Proceedings of the 2011 Winter Simulation Conference
S. Jain, R.R. Creasey, J. Himmelspach, K.P. White, and M. Fu, eds.

THE CONSEQUENCES OF HOW SUBJECT MATTER EXPERT ESTIMATES ARE INTERPRETED AND MODELLED, DEMONSTRATED BY AN EMERGENCY DEPARTMENT DES MODEL COMPARING TRIANGULAR AND BETA DISTRIBUTIONS

Lene Berge Holm

Akershus University Hospital
Health Service Research Centre
P.O. Box 95
Lorenskog, 1478, NORWAY

Mathias Barra

Akershus University Hospital
Health Service Research Centre
P.O. Box 95
Lorenskog, 1478, NORWAY

ABSTRACT

The aim of this paper is to demonstrate empirically the consequences of misinterpreting estimates from subject matter experts (SMEs), and to study the differences between modeling this with triangular and beta distributions. Three estimates which describe the duration of a process; minimum, maximum, and mode, is ideally sufficient as a proxy for the empirical distribution. However, these estimates might be biased when the SMEs confuse the difference between mean and mode. The analysis are performed in an ED model of a Norwegian hospital. When comparing the model output with data from the electronic patient record we see that a model with beta distributions based on the SME estimates outperforms a model with the more frequently used triangular distributions. A triangular distribution will overestimate the mean of the distribution compared to a beta distribution. We therefore encourage the use of beta distributions over triangular for activities with skewed distributions.

1 INTRODUCTION

Data acquisition is a crucial part of simulation modeling. However, when sufficient empirical data is not available modelers often resort to estimates from so-called subject matter experts (SMEs) (Law 2007). They typically provide the modeler with estimates on the duration of different processes included in the model. To account for some variations in these estimates, it is common to have the SMEs give information on the minimum (a), the maximum (b) and the most observed value (c) of time usage of each activity in the model. The most observed value, the mode, is normally expressed by the term m. However in this paper we will refer to it as c when referring to the SME estimate. The three SME estimates are then usually used as parameters for a *triangular distribution*, the min, max and mode of the distribution.

Durations of processes, especially in health care, often exhibit a positive skew (tail to the right). That this phenomenon recurs, is simply due to the fact that there is usually a natural lower bound on how fast it is possible to perform some task, while there is no corresponding “natural” upper bound on time consumption. Unforeseen delays, malfunctions, or lack of information may prolong a process ad infintium, while on the other hand the time it takes to, e.g., transport a patient from A to B is clearly constrained below by the laws of physics.

When using the estimates on min, mode and max from SMEs in a triangular distribution, one is likely to overestimate the tail of the distribution, and hence the mean and, similarly, to underestimate the bulk of the distribution (shown in Figure 1 below). This constitutes a major - and inherent - problem with the triangular distribution, and its impact is evaluated in this paper.

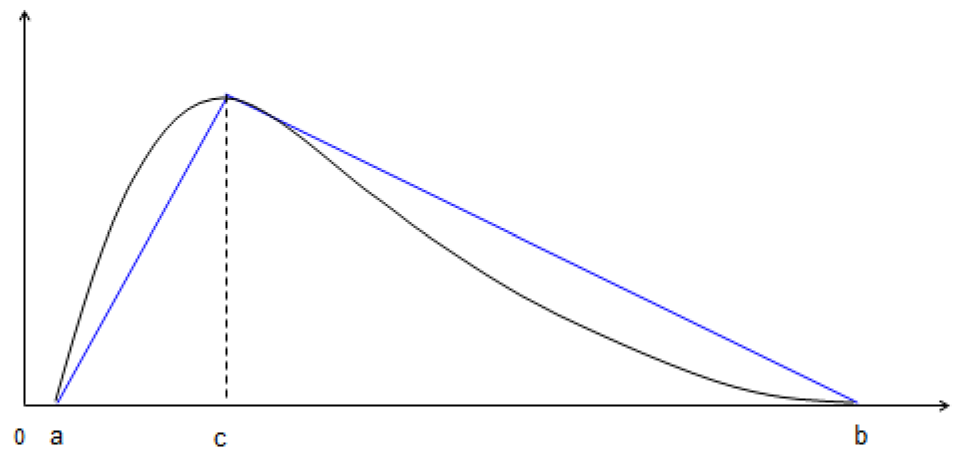

Figure 1: Illustration of a triangular and a beta distribution. a = min, b = max, c = mode.

