by-side MORE plots. Similarly, Figure 5(d) with 5-minute intervals is generated by repeating the exact same 100 replications for 144 scenarios resulting in a total of 14,400 runs of the model.

There are several downsides to this approach:

1. In general, as the number of side-by-side MORE plots increases (as a result of shorter intervals between observations or a longer run length), more scenarios and simulation runs will be needed. This leads to an excessive number of redundant replications requiring significant computational power and time for medium or large models and can easily become infeasible for complex models that run slowly.
2. More importantly, since the analyst needs to decide on the time intervals before running the simulation experiment, a new set of experiments would be needed to assess a different elapsed time between observations making trial and error with various levels of accuracy even more computationally extensive and time-consuming.

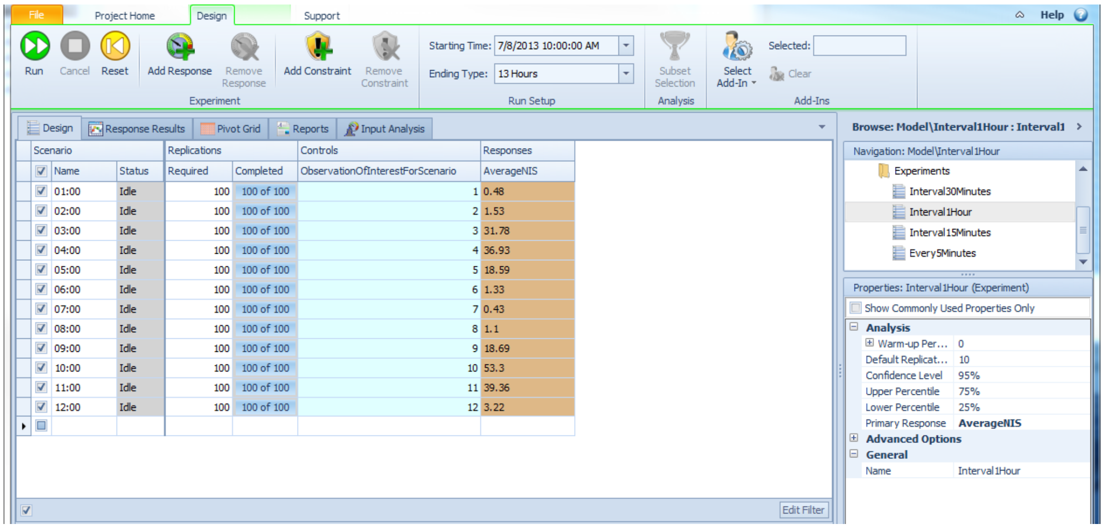


Figure 6: Experiment table in Simio® to generate side-by-side MORE plots in Figure 5(a). Here, all scenarios pertain to the exact same configuration of the model with the only difference being in the simulation time at which NIS is reported (determined by the control variable). Therefore, in order to obtain the twelve hourly MORE plots in Figure 5(a), the same set of 100 replications are repeated twelve times (1,200 replications in total).

3. Simio® and SIMUL8® are the only simulation software that we are aware of that generate MORE plots, but neither support an easy way of generating side-by-side MORE plots for time-dependent statistics.

Therefore, there is a need for a tool that enables generating side-by-side MORE plots for time series data without complicating experimentation while facilitating trial and error with different time intervals (accuracy levels).

4 ADD-MORE: AUTOMATED DYNAMIC DISPLAY OF MEASURE OF RISK & ERROR

We develop an Excel® Add-In that facilitates experimentation and generation of side-by-side more plots. The tool is available online via the tool's webpage (The ADD-MORE Tool 2016). This section describes data collection and user guide for the proposed ADD-MORE tool.

4.1 Data Collection

In order to be able to perform trial and error with different step sizes (i.e., elapsed time between consecutive observation collections), the instantaneous value of the performance measure (in this case NIS) at any point in time in each replication is needed. The data can be obtained by writing the value of the time-dependent statistic whenever it changes along with the respective simulation time at which it occurs to an external file. Fortunately, this can be done easily in the existing simulation packages (for user-programmed models, it can be done by simply adding a *write* step that records the value of the statistic and the current simulation time (in hours) whenever the time-dependent statistic changes). The ADD-MORE tool will then automatically determine the NIS value at any given time t based on the closest NIS value recorded before t in the same replication. This will allow the user to easily experiment with different offset and elapsed time values in the tool rather than in the simulation model eliminating the need to run additional/redundant replications for different values of these parameters. As a result, the tool significantly facilitates the analysis process and the burden on the user when analyzing time-dependent statistics.

