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SIMULATION FUSION

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ABSTRACT

The concept of data fusion (DF) has had a major impact on statistical methodology and practice within the US Department of Defense. In this paper we explore expanding this idea to include simulation modeling and analysis. Simulation fusion (SF) is the concept of combining data, models, experiments, and analyses to improve the outcome of simulation studies. In this paper, we propose an initial framework and discuss various SF schemes. We provide several examples to illustrate these ideas and discuss some of the potential benefits of simulation fusion.

1 INTRODUCTION

A simulation study is a scientific/engineering procedure that transforms input (quantitative or qualitative data, ideas, hypotheses, objectives, etc.) into output (e.g., information, statistics, designs, solutions, and decisions.). A simulation study involves many things: data collection, input modeling, simulation modeling, output analysis, sensitivity analysis, design of experiments, optimizations, synthesis and presentation of results, and so forth. These are referred to in simulation textbooks as the "steps" in a simulation study, with the allusion (illusion) that they provided a distinct, sequential, and orderly progression to a successful outcome. This paper explores some of the consequences of assimilating, or fusing, the notion of simulation study steps into concurrent and complementary simulation study activities. Each activity, step, or sub-step can be carried out by different techniques or approaches—which in this paper we will call *simulation elements*—with the intent of freeing our thinking about simulation from the burdens in the baggage of current jargon. We make no claim that each of the examples of SF we present are in themselves new, indeed, some embody the best practices of experienced simulation users.

Simulation Fusion is a notion, a context for rethinking what we do, not some well-defined methodology or set of algorithms and associated software. Frameworks and concepts, while nebulous and "un-scientific", can have impact (consider "supply chains," "world views," "zero-defects," "six-sigma," etc.). Our hope is that the notion of SF will inspire simulation researchers, educators, and practitioners to view themselves a part of a more holistic society, rather than as researchers specializing in generating random variables, designing experiments, or developing methodology for output analysis, etc.; or as educators teaching within the design constraints and modeling concepts of a particular commercial software package; or as practitioners specializing in modeling or animating the dynamics of factories, hospitals, medical devices, climate change, etc.

To illustrate the idea of SF, consider the following scenario. In a particular simulation study, say, one could use a process-interaction-based simulation model and benefit from its animation capability. Alternatively, one might build a more efficient and flexible event-scheduling-based simulation model. However, it might be more useful to build and simultaneously execute *both* types of models to balance and benefit from both simulation's efficiency and animation capability. One could also combine the outputs of both models in a meaningful way and design experiments to leverage their strengths. For example, one could run very fast event-scheduling experiments to identify and design alternatives while using animations to illustrate the leading contender(s) to interactive decision makers. Simultaneously using multiple simulation approaches allow us to take advantages of the strengths of different approaches.

Simulation fusion is the generalization of this concept to integrate various simulation elements to yield new and better simulation approaches and results. The goals of SF are similar to those of multi-disciplinary teams; each member in the team is an expert in one or more areas. Working *closely* together in the right manner they can combine their strengths and knowledge to be more effectual than the additive impact of the sum of their individual expertise. Exploiting the advantages of combining various types of simulation elements is the goal of SF.

The objective of fusing multiple simulation elements include reducing output uncertainty, gaining simulation efficiency, balancing between speed and fidelity, improving result credibility, getting more robust results, and better decision-making. SF achieves these objectives by taking advantage of using multiple simulation elements that are good at performing particular functions. Selecting the simulation elements in the right combination, in the right way, in the right place, and at the right time is the art of SF. It requires a careful selection and possibly a lengthy and thorough testing to identify the best combination, making it impossible to carry out in a short period of time. Certain guidelines or a framework should be given to pre-define what combinations can lead to what expected results so that users can decide what simulation elements to fuse based on their own situations. Developing these guidelines and framework is the main research task of SF.

Simulation fusion introduces a large degree of flexibility in performing a simulation study, leading to potential new benefits that are difficult to achieve using conventional simulation methods. For example, end-to-end uncertainty can be significantly diminished by reducing the uncertainties at various stages and their compounding uncertainty throughout a simulation study, i.e., input, models, outputs, analyses, and decisions (see Figure 1). These multiple reductions can be done sequentially, concurrently, or repeatedly. For example, outputs from different simulation models can be combined and used by analysis algorithms to improve the overall decision quality (see Figure 2(a)). Decisions (quantitative or qualitative) from different simulation models can also be synthesized to yield more accurate and robust final decisions (see Figure 2(b)). Outputs from a simulation can be used as the input (fully or partially) to another simulation model to further reduce the uncertainty (see Figure 2(c)).