2 TRIANGULAR DISTRIBUTIONS

Triangular distributions are typically used as an approximation to some underlying stochastic variable when no system data are available, or when data acquisition is difficult or too time consuming (Law 2007). SMEs are asked for their educated guesses on the minimum (*a*) and maximum (*b*) value of the duration of a certain process. They are also asked for the most frequently observed value (*c*). In the following *a, b,* and *c* refers to the SME estimates.

What we have experienced in our modeling of the Emergency Department (ED) of a large hospital in Norway (Holm and Dahl 2009, Holm and Dahl 2010) is that the SMEs have difficulty understanding the concept behind the most observed value (the mode) and its use as a parameter in a triangular distribution. However, it might be even more difficult to have them provide an estimate of the mean duration of a process due to the often observed tail of health care process distributions.

If one interprets the SME-c estimate as the mode ($c = m$) the resulting triangular distribution is shown in figure 2a. Note that the mean μ is higher than the mode c. On the other hand, interpreting the SME-c estimate as the mean ($c = \mu$) results in the triangular distribution in Figure 2b, leaving the mode smaller than *c*.

What constitutes the correct interpretation of *c* – as the mode or as the mean – may vary upon context and is an intriguing question which merits further research. In the context of the paper at hand, we note that the, triangular distribution may not even be able to accommodate the second interpretation. This problem arises from the easy observation that for a triangular distribution with end-points *a* and *b*, and with mode parameter $c \in [a , b]$, we have that the mean lies in the middle third of the interval $[a , b]$, shown by equation (1), which may not be the case in many real-world systems.

$$\mu = \frac{a+b+c}{3} \in \left[a + \frac{b-a}{3}, b - \frac{b-a}{3} \right] \qquad (1)$$

One possibility of obtaining a low enough mean with a triangular distribution is to truncate the tail as illustrated in figure 2c. Here a = m, and c = μ is forced by decreasing the maximum value b. This, however, seems a rather contrived and unsatisfactory solution.

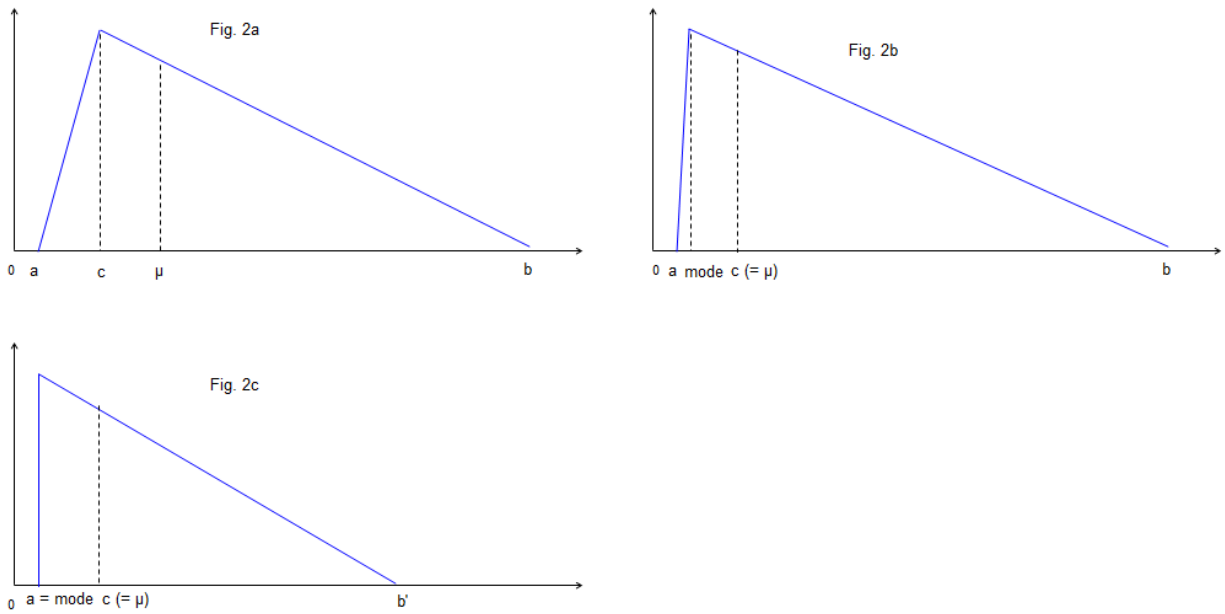

Figure 2: Triangular distributions

3 BETA DISTRIBUTIONS

In this paper we want to study both the effect of interpreting the *c* as the mode (*m*) and as the mean (μ), and we want to study the differences between modeling this as a triangular distribution and as a beta distribution. In a beta distribution it is possible to keep the *b* value and still interpret the *c* as the mean.