4.2 The User-Interface and Implementation of the ADD-MORE Tool

Figure 7 presents the user-interface of the tool. The data from each replication are formatted so that the first column contains the instantaneous NIS and the second column the respective simulation time (in hours) at which NIS changed (as a result of an arrival or departure). The first two columns pertain to the first replication, the second two columns the second replication, and so forth. As shown in the figure, the user specifies an initial *offset* (i.e., the time-tick for which the first MORE plot is generated), the *step* size (i.e., elapsed time between side-by-side MORE plots), the *duration* of time over which the system is analyzed and MORE plots will be generated, upper and lower percentiles of interest, and the confidence level for CIs.

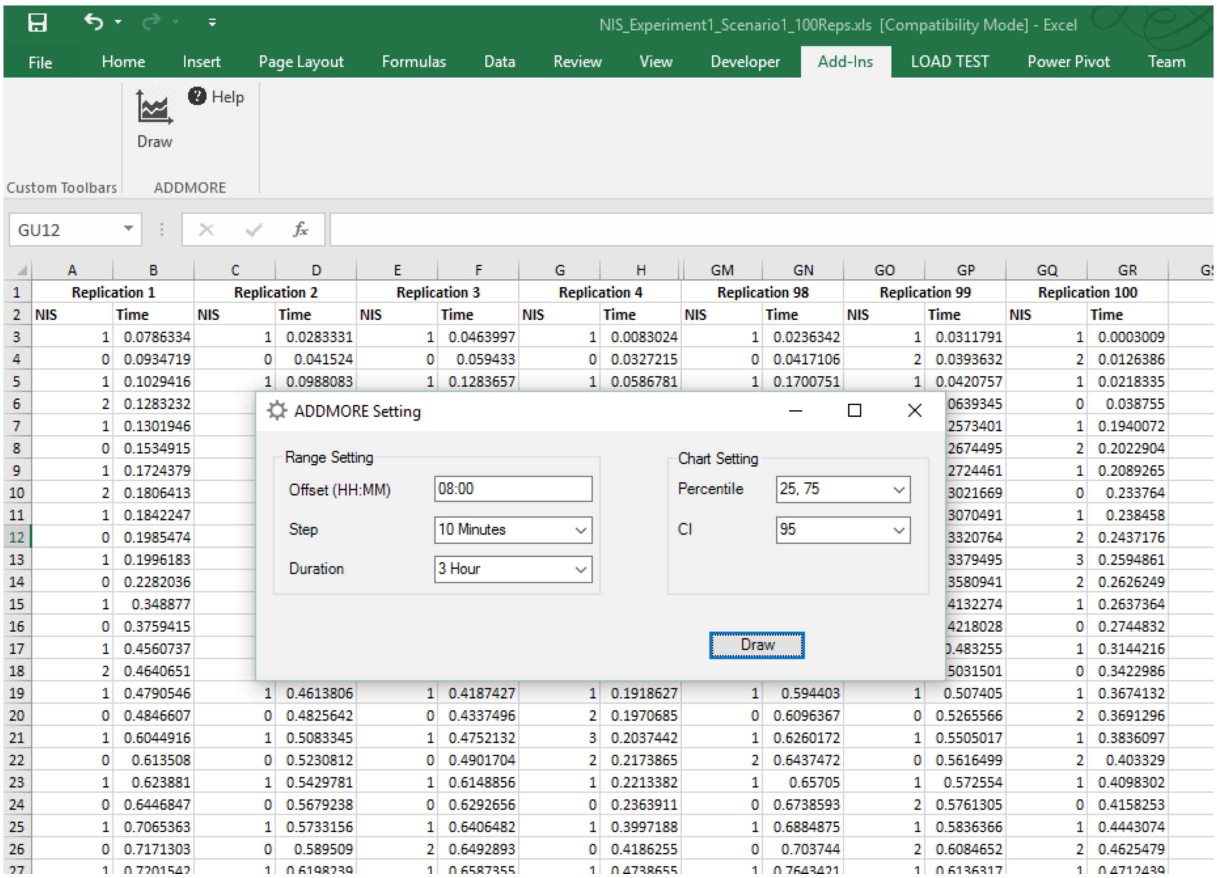
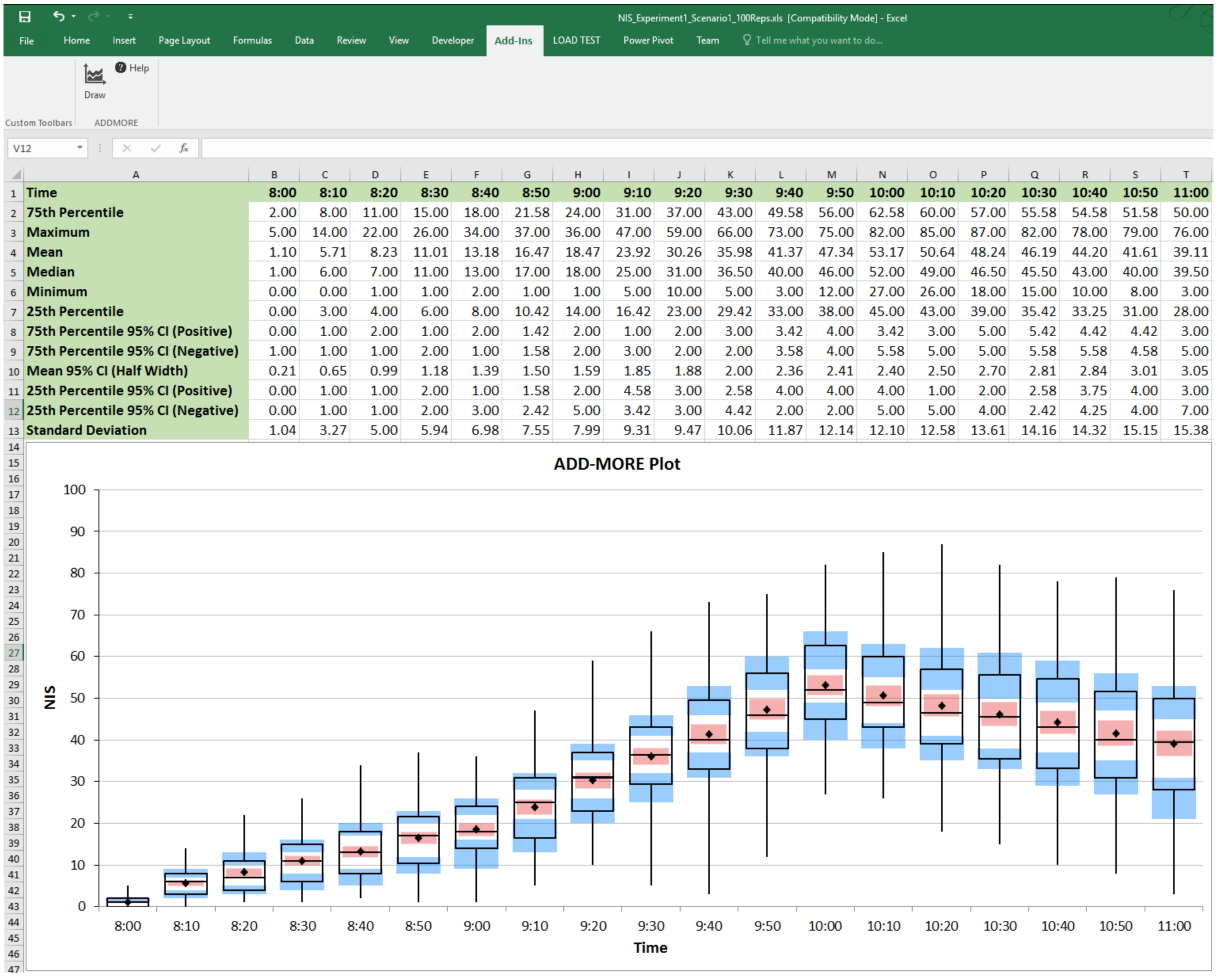


Figure 7: Input data and parameters of the ADD-MORE tool.

With the parameter choices in Figure 7, the time window of interest is the *dinner rush* ($8h:00m-11h:00m$) and thus the offset is set to $08:00$ so the first MORE plot pertains to NIS at $t=8h:00m$. By setting the step size to 10 minutes, MORE plots are generated in 10-minute increments (i.e, the second MORE plot represents NIS at $t=8h:10m$, the third at $t=8h:20m$, and so forth). Since the duration is set to 3 hours, the last MORE plot is created for $t=11h:00m$. Finally, we use the 25^{th} and 75^{th} percentiles to measure risk and 95% confidence intervals to measure the sampling error. After submitting the parameters, the tool reports summary statistics, namely the mean, median, upper and lower percentiles, minimum, maximum, CI half-widths, and the standard deviation of NIS for the specified time ticks and generates the side-by-side MORE plots (Figure 8).



