Proceedings of the 2011 Winter Simulation Conference
S. Jain, R.R. Creasey, J. Himmelspach, K.P. White, and M. Fu, eds.

ANALYZING A STOCHASTIC INVENTORY SYSTEM FOR DETERIORATING ITEMS WITH STOCHASTIC LEAD TIME USING SIMULATION MODELING

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ABSTRACT

We consider an inventory system for continuous decaying items with stochastic lead time and Poisson demand. Shortage is allowed and all the unsatisfied demands are backlogged. Moreover, replenishment is one for one. Our objective is to minimize the long-run total expected cost of the system. Firstly, we have developed the mathematical model with deterministic lead time. Since the stochastic lead time makes the model complex especially when lead time has complicated probability distribution and it is difficult to prove convexity of the objective function, we have applied simulation modeling approach. The simulation model has no limitation on lead time or any other parameters. The simulation model is validated by comparing its outputs and analytical model's results for the deterministic lead time case. Furthermore, we use optimizer module of the applied software to find near optimal solutions for a number of examples with stochastic lead time.

1 INTRODUCTION

In the literature different concepts of deterioration were mentioned. Raafat (1991) categorized deteriorating items into two different groups. Firstly, items which become obsolete simultaneously at the end of planning horizon, such as style goods or classic newsboy problem. Secondly, items which deteriorate during their planning horizon. This category has been divided into two classes by Raafat: (1) items which have a fixed shelf life such as blood and (2) items which have random life time and decay continuously such as radioactive materials. Our model is about items of second group which have random life time or on the other hand continuous decaying items. “In many inventory systems, the deterioration of goods is a realistic phenomenon. It is well known products such as medicine, volatile liquids, blood bank, food stuff and many others decrease under deterioration (vaporization, damage, spoilage, dryness, and so on) during their normal storage period. As a result, while determining optimal inventory policy of that type of products, the loss due to deterioration cannot be ignored” (Dye, Hsieh, and Ouyang 2007).

"The analysis of decaying inventory problems began with Ghare and Schrader (1963)" (Raafat 1991). Since 1963 a lot of research has been conducted in this area. Unfortunately, most of these studies were concentrated on deterministic demand. It could be concluded from the papers that reviewed research from beginning to 2000 (consisting of Goyal and Giri 2001; Nahmias 1982; Raafat 1991) that there were few studies which assume stochastic demand. Although, researchers do not pay sufficient attention to stochastic continuous decaying inventory models, there are lots of studies on different kinds of deterministic demand. Li, Teng, and Wang (2007) and Hsieh, Dye, and Ouyang (2008) are two examples of considering constant demand rate for deteriorating inventory items. Li, Teng, and Wang (2007) developed an EOQ based model with deteriorating items for a supply chain involving a retailer and *N* customers by considering postponement strategy. Hsieh, Dye, and Ouyang (2008) established a model for deteriorating items with constant demand rate and two warehouses (own and rented) to consider capacity limitation. There are also many research efforts that consider time-varying demand in deteriorating inventory models. Papachristos and Skouri (2000), Goyal and Giri (2003), Sana, Goyal, and Chaudhuri (2004), and Chern et al. (2008) are such studies. In addition there are considerable deteriorating inventory models with price-dependent demand. Dye, Hsieh, and Ouyang (2007) and Dye, Ouyang, and Hsieh (2007) provide such models. Furthermore a large number of papers have been published recently for deteriorating items inventory model with stock-dependent demand such as Teng and Chang (2005) and Chang, Teng, and Goyal (2009). But there are few research efforts for deteriorating inventory items with stochastic demand, even though in real world demand is usually stochastic. “Stochastic demand inventory models have received considerably less attention, particularly those papers that have been published after the Goyal and Giri, 2001” (Lodree and Uzochukwu 2008). Lodree and Uzochukwu (2008) developed a production planning model for a deteriorating item with stochastic demand and consumer choice. As their model was developed for an agricultural product which perishes after a known number of periods, they assumed positive procurement lead time. “The complexity of decay models having random demand depends strongly on the lead time assumptions. When lead times are zero, determining optimal order policies is relatively straightforward” (Nahmias 1982). As a result of this fact, not only in all the above mentioned papers (except Lodree and Uzochukwu 2008) but also in a large number of studies lead time have been assumed negligible (zero lead time). There are some stochastic inventory models for deteriorating items in which positive or stochastic lead time are considered. Kalpakam and Sapna (1994) in their paper analyzed an (s, S) exponential decaying system with Poisson demand and exponentially distributed lead time. They assumed demands during stock-out periods are lost. Sivakumar (2009) developed an inventory model for exponential decaying items. In that model a finite number of homogenous sources of demand were considered. (s,S) inventory policy and exponentially distributed lead time were assumed by Sivakumar (2009). In addition, it was considered that demands occurring during stock-out periods enter into an orbit. The orbiting demands send out signal according to exponential distribution to compete for their demand. The system has been analyzed as a Markov process.

