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MEAN-FIELD BASED COMPARISON OF TWO AGE-DEPENDENT SIR MODELS

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ABSTRACT

In this work we compare two structurally different modeling approaches for the simulation of an age-dependent SIR (susceptible, infected, recovered) type epidemic spread: a microscopic agent-based model and a macroscopic integro-partial differential equation model. Doing so we put a newly derived Mean-Field Theorem for mixed state-spaces (continuous and discrete) to the test. Afterwards both models are executed and compared for two abstract scenarios to confirm the derived asymptotic equivalence.

1 INTRODUCTION

Comparing two structurally different models on analytical basis can generally contribute to get deeper insights into either of the two modeling methods. Those insights can furthermore be used to support the validation, parametrization and verification process of either of the two models. In this work we compare two different modeling approaches for the simulation of an age-dependent SIR (susceptible, infected, recovered) type epidemic spread: a (microscopic) agent-based model (ABM) and a (macroscopic) integro-partial differential equation (IPDE) model. As a part of the DEXHELPP project we hope that gained structural knowledge from this comparison can support applied models for the Austrian health-care system.

2 MODELING APPROACHES

The foundation of both compared modeling approaches lies withing the famous SIR model by Kermack and McKendrick (Kermack and McKendrick 1927) and is extended by an idea published by Hoppenstead (Hoppensteadt 1974): First of all the standard progress of the disease ($S \to I \to R$) is supplemented with a death-state D. Second, the contact process, necessary for an infection of an individual, is modeled depending on the ages of the individual and its contact partner. Modeling this feature is justified as e.g. the probability that an individual has contact with other individuals approximately its age is higher, than with individuals about ten years older or younger. Mathematically this leads to the definition of a so called contact kernel κ, indicating how likely contacts between persons with different ages are (see (1)).

$$\kappa(a,b) > \kappa(a,c) \Leftrightarrow \text{Person with age } a \text{ is more likely to have contact with age } b \text{ than } c. \tag{1}$$

Analogously age-dependent mortality μ, recovery γ and infection ξ functions are defined:

$$\left.\begin{matrix} \mu(a) > \mu(b) \\ \gamma(a) > \gamma(b) \\ \xi(a) > \xi(b) \end{matrix}\right\} \Leftrightarrow \text{Person with age } a \text{ is more likely to} \left\{\begin{matrix} \text{die} \\ \text{recover} \\ \text{get infected} \end{matrix}\right\} \text{than with b.} \tag{2}$$

Based on this strategy two structurally completely different models are analyzed. The first one was already stated in (Hoppensteadt 1974) and poses for the IPDE system given in (3)-(6). Hereby S, I and R denote the time- and age-dependent densities for the susceptible, infected and recovered population.

$$\frac{\partial S}{\partial t}(t,a) = -\frac{\partial S}{\partial a}(t,a) - (\lambda(t,a) + \mu(a))S(t,a) \tag{3}$$

$$\frac{\partial I}{\partial t}(t,a) = -\frac{\partial I}{\partial a}(t,a) + \lambda(t,a)S(t,a) - (\mu(a) + \gamma(a))I(t,a) \tag{4}$$

$$\frac{\partial R}{\partial t}(t,a) = -\frac{\partial R}{\partial a}(t,a) + \gamma(a)I(t,a) - \mu(a)R(t,a) \tag{5}$$

$$\lambda(a) := \int_{\mathbb{R}^+} I(t,a)\xi(a)\kappa(a,b)db \tag{6}$$

The second one is an ABM with N agents, wherein each agent's state is given by a tuple (x,y). Hereby x denotes the agent's health state (S,I,R,D) and y denotes the agent's age. Each agent deterministically evolves its age in time, while it can change its state stochastically, according to ξ,γ and μ and its contact partners gained by κ.

3 MODEL COMPARISON

For the analytic comparison we show the asymptotic convergence of the densities of the IPDE and the mean-field of the ABM. Hereby the mean-field O of the microscopic model is defined in (7), wherein $a_i(t)$ denotes the state of agent-based agent i at time t and δ denotes the delta-distribution on $\{S,I,R,D\} \times \mathbb{R}^+$.

$$O(x,t) = \frac{1}{N}\sum_{i=1}^{N} \delta(x - a_i(t)) \tag{7}$$

As a novel approach we hereby proved and applied a Mean-Field Theorem for a hybrid - i.e. continuous (age) and discrete (health state) - state space. The proof by the authors will be available on the poster. Using this theorem we could show that the components of the mean-field asymptotically fulfill the stated IPDE system for $N \to \infty$.

In order to verify the calculated results we executed both models for two fictional scenarios which mainly differed in their initial condition. We used fictional curves for μ,κ,ξ and γ and used 8000 individuals for the ABM. The IPDE model was solved via modified method of lines (MoL).

4 RESULTS AND CONCLUSION

For all tested scenarios quantitatively both model results are almost indistinguishable though slight quantitative errors occur due to the numerics of the MoL. With respect to computation time the ABM is slightly but not significantly slower. As a matter of the approach the results of the agent-based approach are always stable, whereas big step-sizes for the time-integration lead to instable results for the MoL.

As ABMs are getting more and more popular brief analysis of the system behavior of the modeling approach is necessary. Mean-Field analysis as performed here can contribute to this showing relations to other modeling techniques.

REFERENCES

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