The beta distributions $f(x, \alpha, \beta)$ is a family of probability distributions defined on the interval [0,1] with shape parameters α and β. The probability density function is given by equation (2) (where *B* is a normalising constant):

$$f(x;\alpha,\beta) = B(\alpha,\beta)\ x^{\alpha-1}(1-x)^{\beta-1} \tag{2}$$

Figure 3 shows some different shapes of the distribution.

The main difficulty with modeling a process as a beta distribution is to choose the shape parameters. Formulas are readily available, and the investment of efforts on behalf of the modeler is minimal compared to the gained versatility. When $\alpha = \beta > 1$, then $f(x,\alpha,\beta)$ is a symmetric unimodal probability distribution (e.g., $\alpha = \beta = 2$ in Figure 3). However, as discussed previously, it is more likely that the distribution is positively skewed which is obtained when $\beta > \alpha > 1$ (e.g., $\alpha = 2$, and $\beta = 5$ in Figure 3).

Having chosen appropriate shape parameters one can next employ the distribution

$$(b-a) * f(x,\alpha,\beta) + a \tag{3}$$

in place of the triangular distribution. Here the mean μ is given by

$$\mu = (b-a)\frac{\alpha}{\alpha+\beta} + a \tag{4}$$

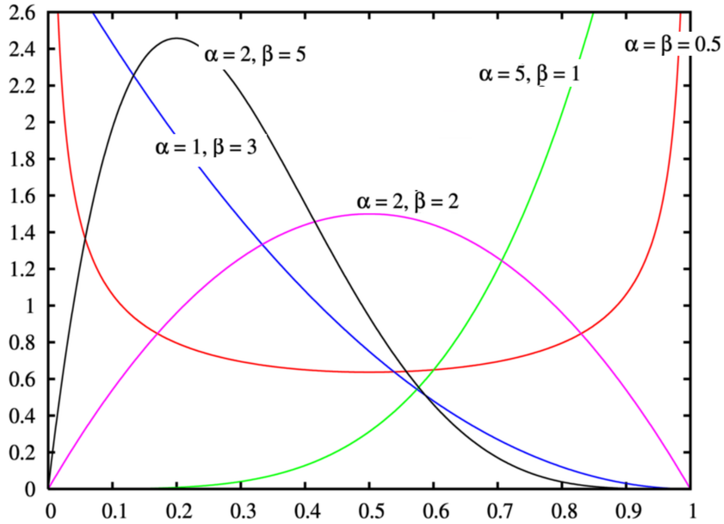

Figure 3: Beta distributions (Wikipedia 2011)

and the mode *m* is given by

$$m = (b-a)\frac{(\alpha-1)}{\alpha+\beta-2} + a \tag{5}$$

The advantage of the beta distribution is that given (SME) endpoints a and b, we may realize any mean in the induced interval by choosing α and β appropriately, and still have some freedom left to adjust, e.g., variance or mode.

In our model we decided on a shape with $\alpha = 2$, and we calculated the corresponding value for β both when c was interpreted as the mode (m), and as the mean (μ):

$$\beta = \frac{(b-a)}{m-a} \tag{6}$$

$$\beta = \frac{2(b-a)}{\mu-a} - 2 \tag{7}$$

The differences between these two distributions are shown in figures 4a and 4b.

By the use of an already validated model of an emergency department of a general hospital in Norway (see section 4) we want to evaluate the differences of modeling the SME estimates with three different distributions as shown in Figure 5.

In the beta distribution 'beta 1' and in the triangular distribution the c is set as the mode. The means (μ) of these two distributions have a higher value than c, but would not be equal for those two distributions. For the distribution 'beta 2' it is the mean which is set to c, and the resulting mode m is lower.