The remainder of the paper is organized as follows. Section 2 reviews relevant literature. Section 3 proposes the SF framework and briefly explains all simulation-fusion schemes. Section 4 provides several examples to illustrate the idea of SF and demonstrates its benefits. Section 5 makes a conclusion and outlines future work.

2 THE PROPOSED SIMULATION FUSION FRAMEWORK

In this section, we outline the proposed SF framework and suggest solutions for creating this framework. It should be noted that because there are certainly many more possibilities for SF, the goal here is to propose the concept and solution methods, rather than to lay out a comprehensive blueprint. Therefore, the presentation will be brief and preliminary. The actual implementation of the framework will probably require a long-term research effort from the whole simulation community.

Simulation fusion can take place at different levels and at various stages during a simulation study. We first describe the framework based on the levels where fusion might be advantageous. Then, we introduce various fusion schemes based on the stages at which fusion can occur.

2.1 Levels of Simulation Fusion

Simulation fusion will be described at six possible levels: data level, statistical level, qualitative level, methodological level, sample path level, and cross-level level. Table 1 lists these levels and their corresponding functions and elements being fused. At the data level, raw data generated from various models or analyses are combined to yield more information-rich data that will be used by subsequent stages during the simulation study. The statistical level deals with fusing statistics such as estimators, aggregated data, or sufficient statistics. The qualitative level is to combine decisions, results, or outcomes from different sources or stages to achieve a more robust and accurate simulation study.

These three levels are similar to the low, intermediate, and high level fusions defined in classical data fusion (Dasarthy 1994). The remaining three levels introduced in the following differentiate SF from classic data fusion. In the methodological level, various simulation methodologies or models are being used sequentially or simultaneously to produce new simulation methods that benefit from the strengths of all the fused simulation elements (see Example 2 in Section 3). At the sample path level, different simulation realizations generated using different random numbers are being fused to yield a set of sample paths or new kinds of meta models (see Example 4 in Section 3). The cross-level, by its name represents the flexibility of SF, which can occur at multiple levels. For example, one can fuse two models and fit the fused model with some statistics (or data) obtained by combining several statistics (or multiple data streams).

Common fusion operators include linear or nonlinear combination (such as weighted sum, min, max, or norm), multiplication, concatenation, voting methods (for binary data/decision), evidence-combination theory, and algorithmic non-closed-form operations.

Table 1: Levels of Simulation Fusion

Fusion Scheme	Functions	Elements Being Fused
Data Level	Multiple sources of data are combined	Raw quantitative data
Statistical Level	Several statistics are amalgamated	Statistical estimates
Qualitative Level	Decisions or results are joined together	High level decisions or scores
Methodological Level	Different simulation models and methodologies are used simultaneously	Models and methods
Sample-Path Level	Multiple simulation realizations are combined	Sample paths
Cross-Level Level	Fusing elements across different levels	Various simulation elements

2.2 Simulation-Fusion Schemes

Simulation fusion can take place at any stage of a simulation study as shown in Figure 1 and Figure 2. Several fusion schemes can be defined based on the stage at which fusion takes place as shown in Table 2. The proposed framework is built on these schemes.

Data Fusion: Data fusion has been an active field for years. Efficient models and algorithms have also been developed. The well-known JDL Data Fusion model developed by the US DoD Joint Directors of Laboratories (JDL) has a major impact in defense-relevant research and applications (Steinberg et al. 1999; Llinas et al. 2004). This JDL Data Fusion model provides guidelines for data fusion in our SF framework. The main difference is that most existing data fusion techniques are developed for object detections and identifications, while the objective of data fusion in SF is to distill or combine different sources of data (including simulated data) to assist in subsequent analyses. Nevertheless, most techniques developed in the data fusion literature can be exploited in SF. It is suggested here that research effort should be put on existing data fusion methods for descriptive analysis (exploratory data analysis) and predictive analysis (regression or classification) and synthesize them to be used by simulation.

Input Fusion: The input to a simulation model could be general input information, such as fused raw data, output from other simulations (e.g., Figure 2(c)), or human input (such as Delphi exercises) or interventions. New protocols and integration techniques are required to synthesize different types of input data.

