

SIMULATION BASED OPTIMIZATION OF MULTI-LOCATION TRANSSHIPMENT PROBLEM WITH CAPACITATED TRANSPORTATION

Banu Yetkin Ekren
Sunderesh S. Heragu

Dept. of Industrial Engineering
University of Louisville
Louisville, KY 40292, USA

ABSTRACT

In this study, a single-item two-echelon inventory system where the items can be stored in each of N stocking locations is optimized using simulation. The aim of this study is to minimize the total inventory, backorder, and transshipments costs, based on the replenishment and transshipment quantities. In this study, transshipments which are the transfer of products among locations at the same echelon level and transportation capacities which are the transshipment quantities between stocking locations, are also considered. Here, the transportation capacities among the stocking locations are bounded due to transportation media or the locations' transshipment policy. Assuming stochastic demand, the system is modeled based on different cases of transshipment capacities and costs. To find out the optimum levels of the transshipment quantities among stocking locations and the replenishment quantities, the simulation model of the problem is developed using ARENA 10.0 and then optimized using the OptQuest tool in this software.

1 INTRODUCTION

Physical pooling of inventories has been widely used in practice to reduce cost and improve customer service. On the other hand, information pooling, which entails the sharing of stock among locations through transshipments, has been less frequent (Ozdemir et al. 2006). Transshipments, the movement of material between locations at the same echelon, provide an effective mechanism for correcting discrepancies between the locations' observed demand and their available inventory. As a result, transshipments may lead to cost reductions and improved service without increasing system-wide inventories. In this study, we focus on a transportation problem which stocking locations share their stocks among other stocking locations and stocking locations

have their replenishment and shipment policies to improve both cost and service.

2 LITERATURE REVIEW

Lateral transshipments between stocking locations are used to enhance cost efficiency and improve customer service in different ways. Two main approaches are used to capture the impact of transshipments between stocking locations. In the first approach, transshipments are realized after the arrival of demand but before it is satisfied. If there is inventory at some of the stocking locations while some have backorder, lateral transshipments between stocking locations can work well. Moreover, pooling the stocks can be viewed as a secondary source of supply for inventory shortages, especially when transshipments between stocking locations are faster and cheaper than emergency shipments from a central depot or backlogging of excess demand. A large portion of the transshipment literature is dedicated to models of such 'emergency' transshipment models. Krishnan and Rao (1965), Herer and Rashit (1999), Herer et al. (2004), Robinson (1990), Tagaras (1989, 1999), and Tagaras and Cohen (1992) analyze such models.

In the second approach, transshipments between stocking locations is considered as a tool to balance inventory levels of stocking locations during order cycles. In this approach, to be able to guarantee a certain level of customer service in all stocking locations, an amount of inventory is carried at each location in balance relative to each other. Inventory levels can become unbalanced due to random variations in demand, where the term imbalance refers to the 'deviation of the inventory position of stocking locations from the average inventory position' (Diks and de Kok 1996). The system stock is redistributed before demand is observed when the transshipments between locations during the system order

cycle is allowed. It is expected that such redistribution will decrease the total shortages and will increase the service level. In emergency models, transshipments respond to actual shortages. However, the purpose of redistributing inventory before the realization of demand is to reduce the risk of possible future shortages. Tagaras (1999), therefore, refers to these models as preventive models. Bertrand and Bookbinder (1998), Diks and de Kok (1996), Hoadley and Heyman (1977), and Jonsson and Silver (1987) analyze such models.

In Krishnan and Rao (1965), they permit transshipments between identical retailers after demand is observed but before demand is satisfied. They consider a single-item inventory distribution system where the item can be stored in each of the N stocking locations that are supplied by an upper echelon common source with infinite capacity. They model the transshipment problem with independent stochastic demand for infinite horizon. Their model shows the pooling advantage of transferring excess inventory of one retailer to the one with excess demand.

Hoadley and Heyman (1977) extend the identical cost model of Krishnan and Rao (1965) to a two-echelon model. Their model assumes returns and transshipments, where preventive transshipments are executed before the realization of demand. Jonsson and Silver (1987) incorporate non-negligible replenishment lead times and transshipment lead times between stocking locations to Hoadley and Heyman's model. Lead times from the central depot to the stocking locations are identical to the transshipment lead time between stocking locations. In addition, in their model there is also a non-zero replenishment lead time from an outside supplier to the central depot.