Time	8:00	8:10	8:20	8:30	8:40	8:50	9:00	9:10	9:20	9:30	9:40	9:50	10:00	10:10	10:20	10:30	10:40	10:50	11:00
75th Percentile	2.00	8.00	11.00	15.00	18.00	21.58	24.00	31.00	37.00	43.00	49.58	56.00	62.58	60.00	57.00	55.58	54.58	51.58	50.00
Maximum	5.00	14.00	22.00	26.00	34.00	37.00	36.00	47.00	59.00	66.00	73.00	75.00	82.00	85.00	87.00	82.00	78.00	79.00	76.00
Mean	1.10	5.71	8.23	11.01	13.18	16.47	18.47	23.92	30.26	35.98	41.37	47.34	53.17	50.64	48.24	46.19	44.20	41.61	39.11
Median	1.00	6.00	7.00	11.00	13.00	17.00	18.00	25.00	31.00	36.50	40.00	46.00	52.00	49.00	46.50	45.50	43.00	40.00	39.50
Minimum	0.00	0.00	1.00	1.00	2.00	1.00	1.00	5.00	10.00	5.00	3.00	12.00	27.00	26.00	18.00	15.00	10.00	8.00	3.00
25th Percentile	0.00	3.00	4.00	6.00	8.00	10.42	14.00	16.42	23.00	29.42	33.00	38.00	45.00	43.00	39.00	35.42	33.25	31.00	28.00
75th Percentile 95% CI (Positive)	0.00	1.00	2.00	1.00	2.00	1.42	2.00	1.00	2.00	3.00	3.42	4.00	3.42	3.00	5.00	5.42	4.42	4.42	3.00
75th Percentile 95% CI (Negative)	1.00	1.00	1.00	2.00	1.00	1.58	2.00	3.00	2.00	2.00	3.58	4.00	5.58	5.00	5.00	5.58	5.58	4.58	5.00
Mean 95% CI (Half Width)	0.21	0.65	0.99	1.18	1.39	1.50	1.59	1.85	1.88	2.00	2.36	2.41	2.40	2.50	2.70	2.81	2.84	3.01	3.05
25th Percentile 95% CI (Positive)	0.00	1.00	1.00	2.00	1.00	1.58	2.00	4.58	3.00	2.58	4.00	4.00	4.00	1.00	2.00	2.58	3.75	4.00	3.00
25th Percentile 95% CI (Negative)	0.00	1.00	1.00	2.00	3.00	2.42	5.00	3.42	3.00	4.42	2.00	2.00	5.00	5.00	4.00	2.42	4.25	4.00	7.00
Standard Deviation	1.04	3.27	5.00	5.94	6.98	7.55	7.99	9.31	9.47	10.06	11.87	12.14	12.10	12.58	13.61	14.16	14.32	15.15	15.38

Figure 8: Summary statistics and side-by-side MORE plots generated by the tool.

5 CONCLUSIONS

Consideration of risk and sampling error is critical when making decisions based on predictions from simulation. When the primary performance measure in a non-steady-state situation is a time-dependent statistic that changes over time (e.g., WIP in an assembly line, number of customers on hold in a call center, monthly sales of a new product after its launch, inventory level for a product in a department store), an assessment of how risk and error change over time is necessary to be able to make statistically reliable decisions. In this paper, we illustrate that the traditional approaches/plots provided by existing simulation and statistical packages are insufficient for such assessment. We then show how side-by-side MORE plots (each representing measures of risk and error for the performance measure at a certain simulation time) can be used to analyze time-dependent statistics and support decision-making. Finally, we developed the ADD-MORE tool that facilitates the generation of side-by-side MORE plots where the user can easily perform trial and error with different step sizes between time ticks for which the MORE plots are generated and visually observe how risk and error change over simulated time. In other words, the tool allows for achieving any level of accuracy (as specified by the user) without having to perform additional simulation runs. We also show how the choice of the elapsed time can affect the decision through an example of a two-station serial line with passing. The ADD-MORE tool is implemented as an Excel® Add-In and is available online via the tool's webpage (The ADD-MORE Tool 2016).

