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OPTIMAL COMPUTING BUDGET ALLOCATION WITH EXPONENTIAL UNDERLYING DISTRIBUTION

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ABSTRACT

In this paper, we consider the simulation budget allocation problem to maximize the probability of selecting the best simulated design in ordinal optimization. This problem has been studied extensively on the basis of the normal distribution. In this research, we consider the budget allocation problem when the underlying distribution is exponential. This case is widely seen in simulation practice. We derive an asymptotic closed-form allocation rule which is easy to compute and implement in practice, and provide some useful insights for the optimal budget allocation problem with exponential underlying distribution.

1 INTRODUCTION

The simulation-optimization (SO) problem is a non-linear optimization problem, which is often too complex to be evaluated analytically due to the uncertainty and dynamic relationships between the parts involved. Therefore, stochastic simulation becomes a powerful modeling and software tool for analyzing modern complex systems. Although the advance of computer technology has dramatically increased computational power, efficiency is still a significant concern because 1) simulation experiment is usually time consuming; 2) many simulation replications are typically required for an accurate estimate of performance (Lee et al. 2010).

In order to address this concern, Ranking and Selection (R&S) problems are widely studied in order to intelligently allocate the simulation budget and improve simulation efficiency. The indifference-zone (IZ) approach aims to provide a guaranteed lower bound for the probability of correct selection (PCS), assuming that the mean performance of the best design is at least δ^* better than each alternative, where δ^* is the minimum difference worth detecting (Dudewicz and Dalal 1975; Rinott 1978; Kim and Nelson 2001; Nelson et al. 2001). Another popular approach is optimal computing budget allocation (OCBA), which allocates the samples sequentially in order to maximize PCS under a simulation budget constraint (Chen et al. 2000). In addition, the optimal selection problem with the expected opportunity cost (EOC), a common quality measure other than PCS, was considered in Gao and Shi (2015). OCBA highly improves the efficiency of budget allocation by intelligently controlling the number of simulation replications based on the mean and variance information (Chen and Lee 2011). Chen et al. (2008), and Gao and Chen (2015) further extended the OCBA method to optimal subset selection problem. For a comprehensive review of this field, see Branke et al. (2007), and Kim and Nelson (2007).

The OCBA method is widely used because of its high efficiency as well as that it establishes a simple, intuitive and closed-form expression to formulate the budget allocation problem. The OCBA method is developed under the assumption that the underlying distribution is normal. However, the normal distribution assumption may not always reflect practice when the sample size is not large enough. Glynn and Juneja (2004), Hunter and Pasupathy (2013), and Pasupathy et al. (2015) extended the budget allocation method by employing large deviations (LD) approach for non-normal distribution context. Broadie et al. (2007) gave some analyses of the algorithm provided in Glynn and Juneja (2004) in the setting of heavy-tailed systems. Moreover, as in Broadie et al. (2007), even though the use of the LD approach provides the flexibility for the underlying distribution to be general, it is computationally intensive and difficult to implement in practice. A closed-form allocation function is still hard to be derived. This opens up the question which is the main topic of this paper.

In this paper, we consider the problem of optimal allocation of computing budget to maximize the PCS when the underlying distribution is exponential. Exponential distribution occurs naturally when describing the lengths of the inter-arrival times in a homogeneous Poisson process, and it is widely used in queuing network (Asmussen 2008). In this paper, we derive an asymptotic closed-form simulation budget allocation rule, called OCBA-exp, based on large deviations theory for exponential distribution problem. The proposed OCBA-exp is easy to compute and implement in practice, and can provide some useful insights for the exponential distribution problem.

The rest of the paper is organized as follows: in the next section, we derive an allocation scheme based on large-deviations theory and then carry out an asymptotic analysis. The performance of the proposed method is illustrated with numerical examples in Section 3. Section 4 concludes the paper.

2 EFFICIENT SIMULATION BUDGET ALLOCATION

In this section, we formulate the budget allocation problem when the underlying distribution is exponential and provide some useful insights for them.