Different from the Krishnan and Rao's model Robinson (1990) also assumes finite horizons and proves the optimality of the base stock ordering policy. Herer and Rashit (1999) solve the single-period model for two stocking locations with non-identical cost structures taking into consideration fixed replenishment costs. Herer and Tzur (2001, 2003) develop optimal and heuristic algorithms for the dynamic transshipment problem incorporating fixed replenishment and transshipment costs with a deterministic demand structure for finite horizon. Ozdemir et al. (2006) incorporate transportation capacity such that transshipment quantities between stocking locations are bounded due to transportation media or the location_s transshipment policy. They develop a solution procedure based on infinitesimal perturbation analysis to solve the stochastic optimization problem, where the objective is to find the policy that minimizes the expected total cost of inventory, shortage, and transshipments.

In this study, we consider five scenarios with N non-identical stocking locations. Similar to Ozdemir et al.

(2006), we propose a capacitated optimization approach for solving the multi-location transshipment problem. The data and scenarios are used from their papers for optimizing the expected cost function. In the model, the fixed costs are ignored and the optimal replenishment policy is an order-up-to S policy to minimize the expected long-run average cost (Herer et al. 2004). Different from Ozdemir et al. (2006), we use a simulation-based optimization method. For this simulation-based optimization procedure, we use OptQuest, which combines the meta-heuristics of Tabu Search, Neural Networks, and Scatter Search into a single search heuristic in ARENA software. While using this methodology, we keep the complex features of the system and we calculate the optimal values of base stock levels and capacity levels. In addition, in the study we define the transportation problem with non-identical costs and stochastic demand structures. The contribution of this paper is twofold. The first is the incorporation of capacity concerns into the traditional emergency transshipment model. To our knowledge, capacitated systems with transshipments have not been addressed in the literature so much. We formulate possible cases of transshipment capacity as a capacitated network flow problem embedded in a stochastic optimization problem. Second, we use the popular OptQuest method which was not used before for this problem which is a stochastic optimization problem.

3 PROBLEM DESCRIPTION AND SIMULATION

In this study, we consider a single-item, two-echelon inventory system where the item can be stored in each of N stocking locations that are supplied by an upper echelon supplier with infinite production capacity. We assume that orders are placed at the beginning of the each period and the replenishment lead time is negligible. The goods are ordered in period $t-1$ and received from the central depot (or the supplier) at the beginning of period t . After backorders are satisfied, each stocking location has S_i unit of items in its stocking place at the beginning of each period. In the problem, the ordering amount and the ordering cost are not taken into consideration. After having S_i unit of items in each location, new stochastic demands, d_i , arrive. If the demand exceeds the current inventory level at one or more of the stocking locations, we have backordering, otherwise we build inventory. If there are any stocking locations whose inventory level exceeds demand, then lateral transshipments take place among the locations having backorders, to decrease the overall overage and underage items in the system. Here, the transshipments are assumed to be instantaneous. Different from the previous studies in the literature, we assume that there exists a constraint on the number of

units of stocking items which location i can ship to location j in each period.

3.1 Parameters

In the simulation model, we use the following parameters:

- c_i : total cost of holding, backordering and transshipment at stocking location i at the end of a period;
- t_{ij} : direct transshipment cost per unit shipped from stocking location i to stocking location j ;
- tr_{ij} : transshipment amount from stocking location i to stocking location j .
- h_i : holding cost incurred at stocking location i per unit held per period;
- b_i : backorder cost incurred at stocking location i per unit backlogged per period;
- d_i : demand for stocking location i ;
- I_i : inventory level at stocking location i , at the end of each period;

Here, at the end of each period c_i , total cost of stocking location i , is calculated as:

$$c_i = \sum_{i=1}^N \left[(h_i * I_i) + b_i * \max \left(0, (S_i - d_i + \sum_{j=1}^N tr_{ij}) \right) + (t_{ij} * \sum_{j=1}^N tr_{ij}) \right]$$

Here, the objective function is aimed to optimize the average total cost of all the stocking locations in a period:

$$AverageTC = \frac{\sum_{i=1}^N \left[(h_i * I_i) + b_i * \max \left(0, (S_i - d_i + \sum_{j=1}^N tr_{ij}) \right) + (t_{ij} * \sum_{j=1}^N tr_{ij}) \right]}{\sum Period}$$

3.2 Decision Variables

In our study, two decision variables are chosen for optimization. These are transshipment and replenishment quantities of each stocking location.