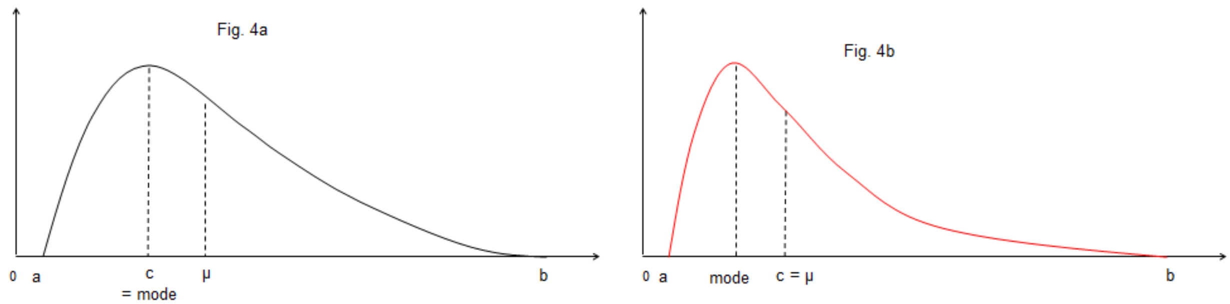

Figure 4: Beta distributions satisfying $c = \mu$ (4a) and.$c = m$ (4b)

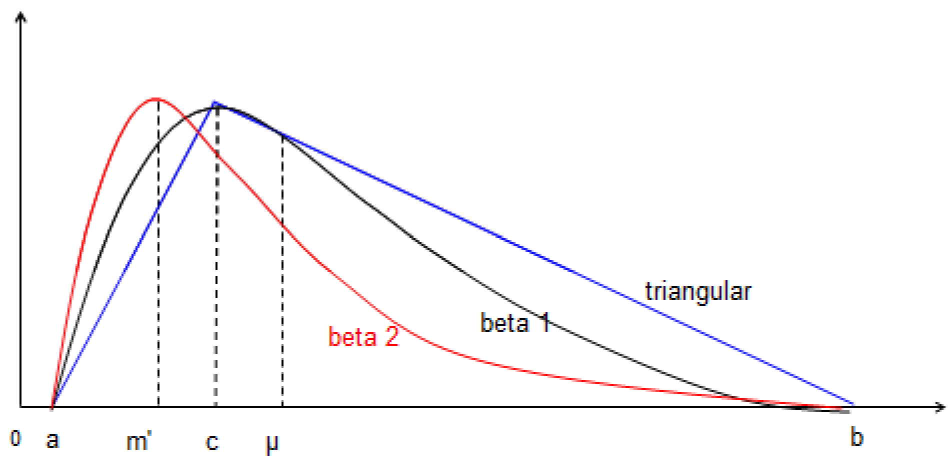

Figure 5: Three distributions to model the SME estimates

4 THE DES MODEL OF THE EMERGENCY DEPARTMENT AT AHUS

4.1 The Model

In Holm and Dahl (2010) a discrete event simulation model of the emergency department (ED) of a Norwegian hospital (Akershus University Hospital, Ahus) has been developed in the simulation software Flexsim Healthcare. The process map of the model is shown in Figure 6.

The inputs for this model are based on a combination of SME estimates, manual data gathering, and data from the electronic patient database of 2009. In this paper we have reused the model with some updates in estimates given by SMEs and studied the differences in outcome when using triangular and beta distributions based on the SME estimates. The output is studied after a model run of 15 months.

The processes included in the model and the data input source are shown in Table 1. Please refer to Holm and Dahl (2010) for detailed information on resources and further model description.

As many of the processes in this model are based on estimates from SMEs it is important to be able to model them as faithfully as possible. The model has been validated thoroughly and reflects the real world well (Holm and Dahl 2010).

4.2 Data Input

Table 2 contains the minimum a, the mode c and the maximum b durations as estimated by the SMEs . The three described distributions and their model terminology are shown in Table 3.