2.1 Notation

In this research, the best design is defined as the design with the smallest mean performance (the largest mean performance could be handled similarly). The simulation output samples are exponentially distributed and independent from replication to replication, as well as independent across designs. We introduce the following notation:

n: total number of simulation replications (budget);
k: total number of designs;
$X_{i,j}$: output of the j-th simulation replication for design i;
μ_i: mean of design i, i.e., $\mu_i = E[X_{i,j}]$;
σ_i^2: variance of design i, i.e., $\sigma_i^2 = Var[X_{i,j}]$;
α_i: proportion of the total simulation budget allocated to design i;
n_i: number of simulation replications allocated to design i, i.e., $n_i = \alpha_i n$;
$\bar{X}_i$: sample mean of design i, i.e., $\bar{X}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} X_{i,j}$;
S_i^2: sample variance of design i, i.e., $S_i^2 = \frac{1}{n_i-1}\sum_{j=1}^{n_i}(X_{i,j}-\bar{X}_i)^2$.

We let the real best design $t = \arg\min_{i\in\{1,2,\ldots,k\}} \mu_i$. In this paper, we ignore the minor technicalities associated with n_i's not being an integer.

Let $\Lambda_i(\theta) = \log E[\exp(\theta X_{i,j})]$ denote the log-moment generating function of $X_{i,j}$ and $I_i(\cdot)$ be the Fenchel-Legendre transform of Λ_i, i.e.,

$$I_i(x) = \sup_{\theta\in\mathbb{R}}(\theta x - \Lambda_i(\theta)).$$

As presented in Glynn and Juneja (2004), rate function $G_{t,i}(\alpha_t,\alpha_i) = \inf_x(\alpha_t I_t(x) + \alpha_i I_i(x))$. Let $x(\alpha_t,\alpha_i)$ be the unique solution to $\alpha_t I_t'(x) + \alpha_i I_i'(x) = 0$. Since $\alpha_t I_t'(x) + \alpha_i I_i'(x)$ is strictly convex, the infimum is obtained at $x(\alpha_t,\alpha_i)$, i.e., $G_{t,i}(\alpha_t,\alpha_i) = \alpha_t I_t(x(\alpha_t,\alpha_i)) + \alpha_i I_i(x(\alpha_t,\alpha_i)),\ i = 1,2,\ldots,k$, and $i \neq t$.

Note that, $\frac{\partial G_{t,i}(\alpha_t,\alpha_i)}{\partial \alpha_i} = I_i(x(\alpha_t,\alpha_i))$ and $\frac{\partial G_{t,i}(\alpha_t,\alpha_i)}{\partial \alpha_t} = I_t(x(\alpha_t,\alpha_i))$, where $i = 1,2,...,k$, and $i \neq t$.

2.2 Optimal Allocation Strategy

We consider the problem of selecting single best design from k alternative designs when the underlying distribution is exponential. The goal is to find a simulation budget allocation that maximize the probability of correct selection (PCS) or minimize the probability of false selection ($\text{PFS} = 1 - \text{PCS}$) with $\sum_{i=1}^{k} \alpha_i = 1$. According to Glynn and Juneja (2004), large deviations approach is used to asymptotically minimize the probability of false selection. In that study, the budget allocation problem is formulated as:

$$\begin{aligned} \min \quad & \lim_{n\to\infty} \frac{1}{n} \log PFS \\ s.t. \quad & \sum_{i=1}^{k} \alpha_i = 1. \end{aligned} \tag{1}$$

As presented in Glynn and Juneja (2004), optimality conditions for general underlying distribution are as follows:

$$\sum_{i=1, i\neq t}^{k} \frac{\partial G_{t,i}(\alpha_t,\alpha_i)/\partial \alpha_t}{\partial G_{t,i}(\alpha_t,\alpha_i)/\partial \alpha_i} = 1, \tag{2}$$

$$G_{t,i}(\alpha_t,\alpha_i) = G_{t,j}(\alpha_t,\alpha_j), \text{ for } i,j = 1,2,...,k \text{ and } i \neq j \neq t. \tag{3}$$

Apply these optimality conditions for the context of exponential underlying distribution. The probability density function of exponential distribution

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0, \\ 0 & x < 0. \end{cases}$$

Thus,

$$I_i(x) = \lambda_i x - 1 - \log(\lambda_i x), \ i = 1,2,...,k,$$

$$x(\alpha_t,\alpha_i) = \frac{\alpha_t + \alpha_i}{\alpha_t \lambda_t + \alpha_i \lambda_i}, \ i = 1,2,...,k \text{ and } i \neq t,$$

$$G_{t,i}(\alpha_t,\alpha_i) = -\alpha_t \log \frac{\lambda_t(\alpha_t + \alpha_i)}{\alpha_t \lambda_t + \alpha_i \lambda_i} - \alpha_i \log \frac{\lambda_i(\alpha_t + \alpha_i)}{\alpha_t \lambda_t + \alpha_i \lambda_i}, \ i = 1,2,...,k \text{ and } i \neq t,$$

where λ_i denotes the rate parameter of exponential distribution for design i.