- S_i : target inventory level (or order-up-to level) at stocking location i at the beginning of each period;
- C_{ij}^{Tr} : number of items transshipped from stocking location i to stocking location j in period k ($P1$);
- $C_{i,out}^{Tr}$: number of items transshipped from stocking location i to all stocking locations in period k ($P2$);

For our experiments, we define two policies, $P1$ and $P2$, which change according to the transshipment policies between stocking locations. In $P1$, the transshipment quantity is capacitated between stocking location i and stocking location j . In other words, C_{ij}^{Tr} represents the total truckload dedicated to transshipment from location i to location j where transshipment between any two stocking locations is restricted to a certain quantity. C_{ij}^{Tr} can be identical or different for each (i, j) arc. We assume that each C_{ij}^{Tr} is same, here.

In $P2$, the transshipment quantity is also capacitated but this time each stocking location allocates only a portion of its starting inventory for total transshipment to the other stocking locations. In other words, the outbound quantity from each location to the other locations is limited by a certain number which is the proportion of the order-up-to inventory level, S_i , for each period ($C_{i,out}^{Tr} = \beta * S_i$). This practice is referred to as partial pooling. Instead of sharing all available on-hand inventories, the stocking locations share only a certain amount of their inventory for transshipment such that the total quantity shipped from location i to all other locations will be less than a pre-determined quantity.

3.3 Simulation Modeling and Experiments

To illustrate the impact of different factors on a network with capacitated stocking locations, we consider five different scenarios of 10 stocking locations. Each scenario is applied for both systems, $P1$ and $P2$. In all scenarios, we consider stocking locations with identical cost parameters. In particular, we set the holding cost to $h_i = \$1$ and backorder cost to $b_i = \$4$ for all 10 locations. Each location faces an independent demand uniformly distributed between $(0, 200)$. The simulation model of the problem is developed using ARENA 10.0 software.

As illustrated in Table 1, we consider five alternative system configurations with different unit transshipment costs, t_{ij} , from stocking location i to location j . Here, $t_{ij} = \infty$ implies that transshipments are not allowed between locations i and j .

Table 1: System scenarios

Scenario	t_{1i}	t_{i1}	t_{ij}
1	∞	∞	∞
2	0.5	∞	∞
3	0.5	0.5	∞
4	0.5	0.5	1.0
5	0.5	0.5	0.5

According to Table 1, Scenario 1 represents that no material movement is allowed among stocking locations. In scenario 2, only the first stocking location can transship to the other stocking locations. In scenario 3, transshipments from all stocking locations to the first stocking location are also allowed. Scenarios 4 and 5, material movement in all directions whereas transshipments between any two stocking locations except location 1, are \$1 unit cost in Scenario 4, it is \$0.5 in scenario 5. It should be noted that, in scenarios 2, 3, and 4, we can view the first stocking location as a distributor for the entire network, while in scenarios 1 and 5, all locations are identical.

To be able to analyze the impact of capacitated transshipments on the cost, we consider two sets of scenarios concerning capacity quantities for $P1$ and $P2$. In $P1$, each arc connecting any two stocking locations is capacitated by four different arc capacities namely, 25%, 50%, 75%, and 100% of the average demand faced by each stocking location. Because average demands are 100 items, we experiment with capacity values of 25, 50, 75 and 100. In the second policy of scenario, $P2$, the total amount of items that can be shipped from any stocking location is limited to a certain percentage of the order-up-to levels. We experiment with four different capacity limits which are 25%, 50%, 75% and 100% (β) of initial inventory level of stocking locations.

Experimenting with all alternative possible values for decision variables is impossible in simulation because it takes a lot of time. Therefore, to find out the optimum levels of the transshipment and the replenishment quantities, the OptQuest method which combines the meta-heuristics of Tabu Search, Neural Networks, and Scatter Search into a single search heuristic in ARENA software is used (Kleijnen and Wan 2007).

To complete the simulation modeling, first, we run all the simulation models for each scenario and policy (a total 9 models). Second, warm-up periods are determined for each scenario. Third simulation models are run for 2000 periods. And finally, OptQuest optimization tool is run for 1000 number of simulations with 3 replications for each scenario. A tolerance value of 0.01% is used to stop the optimization. And 95% confidence interval is used for the average output.