To the best of our knowledge, there is no research in the field of deteriorating items inventory models with stochastic demand and lead time unless lead time is exponentially distributed. Analyzing models with stochastic demand and lead time is necessary for real world problems. In this article we develop a stochastic inventory model for continuous decaying items with stochastic lead time. We assume a warehouse where customers enter the system according to Poisson process and the warehouse inventory policy is base-stock policy (s-1,s). Shortage is allowed and unsatisfied demands are backlogged. Lead time is assumed to be stochastic and we try a number of different probability distribution as our model's lead time. As it is stated before, stochastic lead times make models complex, especially when demand is stochastic too. We develop a mathematical model but it is complicated to obtain an exact solution and it is too difficult to prove convexity of the model. Thus, we solve the problem by establishing a simulation model. We show the simulation model's solution is near to exact solution when lead time is deterministic as validation for the simulation model. After validating the model, we start finding near optimal solution for stochastic lead times.

In our analytical model we take advantage of the approach which is offered by Axsäter (1990). Axsäter (1990) developed a two echelon model for non-deteriorating items in which expected cost per item is calculated. Other assumptions of Axsäter's model are exactly same as our model except deterministic lead time.

2 ANALYTICAL MODEL

2.1 Model Description

As it is stated before, in this model we have a warehouse where customers enter the system on the basis of Poisson distribution. Each customer needs just one unit of the product. Since the items are deteriorating according to exponential distribution, each time a customer orders one unit of the product, the warehouse orders an outside supplier (1+α) unit. After L_0 time unit warehouse's order is delivered. We will discuss more about the extra amount (α) later. Shortage is allowed and all unsatisfied items are backlogged. The inventory system is demonstrated in Figure 1:

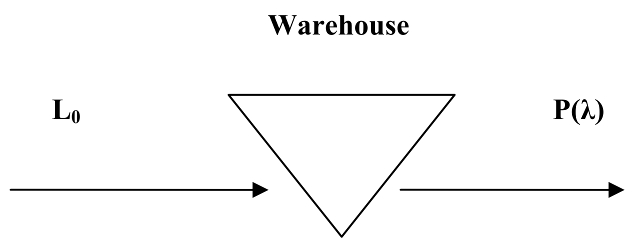

Figure 1: The inventory system

The analytical model of the problem is developed on the basis of the following assumptions:

- Customers are served on the basis of first come, first served rule.
- Replenishment is one for one.
- Holding cost is calculated on the basis of amount of the product which is delivered to the warehouse.
- Shortage is allowed and all the unsatisfied demands are backlogged.
- Deterioration rate is constant.

We introduce the following notations for the model's parameters:

L_0: lead time
λ: demand intensity at the warehouse
h_0: holding cost per unit per time unit at the warehouse
β: shortage cost per unit per time unit at the warehouse
γ: deterioration cost per unit
θ: deterioration rate which is constant and $0<\theta<1$

Notations for the model's decision variables are as follows:

S_0: inventory position at the warehouse
α: extra amount of the product ordered by warehouse

When customers enter the warehouse according to Poisson distribution with a rate λ, distribution of the time elapsed between the placement of an order and occurrence of its assigned demand will have Erlang distribution with parameters (λ, S_0). We show the density function of the Erlang distribution by $g^{S0}(.)$:

$$g^{S_0}(t) = \frac{\lambda^{S_0} t^{S_0 - 1} e^{-\lambda t}}{(S_0 - 1)!}.$$

We show the corresponding cumulative distribution function by $G^{S0}(t;\lambda)$:

$$G^{S_0}(t;\lambda)=\sum_{k=S_0}^{\infty}\frac{(\lambda t)^k}{k!}e^{-\lambda t}.$$