Model Fusion: By fusing different simulation models, one can increase overall model fidelity and reduce the simulation output uncertainties, perhaps by allowing models to be biased. For example, when simulating a system with thousands of aircraft, one can exploit the trade offs between fidelity and efficiency by modeling several aircraft in high fidelity and the remainder in low fidelity. Model details are focused where needed to gain execution speed. The low-fidelity models can also be fine tuned by using output from the high-fidelity models. Two different types of aircraft models, one with more favorable maintenance parameters and the others with less favorable maintenance parameters, can also be created to bound uncertainty in the overall lifetime estimate. Such an exploitation of dependence is similar to the antithetic variate approach (Law 2006). See also Example 3.2 in Section 6.

Output Fusion: Output fusion combines output data or information from multiple sources to create a coherent decision-making process. Given multiple sources of uncertainty, the resulting modeling process is vulnerable to error. One such source, based on statistical hypothesis testing, are Type I and Type II errors. A strategy to reduce one type of error typically increases the other error. It is suggested that output fusion methods be explored that exploit the structure of discrete optimization (DO) models designed to mitigate such error propagation. In particular, new methods for synthesizing output data should be investigated using DO models to improve decision-making capabilities while simultaneously reducing the two classical types of errors. This will require new criteria to combine multiple data sources, such as ordinal rankings and order statistics. Grid search, local search, greedy algorithms, and dynamic programming algorithms are also potential candidates for addressing these models, with the objective of obtaining both practical procedures that are applicable to large-scale, real-world problems and provable performance convergence results (see, e.g., Figure 2(a)).

Analysis Fusion: To gain the maximum benefit from large-scale, multi-resolution simulations, experimental design and analysis tools should support various needs, from sensitivity analysis to optimization to real-time control. Measuring, controlling, and propagating *uncertainty* across heterogeneous simulation models makes this difficult. The design and analysis of (deterministic) computer experiments (DACE) community has emphasized uncertainty due to model risk (Santner et al. 2003), while the discrete-event stochastic simulation community has focused on sampling noise (Ankenman et al. 2009).

Analysis methods should be considered that fuse both intrinsic (noise) and extrinsic (model risk) uncertainties into a unified analysis framework, since both are critical to robust decision making and control. Existing work in the literature should be leveraged to advance traditional experimental design and analysis techniques so that they facilitate sequential, fully adaptive, and multi-resolution output analysis of simulation models.

Decision Fusion: In decision fusion, decisions from multiple simulations are scored and weighted based on criteria such as model fitness, data source quality, and so forth. Final decisions are then made based on these weighted scores using various rules, e.g., sum, min, max, or algorithmic operations (Figure 2(b)). It is suggested that research be conducted to explore the use of these rules under the simulation-fusion framework.

Sample-Path Fusion: Sample-path fusion combines correlated or uncorrelated sample paths to create new sample paths or meta models. Full details including event relationships are preserved in the fused sample paths. Such an information-rich fusion makes sample-path fusion different from conventional variance reduction techniques, such as common random numbers, where statistics (rather than sample paths) from correlated sample paths (rather than uncorrelated and correlated), random variables, or systems are combined to reduce estimation variation. See Example 3.4 in Section 3.

Bootstrap Fusion: The bootstrap-based fusion scheme is to remedy the bias caused by intentionally fusing specific simulation elements. The idea is to randomly select simulation elements and analysis methods to create a series of *randomized* simulation systems. Studies have shown that input bootstrapping makes a major impact on reducing simulation output errors (Barton and Schruben 1993, 2001).

Duality Fusion: Many dynamic system models have their duals, which provide valuable complementary information to the original systems. One example is a tandem queue and its reversed queue. A reserve queue is the same system as the tandem queue except that jobs travel in reverse order. These two systems are dual to each other in the sense that they have the same throughput. Fusing simulation data from these two systems could reduce uncertainty in estimation (see Example 3.3 in Section 3 and Chan and Schruben 2008a). This fusion scheme is a not-fully exploited, yet promising, opportunity in uncertainty reduction.

Multi-Scheme Fusion: To the extreme, multiple fusion schemes can be combined, integrated, aggregated, or propagated in the form of series, parallel, hierarchical, embedded, fork-join, or feedback schemes. It is recommended that studies be carried out to investigate which schemes are better in uncertainty management. Discussions on the effectiveness of combining simulation modeling, experimenting, and analysis can be found in Schruben (2009).

