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ANALYSIS OF MARKET RETURNS USING MULTIFRACTAL TIME SERIES AND AGENT-BASED SIMULATION

James R. Thompson and James R. Wilson

Edward P. Fitts Department of
Industrial and Systems Engineering
North Carolina State University
Raleigh, NC 27695-7906, USA

ABSTRACT

Many types of financial time series, most notably market returns, have been found to exhibit long-range memory as well as dramatic day-to-day swings that cannot be adequately represented by light-tailed distributions such as the normal distribution. In particular, this means that for such time series, the usual variance parameter (i.e., the sum of covariances at all time lags) is not defined because the covariance function does not converge to zero fast enough as the time lag increases. Moreover in such time series, often the tails of the marginal density converge to zero so slowly that higher-order marginal moments such as skewness and kurtosis fail to exist. Therefore conventional methods for analyzing simulation-generated time series cannot generally be applied to high-fidelity simulations of financial markets.

Building on earlier work in fractal geometry and fractal time series, Mandelbrot et al. proposed the multifractal model of asset returns (MMAR) as an alternative to the ARCH models for analyzing time series exhibiting volatility clustering, long-range dependence, and heavy-tailed returns (Mandelbrot et al. 1997). They defined the multifractal spectrum as the renormalized probability density function of the Hölder exponents observed in the time series; and they used the multifractal spectrum to measure the ability of MMAR to match the statistical properties of real data. In 2002 Kantelhardt et al. formulated multifractal detrended fluctuation analysis (MF-DFA), an algorithm for extracting the multifractal spectrum from a time series (Kantelhardt et al. 2002).

Many economists have recently adopted agent-based simulation for modeling financial markets. Fads based on new products, shocks related to world events, scandals involving company leaders, or outright criminal activity can drastically change the processes and relationships governing a market. Already there is evidence that agent-based models yield more accurate approximations to the true or observed behavior in financial markets. Agent-based models exhibit emergent behaviors that have been linked to non-Gaussian interaction metrics and singularities in the time series they generate (Chan 2011). Farmer et al. constructed a "zero-intelligence" agent-based model that is capable of exhibiting many of the statistical properties observed in real financial data simply due to the rules governing a double auction order book (Farmer et al. 2005).

During the 20th century, economists relied heavily on Brownian motion to construct theoretical models of finance. The advantages of using Brownian motion stem from its foundation in the Gaussian distribution; and with advances in stochastic calculus and differential equations, it became a staple in models of empirical finance. The well-known Black-Scholes model for options pricing is a prime example of this application. Multifractal analysis has the ability to illuminate the underlying difference between a monofractal like Brownian motion and a considerably more diverse construction like the multifractal binomial measure. Thus, combining multifractal analysis with an agent-based model has the potential to provide insight into the underlying processes and behaviors of a dynamic economic system that is constantly changing and evolving.

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To analyze market-return time series exhibiting volatility clustering, long-range dependence, or heavy-tailed marginals, we exploit multifractal analysis and agent-based simulation. We develop a robust, automated software tool for extracting the multifractal spectrum of a time series based on MF-DFA. Guidelines are given for setting MF-DFA's parameters in practice. The software is tested on simulated data with closed-form monofractal and multifractal spectra as well as on observed data, and the results are analyzed. We also present a prototype agent-based financial market model based on the zero-intelligence model of Farmer et al. and analyze its output using MF-DFA. The figure below compares the multifractal spectrum of the output from the prototype model with that of the multifractal binomial measure. The ultimate objective is to expand this model to study the effects of microlevel agent behaviors on the macrolevel time series output as analyzed by MF-DFA. Finally we explore the potential for validating agent-based models using MF-DFA and thus being able to "tune" these models to the multifractal spectrum of empirical data.

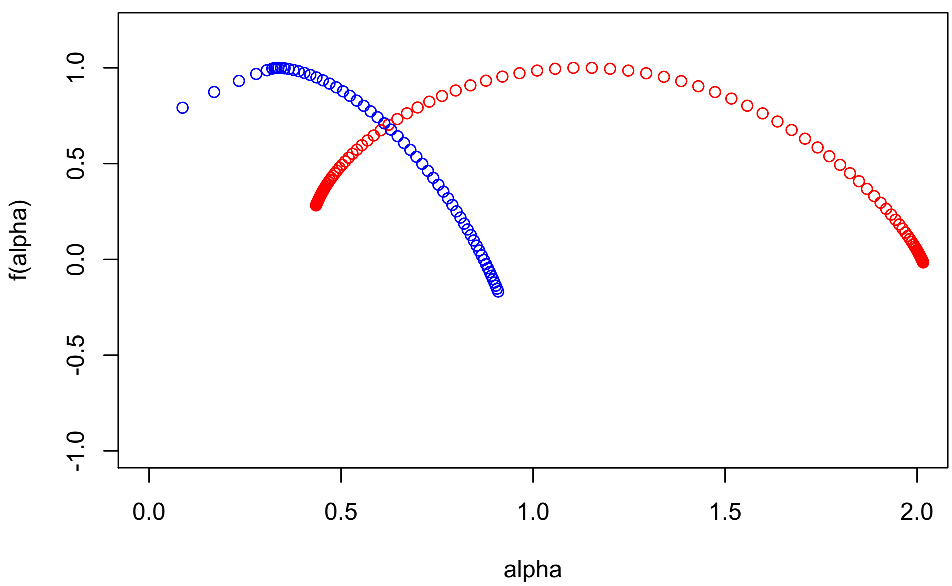

Figure 1: Comparison of the multifractal spectra from the zero-intelligence model (blue) and a multifractal binomial measure with $m_0 = 0.25$ (red)

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