When a demand occurs at the warehouse, (1+α) new unit is immediately ordered from the warehouse to the outside supplier. We designate this time as time zero. Note that the warehouse orders more than one unit for deterioration of the items during the lead time. If customers order while the warehouse is out of stock, the demand is satisfied with delay. As soon as units are again available at the warehouse, customers are served according to first come, first served rule. Furthermore, when a customer orders during stock out period, in fact, the related unit is virtually assigned to the customer. The below lemma which is similar to what is stated in Axsäter (1990) is necessary for understanding our modeling approach:

Lemma. *Any unit ordered by a specific customer is used to fill the S_0th demand following this order, hereafter, referred to as its demand.*

The lemma is an obvious consequence of the ordering and delaying policy (first come, first served).

An order placed by a specific customer arrives after L_0 time units. If the order arrives before its assigned demand, it is considered as inventory stocks and holding cost is incurred. Note that (1+α) units is originally ordered and this amount is decreasing during the period between ordering and delivering times because of deterioration. If the order arrives after its assigned demand, this customer's order is backlogged and shortage cost is incurred. Let H, Π and D denote the expected warehouse inventory holding cost, shortage cost and deterioration cost respectively, incurred to fill a customer's demand.

2.2 Model Formulation with Deterministic Lead Time

When lead time is deterministic H, Π and D are given by

$$H=\int_{L_0}^{\infty}h_0 g^{S_0}(s)(s-L_0)(1+\alpha)e^{-\theta L_0}=$$

$$(1+\alpha)h_0e^{-\theta L_0}\left[\frac{S_0}{\lambda}(1-G^{S_0+1}(L_0,\lambda))-L_0(1-G^{S_0}(L_0,\lambda))\right]. \quad (1)$$

Note that *$(1+\alpha)e^{-\theta L_0}$* is the amount of warehouse order ((1+α)) which remains after the lead time has elapsed. In the above expression, holding cost is calculated on the basis of proportion of (1+α) units that is delivered to the warehouse.

$$\prod=\beta\int_0^{L_0}(L_0-s)g^{S_0}(s)ds. \quad (2)$$

$$\int_0^t g^{S_0}(u)udu=\int_0^t g^{S_0+1}(u)\frac{S_0}{\lambda}du=\frac{S_0}{\lambda}G^{S_0+1}(t,\lambda). \quad (3)$$

As mentioned before, for s≤ L_0 shortage is incurred. According to expressions (2) and (3) we can obtain expected shortage cost (Π). Note that each customer orders just one unit of the product. Thus, when warehouse can not satisfy one unit demand of a specific customer, shortage is incurred.

$$\prod=\beta\int_0^{L_0}(L_0-s)g^{S_0}(s)ds=\beta\left[L_0G^{S_0}(L_0,\lambda)-\frac{S_0}{\lambda}G^{S_0+1}(L_0,\lambda)\right]. \quad (4)$$

$$D=\gamma\int_0^{L_0}(1+\alpha)(1-e^{-\theta L_0})g^{S_0}(s)ds+\gamma\int_{L_0}^{\infty}(1+\alpha)(1-e^{-\theta s})g^{S_0}(s)ds=$$
$$\gamma(1+\alpha)\left[1-e^{-\theta L_0}G^{S_0}(L_0,\lambda)-(\frac{\lambda}{\lambda+\theta})^{S_0}(1-G^{S_0}(L_0,(\lambda+\theta)))\right]. \quad (5)$$

Note that first integral shows expected deterioration cost when the customer arrives before his assigned demand. And the second one explains expected deterioration cost when the customer arrives after the assigned demand. Note that in the analytical model we have just one decision variable, S_0.