According to Gao and Shi (2016),

$$I_i(x(\alpha_t,\alpha_i)) = \frac{\partial G_{t,i}(\alpha_t,\alpha_i)}{\partial \alpha_i} = \frac{\lambda_i(\alpha_t + \alpha_i)}{\alpha_t \lambda_t + \alpha_i \lambda_i} - 1 - \log \frac{\lambda_i(\alpha_t + \alpha_i)}{\alpha_t \lambda_t + \alpha_i \lambda_i}, \ i = 1,2,...,k \text{ and } i \neq t. \tag{4}$$

$$I_t(x(\alpha_t,\alpha_i)) = \frac{\partial G_{t,i}(\alpha_t,\alpha_i)}{\partial \alpha_t} = \frac{\lambda_t(\alpha_t + \alpha_i)}{\alpha_t \lambda_t + \alpha_i \lambda_i} - 1 - \log \frac{\lambda_t(\alpha_t + \alpha_i)}{\alpha_t \lambda_t + \alpha_i \lambda_i}, \ i = 1,2,...,k \text{ and } i \neq t. \tag{5}$$

Simplify (4) and (5) using Taylor expansion: $\log(x) = (x-1) - \frac{(x-1)^2}{2} + O((x-1)^3)$, since $0 < \lambda_i < \lambda_t$, $\left| \frac{\lambda_i(\alpha_t+\alpha_i)}{\alpha_t\lambda_t+\alpha_i\lambda_i} - 1 \right| < 1$ and $\left| \frac{\lambda_t(\alpha_t+\alpha_i)}{\alpha_t\lambda_t+\alpha_i\lambda_i} - 1 \right| < 1$,

$$\log \frac{\lambda_i(\alpha_t + \alpha_i)}{\alpha_t \lambda_t + \alpha_i \lambda_i} \approx (\frac{\lambda_i(\alpha_t + \alpha_i)}{\lambda_t \alpha_t + \lambda_i \alpha_i} - 1) - \frac{1}{2}(\frac{\lambda_i(\alpha_t + \alpha_i)}{\alpha_t \lambda_t + \alpha_i \lambda_i} - 1)^2,$$

$$\log \frac{\lambda_t(\alpha_t + \alpha_i)}{\alpha_t \lambda_t + \alpha_i \lambda_i} \approx (\frac{\lambda_t(\alpha_t + \alpha_i)}{\lambda_t \alpha_t + \lambda_i \alpha_i} - 1) - \frac{1}{2}(\frac{\lambda_t(\alpha_t + \alpha_i)}{\alpha_t \lambda_t + \alpha_i \lambda_i} - 1)^2.$$

Therefore,

$$\frac{\partial G_{t,i}(\alpha_t,\alpha_i)}{\partial \alpha_i} \approx \frac{\alpha_t^2(\lambda_i-\lambda_t)^2}{2(\alpha_t\lambda_t+\alpha_i\lambda_i)^2},\ i=1,2,...,k \text{ and } i\neq t,$$

$$\frac{\partial G_{t,i}(\alpha_t,\alpha_i)}{\partial \alpha_t} \approx \frac{\alpha_i^2(\lambda_i-\lambda_t)^2}{2(\alpha_t\lambda_t+\alpha_i\lambda_i)^2},\ i=1,2,...,k \text{ and } i\neq t.$$