Besides the fusion schemes above, uncertainty-reduction and optimization algorithms that iteratively use fusion schemes also warrant a research effort from the community. Caution, however, should also be taken when fusing highly noisy data/methodologies to avoid potential side effects. Table 2 summarizes some possible fusion schemes and Figure 2(a)-(c) depict several examples. Illustrative examples are given in the next section.

Table 2: Various Simulation-Fusion Schemes

Fusion Scheme	Functions	Elements Being Fused
Data Fusion	Multiple data sources are combined	Data
Input Fusion	Data and other information are fused	Data, Information
Model Fusion	Different kinds of models are married	Methodologies, Ideas
Output Fusion	Multiple simulation outputs or other info are integrated	Data, Info, Estimates
Analysis Fusion	Different kinds of simulation information are used for experimental designs or heuristics optimization algorithms, which could also be fused	Data, Information, Algorithms, Approaches, Solutions
Decision Fusion	Decisions from multiple simulations are scored, weighted, and synthesized	Decisions, Estimates, Predictions
Sample-Path Fusion	Correlated or uncorrelated sample paths are combined	Sample Paths
Bootstrap Fusion	Multiple data sources, models, algorithms are bootstrappingly selected and fused	All
Duality Fusion	Dual models or methods are used simultaneously	All
Multi-Scheme Fusion	Combination of two or more of the schemes above	All

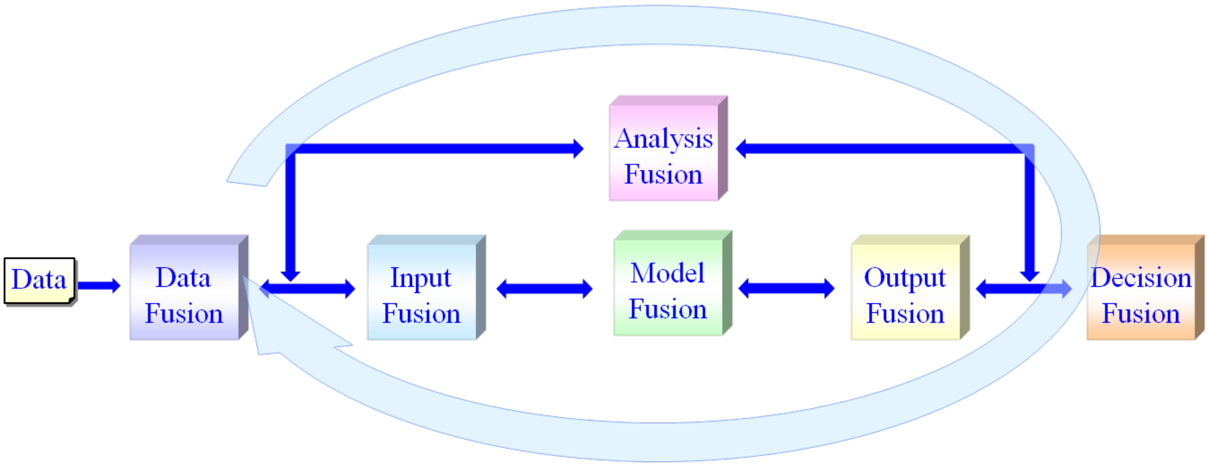
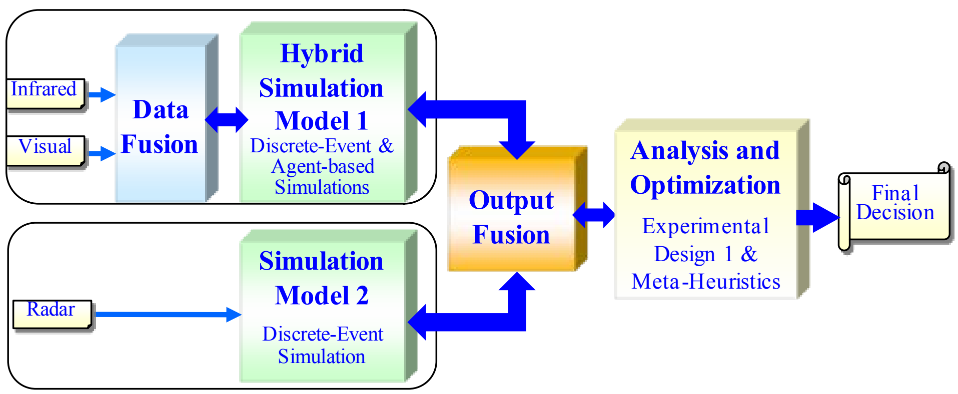
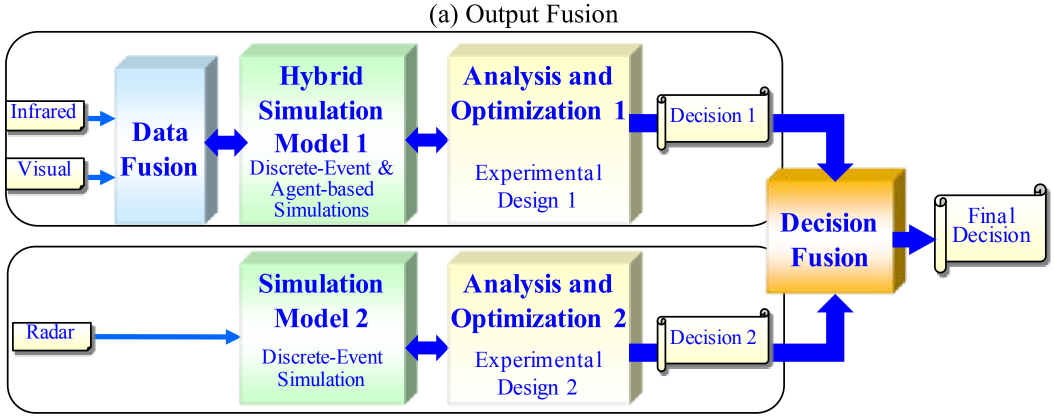