The long-run expected total cost is given by

$$C(S_0) = \lambda\left[H(S_0) + \prod(S_0) + D(S_0)\right].$$

2.3 Model Formulation with Stochastic Lead Time

When lead time is stochastic with a specific density function, which is shown by *w(x)*, holding cost, shortage cost and deterioration cost incurred to fill a customer's demand are given by

$$H = h_0 \int_X w(x) \int_x^\infty g^{S_0}(s)(s-x)(1+\alpha)e^{-\theta x} ds dx =$$

$$(1+\alpha)h_0 \int_X w(x) e^{-\theta x} \left[\frac{s_0}{\lambda}(1 - G^{S_0+1}(x,\lambda)) - x(1 - G^{S_0}(L_0,\lambda))\right] dx. \tag{6}$$

$$\prod = \beta \int_X w(x) \int_0^x (x-s) g^{S_0}(s) ds dx = \int_X w(x) \beta \left[L_0 G^{S_0}(x,\lambda) - \frac{S_0}{\lambda} G^{S_0+1}(x,\lambda)\right] dx. \tag{7}$$

$$D = \gamma \int_X w(x) \int_0^x (1+\alpha)(1-e^{-\theta x}) g^{S_0}(s) ds dx + \gamma \int_X w(x) \int_x^\infty (1+\alpha)(1-e^{-\theta s}) g^{S_0}(s) ds dx =$$

$$\gamma \int_X w(x)(1+\alpha)(1-e^{-\theta x}) G^{S_0}(x,\lambda) dx +$$

$$\gamma \int_X w(x)(1+\alpha)\left[(1 - G^{S_0}(x,\lambda)) - \left(\frac{\lambda}{\lambda+\theta}\right)^{S_0}(1 - G^{S_0}(x,(\lambda+\theta)))\right] dx. \tag{8}$$

As illustrated above, when lead time is deterministic we can obtain holding, shortage and deterioration costs easily. But when lead time is stochastic, these three costs become more difficult to obtain. In addition, in this situation it is difficult to show whether the total cost function is convex and find the optimal solution. Thus, we build the simulation model of this problem and we show how easy we can find solutions in different situations without any limitation on the lead time distribution or any other parameters. In addition we can optimize the problem by using optimization module of the applied software. Firstly, we show that our model is valid and then we solve three different problems with stochastic lead time which is obtained from the literature. In this study we have applied Arena software and its optimizer, OptQuest (Kelton, Sadowski, and Sturrock 2004).

3 SIMULATION MODEL

3.1 Conceptual Model

We built the simulation model according to the analytical model's approach which was explained in the previous section. The conceptual model of our simulation model is illustrated in Figure 2.