(2) becomes: $\sum_{i=1,\ i\neq t}^{k} \alpha_i^2/\alpha_t^2 = 1$, that is

$$\alpha_t^2 = \sum_{i=1,i\neq t}^{k} \alpha_i^2. \tag{6}$$

(3) becomes:

$$\alpha_t \log\frac{\lambda_t(\alpha_t+\alpha_i)}{\alpha_t\lambda_t+\alpha_i\lambda_i} + \alpha_i \log\frac{\lambda_i(\alpha_t+\alpha_i)}{\alpha_t\lambda_t+\alpha_i\lambda_i} = \alpha_t \log\frac{\lambda_t(\alpha_t+\alpha_j)}{\alpha_t\lambda_t+\alpha_j\lambda_j} + \alpha_j \log\frac{\lambda_j(\alpha_t+\alpha_j)}{\alpha_t\lambda_t+\alpha_j\lambda_j},\ i\neq j\neq t,. \tag{7}$$

In order to reduce the total computational cost for identifying the best design, it is advisable to spend more computational effort on good designs. That is, α_t should be increased relative to α_i, for $i=1,2,...,k$, and $i\neq t$. Therefore, according to (6) we can assume $\alpha_t \gg \alpha_i$ and $\alpha_t \log\frac{\lambda_t(\alpha_t+\alpha_i)}{\alpha_t\lambda_t+\alpha_i\lambda_i} \gg \alpha_i \log\frac{\lambda_i(\alpha_t+\alpha_i)}{\alpha_t\lambda_t+\alpha_i\lambda_i}$ for all $i\neq t$ as $n\to\infty$. For more details, please refer to Glynn and Juneja (2004). This assumption enables us to simplify (7) as

$$\frac{\lambda_t(\alpha_t+\alpha_i)}{\alpha_t\lambda_t+\alpha_i\lambda_i} = \frac{\lambda_t(\alpha_t+\alpha_j)}{\alpha_t\lambda_t+\alpha_j\lambda_j},\ i\neq j\neq t.$$

That is,

$$\frac{\lambda_j-\lambda_t}{\alpha_i} = \frac{\lambda_i-\lambda_t}{\alpha_j} + \frac{\lambda_i-\lambda_j}{\alpha_t},\ i\neq j\neq t.$$

We further assume $\alpha_t \gg \alpha_i$ for all $i\neq t$ as $n\to\infty$, then

$$\frac{\alpha_i}{\alpha_j} = \frac{\lambda_j-\lambda_t}{\lambda_i-\lambda_t},\ i\neq j\neq t.$$

For exponential distribution, mean $\mu_i = \frac{1}{\lambda_i}$, variance $\sigma_i^2 = \frac{1}{\lambda_i^2}$. Thus,

$$\frac{\alpha_i}{\alpha_j} = \frac{\sigma_i/(\mu_i-\mu_t)}{\sigma_j/(\mu_j-\mu_t)},\ i\neq j\neq t.$$

By using two approximations: the Taylor approximation of the rate function and the assumption that $\alpha_t \gg \alpha_i$, for all $i\neq t$ as $n\to\infty$, we have the following result:

Theorem 1 Problem (1) can be asymptotically minimized when

$$\alpha_t^2 = \sum_{i=1,i\neq t}^{k} \alpha_i^2, \tag{8}$$

$$\frac{\alpha_i}{\alpha_j} = \frac{\sigma_i/(\mu_i-\mu_t)}{\sigma_j/(\mu_j-\mu_t)},\ \text{for } i,j=1,2,...,k \text{ and } i\neq j\neq t. \tag{9}$$

2.3 Analysis of the Exponential Budget Allocation Rule

We provide some insights for the optimal allocation rules (8) and (9) demonstrated. We conduct a simple numerical experiment to compare the number of simulation replications allocated to each design by OCBA-exp and the traditional OCBA when the underlying distribution is exponential.

The traditional OCBA method allocates the samples sequentially in order to maximize PCS under the assumption that the underlying distribution is normal (Chen et al. 2000, Chen and Lee 2011). In each iteration, it allocates simulation replications to the candidate designs according to

$$\frac{\alpha_t^2}{\sigma_t^2} = \sum_{i=1, i\neq t}^{k} \frac{\alpha_i^2}{\sigma_i^2}, \tag{10}$$

$$\frac{\alpha_i}{\alpha_j} = \frac{\sigma_i^2/(\mu_i - \mu_t)^2}{\sigma_j^2/(\mu_j - \mu_t)^2},\ i \neq j \neq t. \tag{11}$$

It is interesting to find that the proposed optimality conditions (8) and (9) for OCBA-exp method have structural similarities with the optimality conditions (10) and (11) for the traditional OCBA mehtod. (8) and (10) show the relationship between α_t and α_i, while (8) does not have to consider the variance of design during allocation procedure compared with (10). For optimality conditions (9) and (11), $\sigma_i/(\mu_i - \mu_t)$ can be intuitively considered as a noise to signal ratio for design i as compared with the observed best design t. (9) and (11) show the relationship between α_i and α_j ($i \neq j \neq t$), and (9) demonstrates that the allocated computing budget is proportional to the noise to signal ratio instead of the square of the noise to signal ratio.