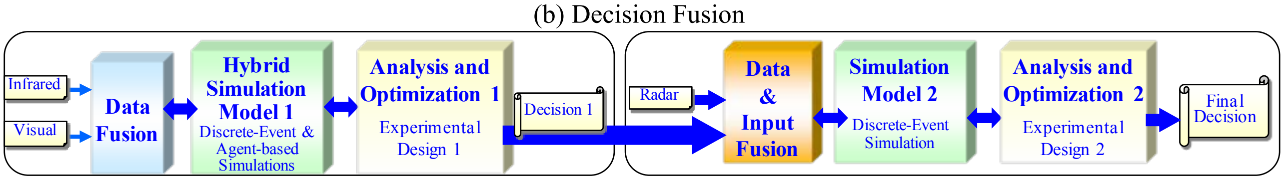
Figure 1: Stages where simulation fusion might take place



(a) Output Fusion



(b) Decision Fusion



(c) Data and Input Fusion (Hybrid Fusion)

Figure 2: Examples of Simulation Fusion at Different Levels, Degrees, and Orders

3 ILLUSTRATIVE EXAMPLES

In this section, we use several examples to illustrate the idea of SF. The first example is related to input fusion.

3.1 Example 1

Suppose we are trying to fuse two input methods to generate input data for a simulation model. The objective of this example is to reduce the uncertainty of the fused data. Let us call these two candidate methods as Method 1 and Method 2. Let X_i and Y_i, $i = 1, 2$ be the input and output of these methods, respectively. The mean and variance of X_i are denoted by μ_i and σ_i^2, respectively. Both methods transform X_i into Y_i according to certain stochastic functions $y = f_i(x), i = 1, 2$. For illustration purposes, we make a simplifying assumption that both functions have a multiplicative form, i.e., $f_i(x) = a_i x$, and that $a_1 = a$ and $a_2 = a + \delta$, where a is the common deterministic factor of the two methods and δ the stochastic factor that differentiates the two methods. X_1, X_2, and δ are all independent.