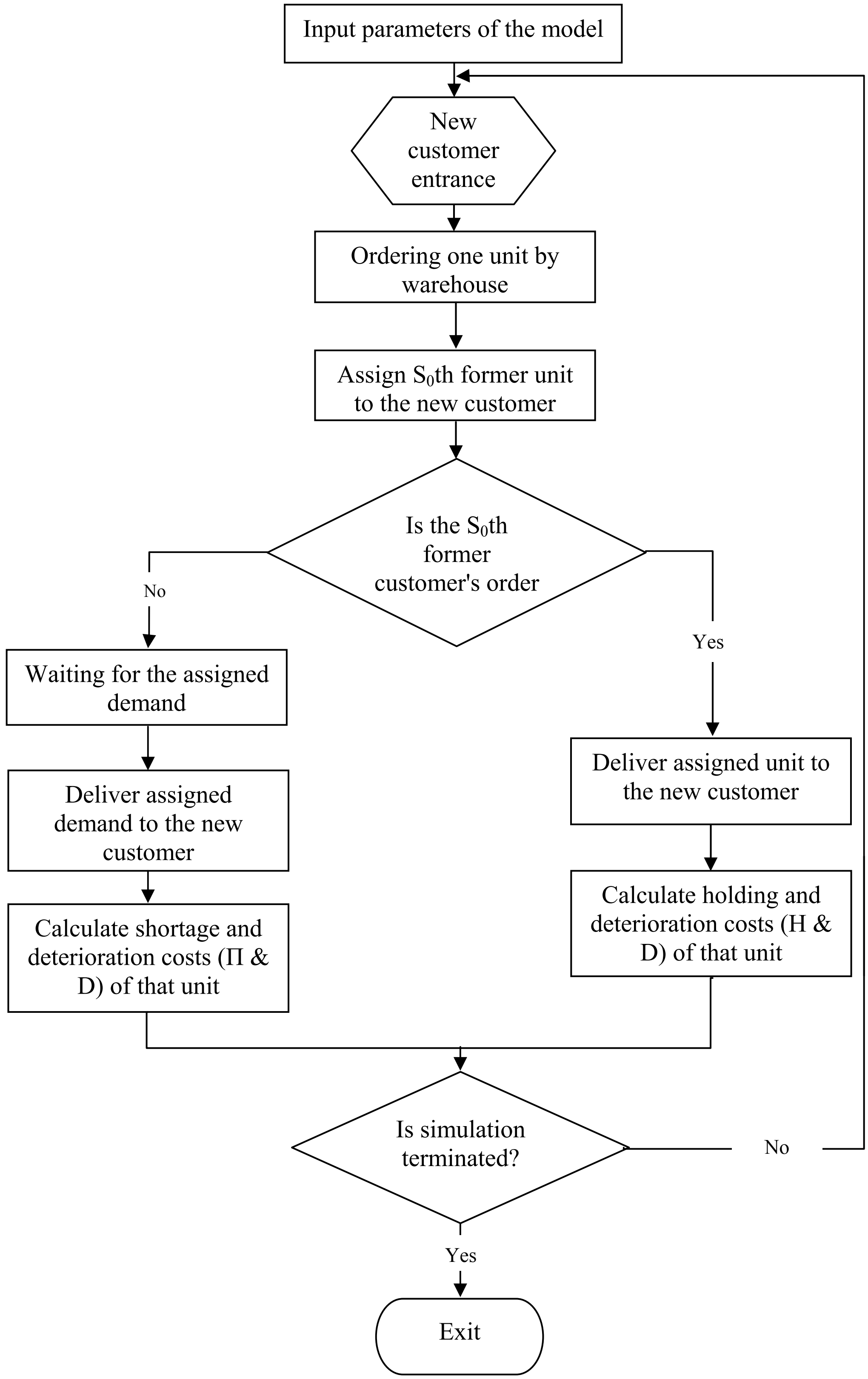

Figure 2: A conceptual model of the simulation model

3.2 Simulation Model Validation

We use deterministic lead time to validate our simulation model. For a number of different experiments we illustrate there are not significant differences between holding, shortage, and deterioration costs of our analytical model and simulation model. Note that the holding cost (H), shortage cost (Π), and deterioration cost (D) of our analytical model are obtained according to expressions (1), (4), and (5) respectively. In addition, we run our simulation model for 50 replications where each replication takes 50,000 hours. Table 1 shows the output results of two models, simulation and analytical, for the eight different configurations where the model parameters are randomly chosen. Note that the confidence level is assumed to be 95%.

Table 1: Validation Experiments

Model Parameters	Model	Cost Holding (H)	 Shortage (Π)	 Deterioration (D)
h_0=3 β=2 γ=15 α=0.1 θ=0.01 L_0=5 S_0=5 λ=3	Simulation	1.5695±0.04	6.6731±0.00	0.8048±0.00
	Analytical	1.6078	6.6674	0.8048
Error		2.382%	0.085%	0.000%
h_0=2 β=5 γ=8 α=0.1 θ=0.01 L_0=12 S_0=17 λ=0.25	Simulation	109.30±0.08	0	4.2830±0.00
	Analytical	109.2686	1.3527e-007	4.2823
Error		0.029%	N/A	0.016%
h_0=5 β=6 γ=17 α=0.2 θ=0.02 L_0=31 S_0=6 λ=1.2	Simulation	0	156.00±0.03	9.4804±0.00
	Analytical	1.4693e-010	156.0000	9.4259
Error		N/A	0.000%	0.578%
h_0=0.6 β=0.4 γ=21 α=0.3 θ=0.00042 L_0=43 S_0=54 λ=10	Simulation	0	15.0402±0.00	0.4886±0.00
	Analytical	0	15.0400	0.4886
Error		0.000%	0.001%	0.000%
h_0=0.25 β=0.3 γ=16 α=0.4 θ=0.00125 L_0=29 S_0=33 λ=0.5	Simulation	12.4901±0.01	0	1.7718±0.00
	Analytical	12.4890	1.1773e-005	1.7717
Error		0.009%	N/A	0.006%
h_0=4 β=3 γ=10 α=0.6 θ=0.03 L_0=5 S_0=30 λ=2.5	Simulation	38.5647±0.01	0	4.8135±0.00
	Analytical	38.5597	1.4309e-005	4.8132
Error		0.013%	N/A	0.006%
h_0=0.32 β=0.25 γ=7 α=0.8 θ=0.00057 L_0=78 S_0=23 λ=30	Simulation	0	19.3084±0.00	0.5479±0.00
	Analytical	0	19.3083	0.5479
Error		0.000%	0.005%	0.000%
h_0=0.55 β=0.67 γ=21 α=0.9 θ=0.001 L_0=10 S_0=97 λ=5	Simulation	9.7205±0.00	0	0.7663±0.00
	Analytical	9.7253	3.4312e-010	0.7665
Error		0.049%	N/A	0.026%