3.4 Results

Figures 1 and 2 represent the optimum average OptQuest cost results for $P1$ and $P2$ policies, respectively. According to the both figures Scenario 1 has the maximum optimum average cost among all scenarios because there is no transportation between two stocking locations. Scenario 2 and Scenario 3 overlap in both figures. Presumably, this means that Scenario 3 is not different from Scenario 2 in practice. In Scenario 3

transportation from all stocking locations to location 1 do not affect the average cost Hence, Scenario 2 and Scenario 3 can be seen to be the same in both policies.

Scenario 4 and 5 were the scenarios where transportation from all stocking locations to other stocking locations was allowed. In Figure 1, scenarios 4 and 5 do not vary with the transshipment capacities because they reach their optimum points earlier than the other scenarios. Therefore, after a certain transshipment capacity the optimum average costs do not change. These optimum points where scenarios of $P1$ have reach minimum costs are illustrated in Figure 4. In Figure 1 and 2, the optimum average costs of Scenario 5 are less than the optimum average costs of Scenario 4. The reason is apparent that the transportation costs between any two stocking locations, except location 1, are less than Scenario 4's transportation costs between any two stocking locations ($\$0.5 < \1). In the, 2nd, 3rd, 4th and 5th, scenarios of Figure 2, when transportation capacities (quantities) increase, the optimum costs decrease. The decrease takes place more rapidly in the first increase of the capacity than the later increases. This means that after 75% of the initial inventory level of stocking location capacity, the average cost is not affected significantly.

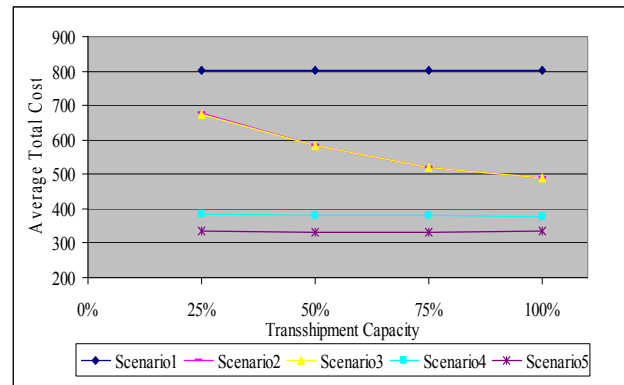


Figure 1: Optimum average costs for $P1$ policy

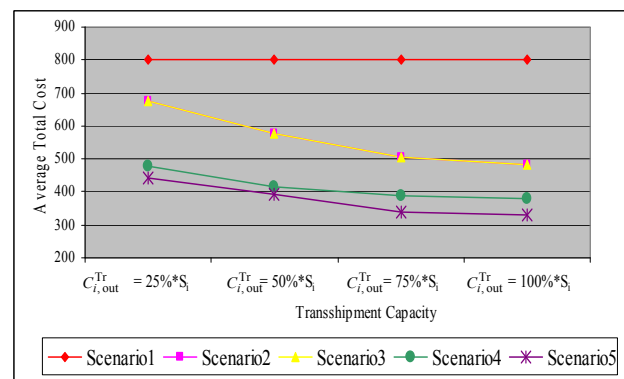


Figure 2: Optimum average costs for $P2$ policy

Scenario 4 and Scenario 5 were the scenarios where transportation from all stocking locations to other stocking locations was allowed. In Figure 1, Scenario 4 and Scenario 5 are not seen variable against the transshipment capacities since they reach their optimum points earlier than the other scenarios. Therefore, after a certain transshipment capacity point the optimum average costs do not change. These optimum points where scenarios of P1 reach have minimum costs are illustrated in Figure 4. Besides, in Figure 1 and 2, the optimum average costs of Scenario 5 are less than the optimum average costs of Scenario 4. The reason is apparent that the transportation costs between any two stocking locations, except location 1, are less than the Scenario 4's transportation costs between any two stocking locations ($\$0.5 < \1). In the, 2nd, 3rd, 4th and 5th, scenarios of Figure 2, when transportation capacities (quantities) increase, the optimum costs decrease. The decrease takes place more rapidly in the first increase of the capacity than the later increases. This means that after 75% of the initial inventory level of stocking location capacity, the average cost is not affected significantly.