Let total simulation budget $n = 10000$ which will be allocated to 10 designs. Design i has a distribution of $Exp((4+i/10)^{-1})$, i.e., $\mu_i = \sigma_i = (4+i/10),\ i = 1,2,...,10$. As μ_i and σ_i are known to us, we can easily calculate the number of simulation replications allocated to each design, according to the optimality conditions for OCBA-exp and OCBA, respectively. The budget allocation strategy for traditional OCBA and the proposed OCBA-exp method is reported in Figure 1.

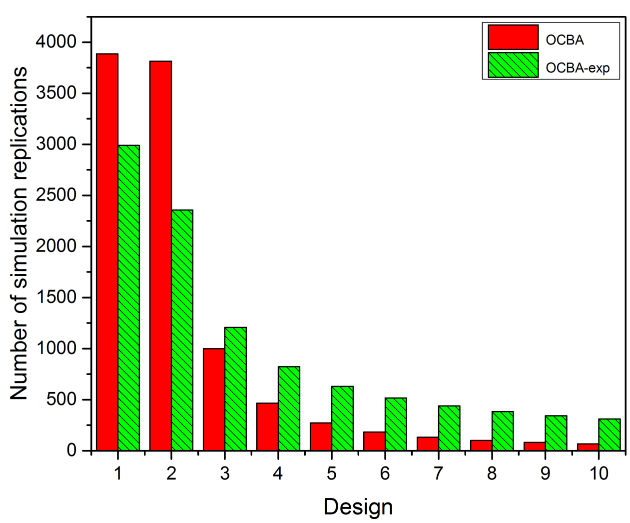


Figure 1: The budget allocation strategy for OCBA method and OCBA-exp method.

From the result, it is observed that the proposed OCBA-exp allocation method allocates more computing budget to the inferior designs compared with traditional OCBA method. Design 3 to design 10 receive more simulation replications calculated by OCBA-exp method compared with the traditional OCBA method. One of the possible reasons is that, with identical means and variances, exponential distribution has a heavier tail than the normal distribution.

2.4 Sequential Budget Allocation Procedure

We develop a sequential simulation budget allocation procedure, called OCBA-exp, to implement the optimality conditions (8) and (9). Each design is initially simulated with n_0 replications, and additional replications are allocated incrementally with Δ replications in each iteration according to optimality conditions (8) and (9). In summary, we have the following budget allocation procedure.

OCBA-exp Procedure

INITIALIZE Iteration counter $l \leftarrow 0$;
Perform n_0 simulation replications for all designs; $n_1^l = n_2^l = ... = n_k^l = n_0$.

LOOP WHILE $\sum_{i=1}^{k} n_i^l < n$ DO

UPDATE First, calculate sample means $\bar{X}_i = \frac{1}{n_i^l}\sum_{j=1}^{n_i^l} X_{i,j}$, and sample variance $S_i^2 = \frac{1}{n_i^l-1}\sum_{j=1}^{n_i^l}(X_{i,j} - \bar{X}_i)^2$, $i = 1,2,...,k$, using the new simulation output; find $\hat{t} = \arg\min_{i\in\{1,2,...,k\}} \bar{X}_i$.

ALLOCATE Increase the computing budget by $\triangle$ and calculate the new budget allocation, n_1^{l+1}, $n_2^{l+1},...,n_k^{l+1}$, according to (8) and (9).

SIMULATE Perform additional $\max(n_i^{l+1} - n_i^l, 0)$ simulations for design i, $i = 1,2,...,k$; $l \leftarrow l+1$,

END OF LOOP

SELECT Select the design with the smallest sample mean.

3 NUMERICAL EXPERIMENTS

In this section, we test the proposed OCBA-exp procedure by comparing it with the traditional OCBA method on two typical selection problems.