Table 1 indicates that there are very small differences between results obtained from simulation model with those of the analytical model. As can be seen in the table, because of running model for 50 long replications (50,000 hours) half width of the simulation model's costs are very small. In other words, the simulation estimators of the system's costs are almost point estimators. Thus, error is calculated for each configuration to show small differences between results of simulation and analytical models. Now we can trust the simulation model and solve problems with stochastic lead times.

4 STOCHASTIC LEAD TIME PROBLEMS

In the previous section we show our simulation model is valid. Now we can run our model with stochastic lead time. To the best of our knowledge all the deteriorating items inventory models with stochastic lead time have focused on exponential distribution which can be analyzed by Markov process. As mentioned before in stochastic demand inventory models of deteriorating items, stochastic lead time makes these models complicated. We also have shown this fact in expression (6), (7) and (8). Furthermore, it is extremely difficult to prove convexity of total cost function. In this inventory model, we have three stochastic parameters: the life time of the inventory items, lead time and the Poisson demand. It will be difficult to analyze this problem with analytical models. On the other hand, we can create the simulation model and choose any distribution for lead time or for other parameters without any limitation. We found in the inventory system literature that the lead time is typically normally or exponentially distributed. In this section we will take advantage of the simulation model for three examples with three different lead time distributions. We will use Arena optimizer module, OptQuest, and try to find near optimal solutions for these examples. OptQuest combines the metaheuristics of Tabu search, Scatter search and Neural Networks into a single, composite search algorithm to provide maximum efficiency in identifying new scenarios (April et al. 2003). In all the three examples we have used the parameters which have been used by Lin and Lin (2007). These parameters are provided in Table 2.

Table 2: Common Parameters of the Numerical Examples

Shortage cost per unit per time unit	β=100
Deterioration cost per unit	γ=10
Holding cost per unit per time unit	h_0=1
Deterioration rate per unit per day	θ=0.02

In addition, we use customer arrival rate which was applied by Kalpakam and Sapna (1994). In their research customer arrival rate (λ) was considered to be 5 (λ=5). In the literature of inventory models we have found normal and exponential distributions for lead time. In the first example we assumed normal distribution for lead time (*w(x)*) with parameters μ=0.03 day and σ=0.06 day, and exponential distribution with parameter b=3.33 for the second one which were used by Maiti, Maiti, and Maiti (2009). For the third example we assumed Erlang distribution with parameters lambda=3 and n=8. Note that for each example parameters of the lead time distribution are given in the associated table. We took advantage of Arena optimizer, OptQuest, to minimize the total cost which is sum of H, Π and D. Our goal is to find near optimal solution for decision variables, extra amount of ordering (α) and the inventory position at the warehouse (S_0). The results of optimizations are illustrated in Tables 3, 4 and 5 for the example number 1 to 3 respectively. These tables show the trend in which the best found solutions are obtained during specified replication number by OptQuest software. Graphical view of each table is illustrated by the graph below these tables (Figures 3, 4 and 5). These figures also show how the value of the objective function improves during simulation runs.