One simple fusion scheme is to average the two Y_i's, i.e., $\bar{Y} = (Y_1 + Y_2)/2$. As our objective is to reduce the uncertainty of the fused $\bar{Y}$, we aim at finding f_i's so that Y_1 and Y_2 are negatively correlated. Because f_i's only differ at the factor δ, this is equivalent to finding δ so that $\rho(Y_1, Y_2) < 0$, where $\rho()$ is the correlation function. The variance of $\bar{Y}$ can be computed as follows:

$$\begin{aligned}
Var(Y_1 + Y_2) &= Var(aX_1 + (a+\delta)X_2) \\
&= a^2\sigma_1^2 + Var((a+\delta)X_2) \\
&= a^2\sigma_1^2 + E[(a+\delta)^2]\sigma_2^2 + Var(a+\delta)\mu_2^2 \\
&= a^2\left(\sigma_1^2 + \sigma_2^2\right) + \left(2aE[\delta] + E[\delta^2]\right)\sigma_2^2 + Var(\delta)\mu_2^2.
\end{aligned}$$

For this variance to be less than $a^2\left(\sigma_1^2 + \sigma_2^2\right)$, we obtain the following equation specifying the relationships between the excepted difference between the two methods and the variance of transformed difference in X_2 in units of the variance of X_2:

$$\begin{aligned}
& \left(2aE[\delta] + E[\delta^2]\right)\sigma_2^2 + Var(\delta)\mu_2^2 < 0 \\
\Longrightarrow\quad & 2aE[\delta]\sigma_2^2 + E[\delta^2]\sigma_2^2 + Var(\delta)\mu_2^2 < 0 \\
\Longrightarrow\quad & 2aE[\delta]\sigma_2^2 + Var(\delta X_2) < 0 \\
\Longrightarrow\quad & E[\delta] < -\frac{Var(\delta X_2)}{2a\sigma_2^2}.
\end{aligned}$$

When $\mu_2 = 0$, the above relationship can be reduced to:

$$\begin{aligned}
& 2aE[\delta] + E[\delta^2] < 0 \\
\Longrightarrow\quad & 2aE[\delta] + Var(\delta) + (E[\delta])^2 + a^2 - a^2 < 0 \\
\Longrightarrow\quad & Var(\delta) < a^2 - (a + E[\delta])^2.
\end{aligned}$$

This inequality provides a guideline for selecting the right methods to fuse the data so that uncertainty is reduced. It roughly says that the uncertainty introduced by the difference factor between the two methods should be less than the amount of uncertainty reduction by using Method 2.

Observe that one can also use the classical antithetic variates (AV) approach to reduce the uncertainty of $\bar{Y}$ (Law 2006). In that case, one would collect X_1 and X_2 in such a way that they are negatively correlated to obtain $Cov(Y_1, Y_2) < 0$ and thus reducing the variance of $\bar{Y}$.

To compare the SF scheme outlined above and the classical AV approach, we note that they are not identical but dual. In AV, one tries to *select* X_i's for *given* f_i's such that $Cov(Y_1, Y_2) < 0$. Dually (i.e., reversely) in SF, one tries to *select* f_i's for *given* X_i's such that $Cov(Y_1, Y_2) < 0$, i.e.,

AV:	*select* X_i's for *given* f_i's s.t. $Cov(Y_1, Y_2) < 0$
SF:	*select* f_i's for *given* X_i's s.t. $Cov(Y_1, Y_2) < 0$

There are many open questions for research in input fusion. For example, what if the function $f_i(x)$ is non-multiplicative? What if more than two models are fused? What if SF is used in conjunction with AV or other common random numbers or control variates approaches?

3.2 Example 2

The second example is about fusing different simulation models to balance between efficiency and fidelity. We use the two parallel-server-queue simulation models given in Schruben (2009) to illustrate the idea. The purpose here is to estimate the mean queue length. The first model, called ERG_1, is a fast model because it records the trajectory of the queue using only a single integer variable and does not keep track of the voluminous individual waiting time data. The second model, called ERG_3, runs orders of magnitude slower than ERG_1 because it provides greater details on the exact individual waiting time data (See Schruben 2009 for a detailed discussion of these two models). It is shown in the literature of *indirect estimation* that the variance of the average waiting time is less than the variance of the average queue length (Law 1975, Glynn and Whitt 1989). This result suggests that instead of estimating the average queue length directly, one should estimate the average waiting time and use the Little's Law to obtain the average queue length indirectly. The amount of variance reductions, which is due to the detailed individual waiting time data, is shown to be about 20%. .