Figure 3, illustrates the optimum S_i levels for Scenario 3 of P1 policy as an example and S_i levels for the first scenario. In the first scenario, optimum S_i levels are almost same and stable for every location. Scenarios, 2, 3, 4 and 5, optimum S_i levels vary because transshipments are allowed between some or all two stocking locations and these transshipments are capacitated. In these scenarios, the first stocking location behaves like a clearing house for all the stocking locations (retailers) as seen in Figure 3. This is probably because its transshipment cost is as half as costly as the holding cost of the stocking locations. And as this means that it is more profitable to ship goods from location 1 to all other locations than holding goods in stocking locations 2-10, due to cheaper transportation cost of location 1 ($\$0.5 < \1). However, when the capacity is tight, stocking location 1 is not capable of capitalizing on cheaper transshipment costs. Therefore, other stocking locations start to hold goods to meet their demands or ship goods to location 1.

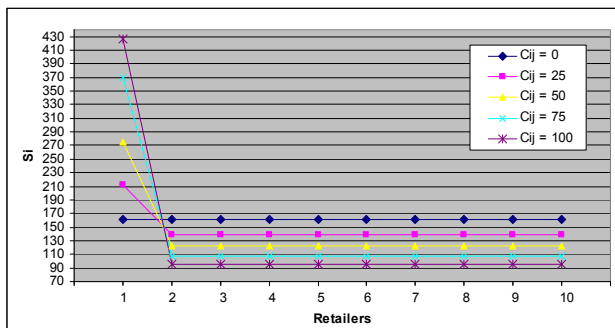


Figure 3: Optimum S_i levels for scenario 3 of P1 policy

Figures 4 and 5 represent the optimum points of capacity levels where all the scenarios of P1 and P2 have their minimum average costs, respectively. As mentioned before, in Figure 1, Scenario 4 and 5 reach their optimum capacity levels earlier than Scenario 2 and 3. It can be seen from Figure 4 that Scenario 4 and Scenario 5 have their minimum costs \$377.826 and \$332.818 at 54 and 50 capacity levels of transshipments, respectively. However, Scenario 2 and Scenario 3 have their minimum costs, \$486.53 and \$486.68 at 125 and 123 transshipment capacities, respectively. So, Scenarios 2 and 3s' optimum average costs should decrease after 100% transshipment capacity levels in Figure 1. From Figures 4 and 5, we can obtain the optimum combinations for P1 and P2. Hence, according to Figure 4, the optimum cost is obtained in Scenario 5 with 50% of the average demand faced by each stocking location.

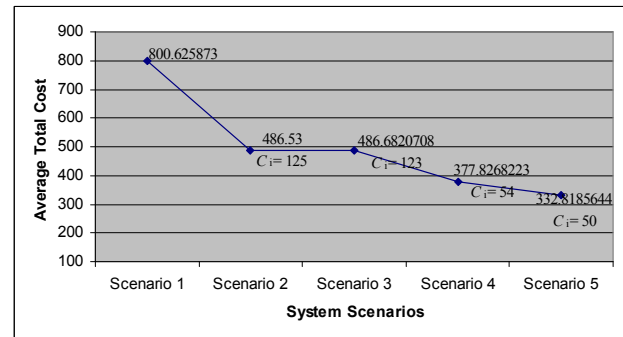


Figure 4: Optimum average costs and capacities for all scenarios of P1

In Figure 5, all scenarios reach their optimum levels at 100% of initial inventory level of stocking locations ($C_{i, out}^{Tr} = 100\% * S_i$). This is due to the limited capacity quantity level of policy P2.

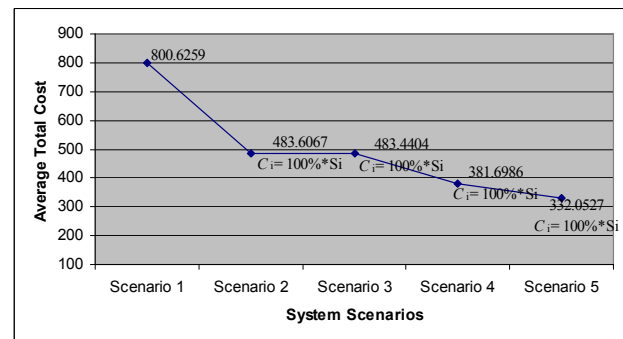


Figure 5: Optimum average costs and capacities for all scenarios of P2

Consequently, all the results obtained by OptQuest can be summarized as below:

In $P1$, the optimum costs of Scenarios 2 and 3 decrease when the transportation capacities increase. However, Scenario 4 and 5s' optimum costs do not change much with the increase in transshipment capacity. This is probably because we allow transshipments from all the stocking locations to the other locations and 25 units of transshipment capacity between two locations is almost enough for every location.