Table 3: Optimization Results Obtained from OptQuest for Normal Lead Time (Example 1)

Lead time: Normal (μ=0.03, σ=0.06)(day)		No. of replication: 100	
Parameters h_0=1 β=100 γ=10 θ=0.02 λ=5			
Simulation Run	S_0	α	Total Cost
1	50	0.500000	17.1760
11	28	0.349305	8.53485
41	8	1.32600	3.87091
82	8	1.30156	3.83164
85	8	1.27711	3.79237
91	8	0.909638	3.20208
98	8	0.718857	2.89561

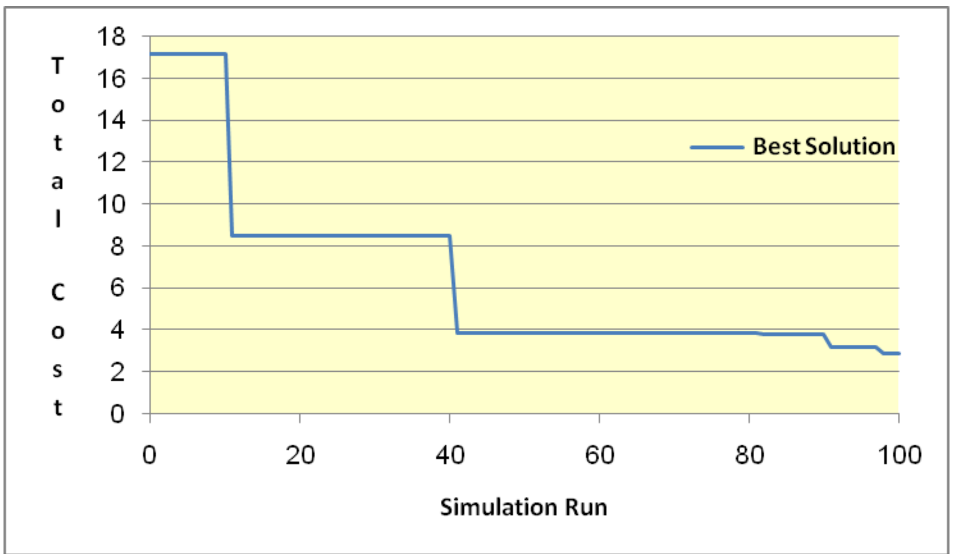

Figure 3: Optimization results obtained from OptQuest for example 1

Table 4: Optimization Results Obtained from OptQuest for Exponential Lead Time (Example 2)

Lead time: Exponential (b=3.33)		No. of replication: 100	
Parameters h_0=1 β=100 γ=10 θ=0.02 λ=5			
Simulation Run	S_0	α	Total Cost
1	50	0.500000	342.937
3	1	0	333.735
5	189	0.574661	114.052
24	200	0.574691	101.173
27	200	0.052606	80.2533
63	200	0.039190	79.7157
65	200	0.025773	79.1781
67	200	0	78.1454

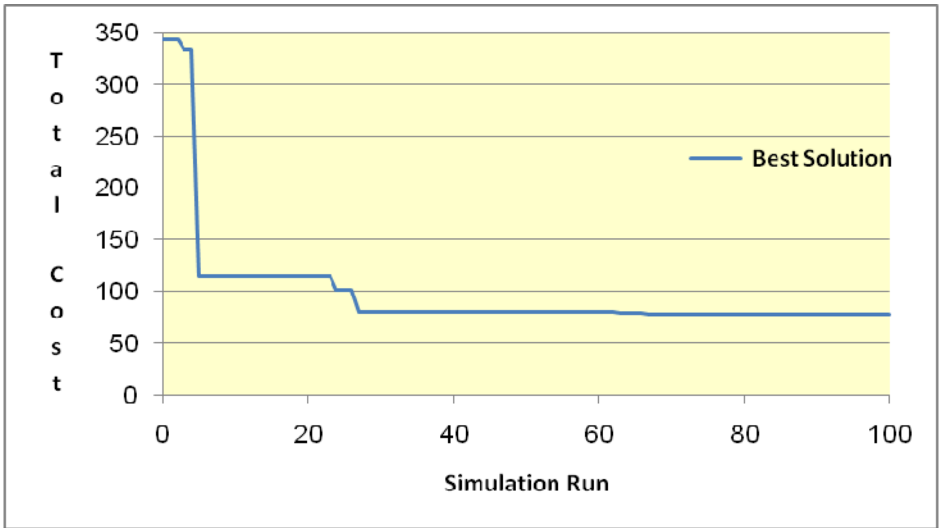

Figure 4: Optimization results obtained from OptQuest for example 2

Table 5: Optimization Results Obtained from OptQuest for Erlang Lead Time (Example 3)