In order to compare the performance of these allocation approaches, we test them empirically on the selection examples below.

Example 1: It has 10 designs. Design i has a distribution of $Exp((4+i/10)^{-1})$, i.e., rate parameter $\lambda_i = (4+i/10)^{-1}$ and $i = 1,2,...,10$.

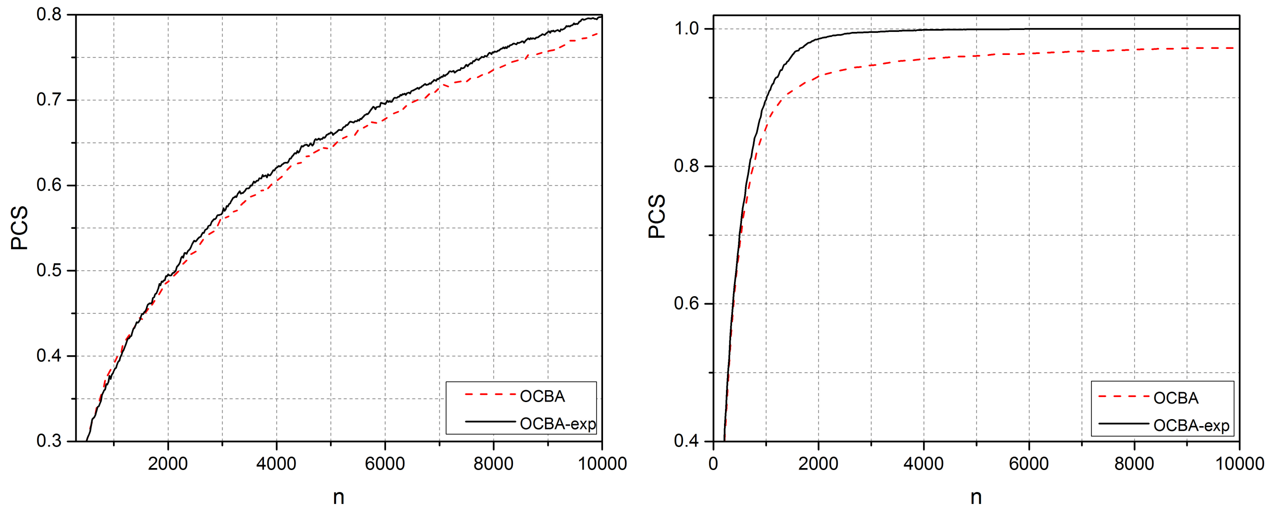
Example 2: It has 10 designs. Design 1 has a distribution of $Exp(4^{-1})$, i.e., rate parameter $\lambda_1 = 4^{-1}$, and design 2 to design 10 have the same distribution of $Exp(5^{-1})$, i.e., rate parameter $\lambda_i = 5^{-1}$ and $i = 2,3,...,10$.

The sequential OCBA and OCBA-exp procedures allocate the computing budget with the objective of selecting the best design, i.e., $t = 1$. We perform 10 initial replications for each design. Incremental budget is 20, which will be allocated to the candidate designs according to (8) and (9) for the proposed OCBA-exp method, and (10) and (11) for the traditional OCBA method. The estimate of PCS is based on the average of 8000 independent replications of each procedure to the problem. The comparison of the two approaches is reported in Figure 2.

From the results, it is observed that the proposed OCBA-exp method works better than the traditional OCBA method when the underlying distribution is exponential. That is, the proposed OCBA-exp method can better adapt to exponential underlying distribution structure. In addition, when dealing with relatively difficult selection problem, the OCBA-exp method seems to demonstrate more advantages compared with the traditional OCBA method.

4 CONCLUSIONS

In this study, an efficient simulation budget allocation rule is presented for exponential underlying distribution. Thanks to its closed-form expression, the proposed OCBA-exp method is easy to compute and implement



(a) Example 1 (b) Example 2

Figure 2: Comparison results of the two methods.

in practice. The objective is to maximize the probability of correct selection within a given computing budget. Numerical testing indicates that the proposed OCBA-exp approach is more efficient than the traditional OCBA method when the underlying distribution is exponential. We also perform some analysis on the budget allocation method and provide some useful insights for determining the best design when the underlying distribution is exponential.

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