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A UNIFIED APPROACH FOR MODELING AND ANALYSIS OF A VERSATILE BATCH-SERVICE QUEUE WITH CORRELATED ARRIVAL

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ABSTRACT

A batch-service queue with Markovian arrival process can be adopted to deal with highly correlated systems which arises in high speed telecommunication networks. Such complex models are usually dealt by evoking simulation method. However, we carry out modeling and analysis of this queue by developing an analytically tractable methodology which is based on bivariate vector generating function. The queue length and server content distribution at service completion epoch are extracted and presented in terms of the roots. The knowledge of both the queue and server content distributions helps the system designer to evaluate the efficiency of the queueing system in a better way. Some significant key performance measures along with illustrative numerical examples are presented in order to show the usefulness of our results. The notable feature of our methodology is that it is easier to follow and implement.

1 INTRODUCTION AND MOTIVATION

The Poisson process is not capable to cope with the correlated and bursty traffic, which usually arises in high-speed teletraffic networks where IP packets, messages, images and data are transmitted. The barriers arising from the Poisson process, used in modeling of correlated arrivals, was broken by introducing Markovian arrival process (*MAP*), which is an excellent representation of bursty nature of traffics with correlated arrival. The *MAP*, a rich class of stochastic counting process, includes most commonly used arrival processes such as the Poisson process, the phase-type (*PH*) renewal process, the Markov-modulated Poisson process (*MMPP*), and interrupted Poisson process (*IPP*).

Batch-service queues have wide range of noteworthy applications in wireless telecommunication to deal with multimedia type of data, manufacturing systems etc. In most of the applications of batch-service queues, it is always better to use the maximum capacity of the server in order to minimize cost or delivery time. This purpose demands the necessity of the server content distribution. This importance motivates us to bring a clear identification about both queue and server content distribution together. In recent years, attention has been paid to obtain both queue and server content distribution by few authors such as (Banerjee, Gupta, and Chakravarthy 2015), (Yu and Alfa 2015), and (Claeys, Steyaert, Walraevens, Laevens, and Bruneel 2013).

In this work, we consider an infinite-buffer batch-service queue where customers arrive according to *MAP*. Using supplementary variable technique we derive the bivariate vector generating function (VGF) of queue length and number with the departing batch in a straightforward way as well as develop the connection between arbitrary and service completion epoch. The significant new results in this work are: (*i*) a bivariate VGF of queue length and number in a served batch is derived at service completion epoch, (*ii*) joint distribution of queue content and size of departing batch has been extracted from the bivariate VGF by methods of roots, (*iii*) arbitrary epoch probability vectors are computed by generating a relation between service completion and arbitrary epoch probability vectors, (*iv*) towards the end, our analytical procedure has been justified by means of assorted numerical examples.

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2 MODEL DESCRIPTION AND ANALYSIS

Customers arrive according to *MAP*, governed by m-state underlying Markov chain (UMC). The transition occurs from state i to j in the UMC without and with an arrival with probability $c_{i,j}$ and $d_{i,j}$, respectively and $1 \le i, j \le m$. The $m \times m$ matrix $\mathbf{C} = (c_{i,j})$ governs transition without arrivals, while the $m \times m$ matrix $\mathbf{D} = (d_{i,j})$ governs transition with arrivals. The single server operates the customers in batches with a predetermined threshold value a and with a maximum capacity b. It remains idle if queue length is less than a and initiate the service as soon as queue length reaches to a. For the queue length r, $(a \le r \le b)$ it takes all the customers for service. The service time of a batch follows general distribution having cumulative distribution function $S(t)$ and mean service time $\frac{1}{\mu}$.

Our main objective is to perceive joint queue and server content distribution at service completion and arbitrary epochs. For this, we first derive a bivariate VGF $[\Pi^+(z,y)]$ of queue length and number with the departing batch $[\pi^+(n,r),\ n \ge 0,\ a \le r \le b]$ and is given by

$$\begin{aligned}\Pi^+(z,y)\left[z^b\mathbf{I}-\mathbf{A}(z)\right] \;=\; & \sum_{n=0}^{a-1}\psi^+(n)\left\{\left(y^b-y^a\right)\overline{\mathbf{D}}^{a-n}\mathbf{A}(z)^2+\left(\overline{\mathbf{D}}^{a-n}y^a z^b-y^b z^n\right)\mathbf{A}(z)\right\}\\ &+\sum_{n=a}^{b-1}\psi^+(n)\left\{\left(y^b-y^n\right)\mathbf{A}(z)^2+\left(y^n z^b-y^b z^n\right)\mathbf{A}(z)\right\}\end{aligned}$$

where $\psi^+(n)$ is the probability vector of only queue length consisting of 'n', $(n \ge 0)$, customers at service completion epoch of a batch, $\mathbf{A}(z)$ is the matrix generating function of $\mathbf{A}_j$'s (the probability that $j,\ j \ge 0$ customers arrive during the service time of a batch), $\mathbf{I}$ is the identity matrix of order m, and $\overline{\mathbf{D}} = (-\mathbf{C})^{-1}\mathbf{D}$.

First we determine the unknowns contained in the above complex bivariate VGF by making use of well known Rouché's theorem. Then, we extract the distribution of both queue content and number with the departing batch $\pi^+(n,r),\ n \ge 0,\ a \le r \le b$, in terms of the roots of the associated characteristic equation.

Finally, utilizing these state probabilities we establish an association between service completion, arbitrary and pre-arrival epoch probability vectors. Moreover, we present some relevant performance measures such as average number in the queue, system and server, waiting time in the queue as well as system, probability that server is idle, etc. For implementation purpose of our method, we adopt service time to be Phase type (*PH*) and matrix exponential (*ME*) distributions.

3 CONCLUSION

A complex correlated arrival queueing model with batch-service and general service time distribution is analyzed where both queue and server content distribution is obtained instead of only queue length distribution. Our methodology can be used in wider range to analyze a more complex queue with batch Markovian arrival process (*BMAP*) and batch-size-dependent service time. At the endmost, it is hoped that the results will be worthwhile to the real system designer.

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