Our objective in this example is to determine a rule to use both models to estimate the mean queue length. In particular, we are interested in estimating the mean queue length, $\bar{Q}(n)$, by using a fused model, called ERG_1_3, where n is the length of the simulation (such as number of jobs simulated). ERG_1_3 generates $\bar{Q}(n)$ by making a simulation run of length cn from ERG_1 and independently another simulation run of length $(1-c)n$ from ERG_3, where $c, 0 < c < 1$, is a constant. Denoted by $\tilde{Q}_1(cn)$ the mean queue length estimator obtained from the simulation of ERG_1 with length cn and $\tilde{Q}_3((1-c)n)$ the similar estimator from the simulation of ERG_3 with length $(1-c)n$. We fuse $\tilde{Q}_1(cn)$ and $\tilde{Q}_3((1-c)n)$ by using the following simple convex combination:

$$\tilde{Q}(n) = c\tilde{Q}_1(cn) + (1-c)\tilde{Q}_3((1-c)n).$$

Let $\tilde{S}_1$ and $\tilde{S}_3$ be, respectively, the speeds of ERG_1 and ERG_3 in executing a simulation run of length n. The speed of ERG_1_3, $\tilde{S}$, is then equal to,

$$\tilde{S} = c\tilde{S}_1 + (1-c)\tilde{S}_3.$$

Because ERG_3 is recommended by the literature, we evaluate the performance of ERG_1_3 against ERG_3. Define the following effectiveness measure (E) that is the weighted average of the effectiveness of speed (ES) and the effectiveness of variance (EV):

$$\begin{aligned} E &= wES + (1-w)EV \\ &= w\frac{\tilde{S}}{\tilde{S}_3} + (1-w)\frac{Var(\tilde{Q}(n))}{Var(\tilde{Q}_3(n))}. \end{aligned}$$

Because ERG_1 is faster than ERG_3 but with a bigger variance, we can assume that $\tilde{S}_1 = \beta\tilde{S}_3$ and $Var(\tilde{Q}_3(n)) = \alpha Var(\tilde{Q}_1(n))$, where $0 < \beta < 1$ and $0 < \alpha < 1$. For simplicity, we also assume independences to obtain $Var(Q(cn)) = Var(Q(n))/c$. Plugging the expressions of all estimators into above expression for E, we obtain

$$E = w[1-(1-\beta)c] + (1-w)[(1-\alpha)c/\alpha + 1]$$

This effectiveness measure should be less than one if the overall performance of ERG_1_3 is better than that of ERG_3. This depends on the values of α, β, c, and w. Based on the reported values in the literature, α is roughly 0.8 and β is about 0.1 or more. Figure 4 depicts the effectiveness measure of ERG_1_3 against ERG_3 under different values of c and w. Only when w is small such as 0.2 can ERG_3 out-performs ERG_1_3. In other cases, ERG_1_3 is superior. This result means that unless one pays more than 80% of attention to the variance reduction (or less than 20% of attention to the speed of the simulation), fusing two simulation models yields a desirable outcome.

As in the first example, there are also many research opportunities in model fusion. For example, can we find the optimal c when fusing the two models under various constraints? What if we fuse more than two models, such as ERG_2, or ERG_4 in Schruben (2009)? What if we consider other factors, such as animations and extendibility, when calculating the effectiveness measure?