Lead time: Erlang (lambda=3, n=8)		No. of replication: 100	
Parameters h_0=1 β=100 γ=10 θ=0.02 λ=5			
Simulation Run	S_0	α	Total Cost
1	50	0.500000	1441.83
2	101	5	912.932
4	200	10	717.507
5	189	0.574661	569.487
14	194	0.747858	565.113
23	200	0.571959	556.463
26	199	0.561300	556.368
42	200	0	546.693

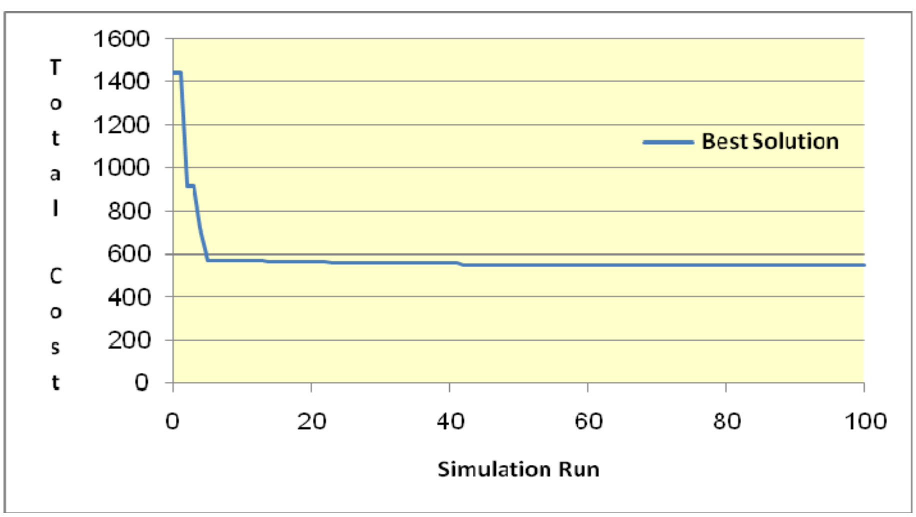

Figure 5: Optimization results obtained from OptQuest for example 3

For the best solutions of each example we run simulation models for 50 replications where each replication takes 3000 days to obtain average of H, Π and D separately. The results are demonstrated in Table 6.

Table 6: The Related Costs for the Best Solutions for Three Examples

Example	S_0	α	H	Π	D	Total Cost
1	8	0.718857	2.2209	0.1259	0.5402	2.8870
2	200	0	35.0566	36.9597	5.4994	77.5156
3	200	0	11.5524	530.45	5.5215	547.53

Note that there is no limitation in lead time or any other parameters in our simulation model which could be a great advantage of the simulation approach. In addition, we can take advantage of simulation optimization as a decision support tool to obtain near optimal values of decision variables (S_0 and α).

5 CONCLUSION

Although, most of the real world cases have stochastic demand and lead time, in the literature of deteriorating items inventory models there are limited number of research efforts with stochastic demand and lead time. We considered an inventory model of deteriorating items with stochastic demand and lead time. For deterministic lead times, we built analytical model and we showed how difficult it would be to solve for stochastic lead time. Since in our model we have three stochastic parameters (items life time, demand and lead time) it is too difficult to build the analytical model of this problem and prove convexity of the model. Hence we built a simulation model of our problem using the Arena simulation software. We have validated our simulation model by comparing its result with analytical model's result for eight random experiments. There were very small differences between two models' results. After model validation we applied OptQuest software to find near optimal solutions with stochastic lead time for three different examples. Although analytical model is too difficult to be established for stochastic lead time especially when the density function of lead time is complex, there is no limitation in our simulation model. In fact, difficult problems could be analyzed by this simulation model and simulation optimization can be used as a decision support tool to find optimal or near optimal solutions.

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