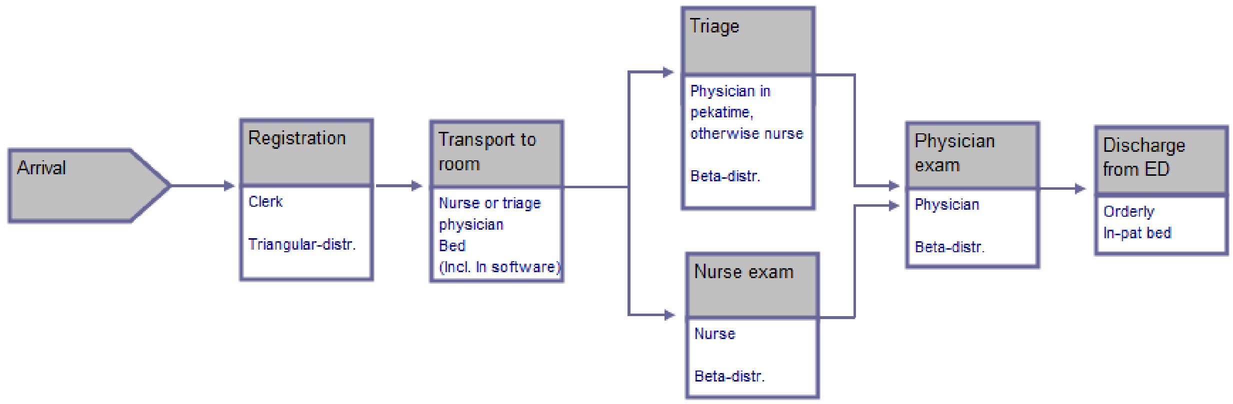

Figure 6: Model process map

Table 1. Processes and data source

Process	Data source
Arrival	Database
Registration	SME
Transport to waiting area	Software (when available resources)
Transport to treatment room	Software (when available resources)
Triage	SME
Nurse examination	SME
MD examination	SME
Observation	SME
Discharge	0 min

Table 2: SME estimates

Process	a	c	b
Registration	6	7	8
Triage	2	6	20
Nurse examination	10	30	200
MD examination + 20 min	10	65	250
Observation	0	55	260

Table 3: Distributions and their model terminology. The beta distribution is given by equation (3) above

Process	Triangular distribution	Beta distribution, c = mean	Beta distribution, c = mode
Registration	Triangular(6,8,7)	2*beta(2,2)+6	2*beta(2,2)+6
Triage	Triangular(2,20,6)	18*beta(2,4.5)+2	18*beta(2,7)+2
Nurse examination	Triangular(10,200,30)	190*beta(2,9.5)+10	190*beta(2,17)+10
MD examination	Triangular(10,250,65)	240*beta(2,4.36)+10	240*beta*(2,6.7)+10
Observation	Triangular(0,260,55)	260*beta(2,4.73)	260*beta(2,7.5)

5 RESULTS

We compared the duration of each SME estimated process implemented in the model with a triangular distribution (Table 4) and beta distributions (Table 5 and 6) as described above. Table 5 shows the results where the SME estimate is interpreted as the mode, while the results when it is interpreted as the mean are shown in Table 6. We have also compared the overall outcome: total length of stay (LOS) at the ED.

Table 4: Triangular distribution, SME c = mode

Process	Mean	Median	95% Confidence Interval
Registration	7.00	7.00	7.00, 7.00
Triage	9.43	8.85	9.38, 9.48
Nurse examination	79.52	72.57	79.01, 80.02
MD examination + 20 min	107.78	100.41	107.17, 108.39
Observation	104.29	95.69	103.63, 104.96
LOS	339.57	304.09	337.71, 341.43

Table 5: Results Beta distribution, SME c = mode

Process	Mean	Median	95% Confidence Interval
Registration	7.00	7.00	7.00, 7.01
Triage	7.65	7.28	7.62, 7.69
Nurse examination	43.33	39.59	43.08, 43.58
MD examination + 20 min	85.32	80.40	84.83, 85.80
Observation	77.43	72.02	76.92, 77.94
LOS	221.84	204.97	220.71, 222.97

Table 6: Results Beta distribution, SME c = mean

Process	Mean	Median	95% Confidence Interval
Registration	7.00	7.00	6.99, 7.00
Triage	6.11	5.74	6.09, 6.14
Nurse examination	30.15	27.6	30.00, 30.31
MD examination + 20 min	85.42	80.70	84.94, 85.90
Observation	54.74	49.25	54.36, 55.13
LOS	199.31	179.75	198.19, 200.43

From the electronic patient record of 2010 we know that the average LOS in the ED is 201 minutes. The results of these analyses therefore show that the duration of the processes and the total time spent in the ED are clearly overestimated when using a triangular distribution. It also shows that interpreting the estimates from the SMEs as the mode or the mean gives significant differences in the results when modeled as a beta distribution. At the ED of Ahus the SME-c value is more likely to equal the mean than the mode.