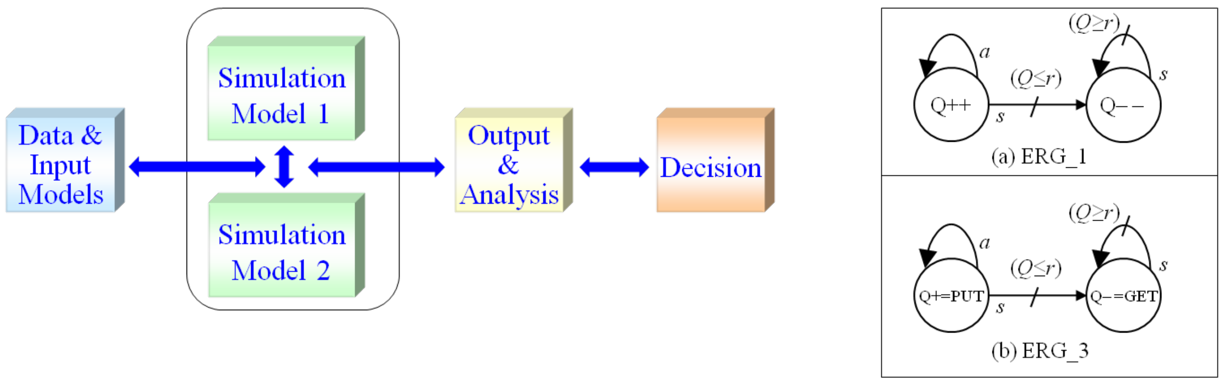


Figure 3: Model Fusion: Fusing Two Simulation Models, ERG_1 and ERG_3

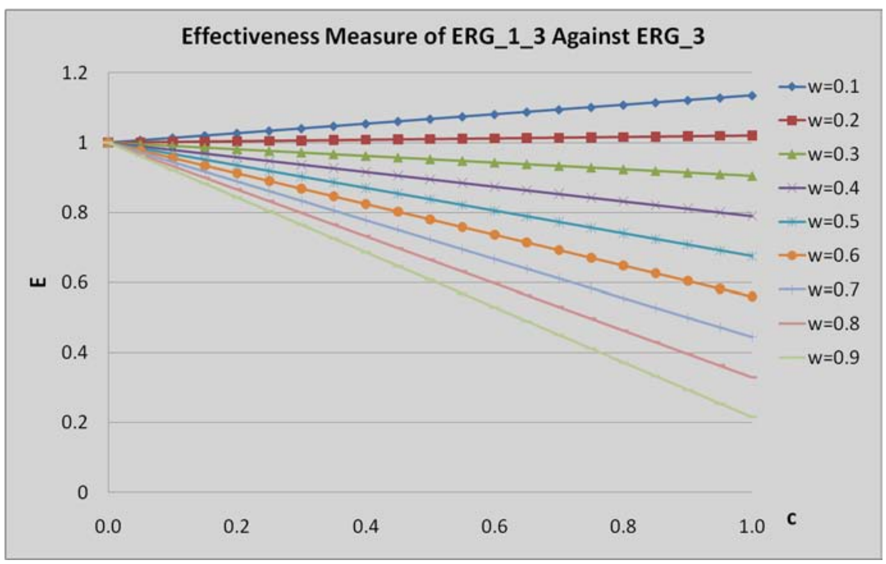


Figure 4: Effectiveness of ERG_1_3 Against ERG_3 under Different Values of c and w

3.3 Example 3

The third example is about duality fusion. Consider a tandem queue that includes m stages connected in series by intermediate finite-size buffers. Jobs enter the system at the first stage, proceed through the system stage by stage, and eventually leave the system from the last stage. It has been shown that such a system is reversible in the sense of having the same throughput when reversing the job flow, that is, having the jobs enter the system from the last stage, going backward through the system, and leaving from the first stage (Yamazaki and Sakasegawa 1975, Chan and Schruben 2008a). This job-flow-reversing system is called the *reverse system* of the original system.

The simulation fusion scheme here is to exploit the use of both systems in achieving various objectives. For example, can one reduce estimation variances by exploiting the correlations of the two systems? Or can one verify the correctness of the two simulation models by comparing their results, which should produce the same throughput as described by theory?.

3.4 Example 4

Combing different samples from a simulation using various rules is also one type of SFs. It can take place at different stages of a simulation study. Importance sampling is example and can occur at the input and output stages (Srinivasan 2002). Sample-path fusion generalizes such an idea to fuse not only statistics from different samples but the entire sample paths, correlated or uncorrelated. Correlated sample paths can be realizations using dependent random numbers or related system configurations. Uncorrelated sample paths can be realizations under different parameter values. Fusing these sample paths provides a way of modeling the response of a system in change of certain parameter values.

Chan and Schruben (2008b) have shown that simulation sample paths can be modeled as linear programs. The main difference between these sample-path linear programs and the usual simulation samples taken in most simulation studies is that these sample path linear programs preserve all information contained in the sample path and have the capability of predicting other sample paths by using duality information (which is free) from the linear program solutions. Such a difference makes it possible to fuse different sample paths along with their duality information to construct new kinds of meta models or response-type models that fully exploit the sample path and duality information.

4 CONCLUSIONS AND FUTURE WORK

There is no single model or method that work perfectly for any systems under any situations. Combining different models or methods is a way to produce new, more robust and efficient models or methods. The spirit of SF is to trade off strengths and weaknesses of various simulation elements. While we have provided a preliminary definition of a general simulation-fusion framework, its full implementation requires a much larger effort from the community. The benefits of SF are vast. Therefore, we suggest and encourage researchers to explore this young and promising research area.

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