PARTIAL LEADING IN PURSUIT AND EVASION GAMES

Chris Arney Elisha Peterson

Department of Mathematical Sciences United States Military Academy West Point, NY 10996, USA

ABSTRACT

Pursuit and evasion games encompass a large class of games in which one or more "pursuers" attempt to find and/or capture one or more "evaders". These games have immense practical importance, yet their mathematics is not fully-understood outside of a limited number of simple cases. This paper introduces <code>PursuitSim</code>, a simulation platform for pursuit and evasion games in which the user interactively explores these games by dynamically adjusting algorithm parameters. The dynamic and exploratory nature of the platform allows the user to quickly ascertain broad patterns and test hypotheses. We discuss insights gained using the platform on the efficacy of "leading" strategies in situations where the pursuer can make reasonable assumptions about the path of the evader.

1 INTRODUCTION

Pursuit and evasion games are one of the most ubiquitous games found in the "real world", and have been for thousands of years. One can see these games played out in the natural world with predator and prey, in the human world with a typical "cops and robbers" chase, a football game, or even a simple children's game of tag, and in the artificial world with missile routing and robotic algorithms.

Despite their prevalence, they are not very well understood from a mathematical point-of-view. The earliest rigorous mathematical treatment of such games focused on finding optimal solutions to specific cases using the theory of differential games (Isaacs 1965), but the techniques do not apply to all varieties of pursuit games, and in many cases the optimal solutions are impossible to find or mathematically intractable. In many cases, closed-form mathematical solutions are known to be impossible to find (Nahin 2007), making simulation one of the best tools for studying these games. The understanding of multipursuer, multi-evader games is especially limited; in particular, relatively little is known about what it means for players to *cooperate* in these scenarios.

Two-player pursuit and evasion games involve a single pursuer trying to catch a single evader. In the absence of any constraints, the optimal strategies in this case are simple: the pursuer heads directly to the evader, and the evader heads directly away from the pursuer (Isaacs 1965). Capture occurs if and only if the pursuer is faster than the evader. On the other hand, if the evader's path is known precisely, the pursuer's optimal strategy is also quite simple: head to the location where the pursuer can first catch the evader. There are some difficulties with the assumption of full knowledge in this scenario. In his landmark work on differential games, Isaacs goes as far as to say that any predictive tactic is a "policy outside the optimal strategy", because the pursuer "has no grounds for his prediction" (Isaacs 1965).

Despite Isaacs statement, there are numerous scenarios in which a pursuer has grounds for some kind of prediction. In sports, a football player is reliably known to be heading toward the endzone. In the natural world, the flight patterns of hoverflies and dragonflies have been observed to follow *motion camouf*-