6 DISCUSSION AND CONCLUSION

Triangular distributions based on estimates from SMEs are usually used in simulation models where there are no other sufficient data. This paper discusses two possible downsides of this. Firstly, it is not always clear what the estimate *c* from SMEs really represents, the mode or the mean. When the SME-c is interpreted as the mode, a simulation model will result in a higher duration of the activities than when the SME-c is interpreted as the mean. It is therefore very important to educate the SMEs well before using their estimates, so that they really understand the concept of the mode compared to a mean value. Interpreting the SME-c as mean or mode may vary upon context and is an intriguing question which merits further research. What is important to acknowledge is that it is not possible to use a triangular distribution where the SME-c value is interpreted as the mean for heavy positively skewed distributions because the mean in a triangular distribution must lie in the middle third of the interval [a, b]. The second disadvantage with the triangular distribution is that it cannot have a long positive tail, as is often the case with probability distributions approximating the time to perform some real world task (Law 2007).

A beta distribution has two shape parameters, α and β, and it is both possible to model the SME-c estimate as mean and mode. Furthermore, the beta distribution is smooth, which is not an unrealistic assumption on the probability distribution of the durations of many real-world processes. Hence, a beta distribution may represent the system better than a triangular distribution. When we used the beta distribution in our model, we decided on an α-value of 2 which would yield a positively skewed distribution when $\beta > \alpha$. However, instead of deciding the value of one of the shape parameters, one could assume that the variance of the beta distribution should equal the variance of the original triangular distribution and thereby calculate both shape parameters α and β.

The analyses presented in this paper show that which of the two distributions is chosen to model the SME estimates yields significant differences in the outputs. The triangular distribution overestimates the tail, and hence the mean, of the distribution compared to the beta distribution, as is evident in Figure 1. It is the second beta distribution where SME-c estimates are interpreted as the mean that fits best with real world data from the electronic patient record in our ED model. The aim of the paper is not the results of the analysis themselves, but to show how different the results can be depending on how the estimates given by the SMEs are interpreted.

In conclusion, rather than automatically opting for a triangular distribution, we recommend considering using a positively skewed beta distribution when there is reason to believe that the underlying distribution exhibit a long tail. It is also important to make sure the SME estimates given is valid for their use in the model. If possible, validation of SME estimates is recommended. We also believe that research should be undertaken to shed more light upon when SMEs can be expected to produce sound means and modes, and also to investigate how skewedness and other properties affect their ability to give good guesstimates.

REFERENCES

Holm, L. B., and F. A. Dahl. 2009. "Simulating the Effect of Physician Triage in the Emergency Department of Akershus University Hospital." In *Proceedings of the 2009 Winter Simulation Conference*, edited by M. D. Rossetti, R. R. Hill, B. Johansson, A. Dunkin and R. G. Ingalls, 24-33. Piscataway, New Jersey: Institute of Electrical and Electronics Engineers, Inc.

Holm, L. B., and F. A. Dahl. 2010. "Simulating the Influence of a 45% Increase in Patient Volume on the Emergency Department of Akershus University Hospital." In *Proceedings of the 2010 Winter Simulation Conference,* edited by B. Johansson, S. Jain, J. Montoya-Torres, J. Hugan and E. Yucesan, 2455-2461. Piscataway, New Jersey: Institute of Electrical and Electronics Engineers, Inc.

Law, A. 2007. *Simulation Modeling and Analysis.* 4th ed. New York: McGraw-Hill, Inc.

Wkipedia. 2011. "Beta distribtution." Accessed March 28. http://en.wikipedia.org/wiki/Beta_distribution.

AUTHOR BIOGRAPHIES

LENE BERGE HOLM is working for the Health Services Research Centre in the Research Department of Akershus University Hospital and is currently working toward her PhD degree in simulating patient flow in Norwegian Healthcare. She holds a Master's Degree in Pharmacy from the Danish University of Pharmaceutical Science with a specialization in Social Pharmacy. Prior to her PhD she worked with health economy and pharmacoeconomics. Her research interests include medical and pharmaceutical science, patient flow, and health economic analysis. Email can be directed to: lene.berge.holm@ahus.no.

MATHIAS BARRA is working for the Health Services Research Centre in the Research Department of Akershus University Hospital. He holds a PhD degree in mathematical logic. Email can be directed to: mathias.barra@ahus.no.