It is important to note that the current version of the tool supports the analysis of *time-dependent statistics* (e.g., number in system, number in queue, utilization, etc.); however, the analysis of *observational statistics* (e.g., time in system or time in queue) is more involved. Suppose simulation is used to estimate the waiting time of customers at a Department of Motor Vehicle (DMV) office for different arrival times. Conceptually, in order to generate the MORE plot for waiting time given that the customer arrives at a particular (simulated) time t, we need to insert and track a customer arrival exactly at time t and then use the recorded waiting times for the marked customer across the n replications (which are i.i.d.), to evaluate measures of risk and error. While this method works well for a few fixed simulated times, if we want to assess waiting time throughout the day, inserting marked arrivals every few minutes (e.g., every 5 minutes) would substantially alter the actual arrival process making the model invalid. Smith and Nelson (2015) address this problem in the context of passenger check-in process at an airport. More specifically, they assume i.i.d. waiting times within time buckets and develop a bootstrap method to estimate the variance of the waiting time at any simulated time (used for calculating the mean CI) while the CI for percentiles are obtained through a generalization of the normal approximation to the standard nonparametric confidence interval for a quantile based on the binomial distribution (Banks et al. 2000). Therefore, incorporating their computational logic will be an important extension to the current ADD-MORE tool to facilitate analysis of observational statistics.

As another important extension, statistical tests could be incorporated to identify the time ticks at which the underlying distribution of the performance measure changes (statistically). More specifically, the Welch's t-test can be used to identify significant statistical differences between sample averages for adjacent MORE plots while non-parametric tests (such as bootstrap methods) would help compare percentiles from one time tick to another. The tool can be further extended to automate the process of identifying the *exact* time at which statistical changes in measures of risk and error occur as follows. Suppose a significant change is determined for two adjacent time ticks in the hourly side-by-side MORE plots. The *Bisection* method (or any other related numerical method) can be used to find the exact time(s) in this 1-hour interval (at a given level of accuracy) at which the underlying distribution changes. This feature could potentially help characterize the transient effect and determine an appropriate warm-up period in steady-state simulations.

Finally, while our focus here is mainly on the application of the ADD-MORE tool for simulation output analysis, the tool can be potentially useful in any non-simulation application area that requires analysis of historical data on time series (i.e., multiple observations of a variable at different times). Examples include but are not limited to: (1) quality control (e.g., analyzing the $\overline{X}$-charts obtained from 50 days of operation of a semiconductor fab); (2) transportation (e.g., analyzing the number of cars traveling on each bound of a major bridge at different hours to manage lane configurations through the use of movable median barriers); (3) inventory management (e.g., analyzing seasonal demand for a product to determine an appropriate level of safety stock); (4) wireless network (e.g., analyzing historical data on bandwidth usage at different hours to determine an appropriate bandwidth/capacity for the network); (5) electric power grids (e.g., optimizing power distribution and transmission strategies based on historical data on hourly electricity consumption of different demand sites). In all of the above examples, the historical data contain multiple trajectories of a time-dependent statistic and the ADD-MORE tool would provide useful information about risk and sampling error and thus support effective decision-making. It is worth noting that some modifications may be necessary to set up the tool to support the different types of data used in such non-simulation applications.

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AUTHOR BIOGRAPHIES

ASHKAN NEGAHBAN is an Assistant Professor of Engineering Management at Pennsylvania State University, School of Graduate Professional Studies. He has a PhD and MS from Auburn University and a BS from University of Tehran (all in Industrial Engineering). His research interest is in the application of agent-based, discrete event, and Monte Carlo simulation in manufacturing systems and operations management. His email and web addresses are anegahban@psu.edu and http://ashkannegahban.wordpress.com.

MOHAMMADNASER ANSARI is a teaching assistant and PhD student in the Department of Industrial and Systems Engineering at Auburn University. He received his B.Sc. in Industrial Engineering from University of Tehran, Iran. His research interests are in simulation, visualization analytics, supply chain management, and distribution center design and operations. His email address is: ansarim@auburn.edu.

JEFFREY S. SMITH is the Joe W. Forehand Professor of Industrial and Systems Engineering at Auburn University. He has served as the WSC Business Chair (2010) and General Chair (2004) and is currently on the WSC Board of Directors. He has a BIE from Auburn University and a MS and PhD (both in Industrial Engineering) from Penn State University. His email and web addresses are: jsmith@auburn.edu and http://jsmith.co.