In all scenarios the optimum costs decrease with the increase in transshipment capacities. Because the outbound quantity from each location to the others is limited, this causes $P2$ to have tight transportation capacity relative to $P1$. Therefore, an increase in the transportation capacity may affect the optimum cost, significantly. However, it should be noted that with a high transportation capacity the amount of decrease is lower than the low transportation capacity. And in the high capacities the optimum costs become almost the same in scenarios $P1$ and $P2$ because $P2$ behaves like $P1$ with high capacities.

In both policies, Scenarios 2 and 3 behave as if they are the same scenarios. Because all stocking locations can get their backorders only from the first stocking location and its transportation cost is cheaper than the holding costs in their stocking locations, the first stocking location becomes a supplier for locations 2-10. Besides, in both policies when the transportation capacity decreases from 100% through 25%, the optimum amount of S_i levels in locations 1 decrease. To be able to meet the demands of all other locations, S_i levels of locations 2-10 increase.

In $P1$ of Scenario 4, stocking location 1 still plays a supplier role for the other locations because its transportation cost is cheaper than the others' holding and transportation costs of the others. In $P1$ of Scenario 5, because all locations have the same transportation costs between any two locations the optimum amount of S_i values obtained are nearly the same for all the stocking locations.

In $P2$ of Scenario 4 only in the high transportation capacities the location 1 behaves as the clearing house while the others have early the same S_i values. However, in Scenario 5 of $P2$, all the stocking locations have nearly equal optimum S_i values.

From Figures 4 and 5 the optimum capacity levels can be seen for both policies, $P1$ and $P2$. It should be noted that for both policies, optimum average costs are almost the same because after a certain, optimum point, both policies become the same. As a result we can say that for policy 1, the optimum combination is obtained as Scenario 5 with transshipment capacity level, 50% of the average demand faced by each stocking locations which is 50 units, here. And for policy 2, the optimum combination is obtained as Scenario 5 with transshipment capacity

level, 100% of initial inventory level of stocking locations which are obtained for all retailers as 200, 105, 107, 113, 115, 112, 114, 113, 112, 104 units, respectively.

4 CONCLUSION

In this study, a multi-location transshipment problem with capacitated transportation problem is optimized based on two policies and five scenarios, using OptQuest tool in ARENA 10.0 simulation software. Consequently, we find out that the stocking location having cheaper transportation cost has a clearing house role in the transportation problem. And when transportation capacity increases the optimum cost decreases drastically until a certain amount of the transshipment capacity value. After a certain level of the transshipment capacity, no decrease occurs. To find out these points of the transportation capacity levels where the optimum costs reach their steady state values for all scenarios, we optimize the all scenarios of both policies, by taking into consideration the percentages and β values. Note that, we did not fix the percentages and β values, as 25%, 50% etc. for the simulation runs. We take it as a decision variable in the optimization. As a result, for $P1$ the optimum transshipment capacity levels for Scenario 2, Scenario 3, Scenario 4 and Scenario 5 are obtained as 125%, 123%, 54%, 50% of the average demand faced by each stocking location, respectively. And for $P2$ the optimum transshipment capacity levels for Scenario 2, Scenario 3, Scenario 4 and Scenario 5 are obtained as 100% of initial inventory level of the stocking locations. In both policies the minimum average cost values are obtained as almost the same as 332\$.

REFERENCES

- Bertrand, L. P., and J. H. Bookbinder. 1998. Stock redistribution in two-echelon logistics systems. *Journal of Operational Research Society* 49:966–975.
- Diks, B. E., and A.G. de Kok. 1996. Controlling a divergent 2-echelon network with transshipments using the consistent appropriate share rationing policy. *International Journal of Production Economics* 45:369–379.
- Herer, Y. T., and A. Rashit. 1999. Lateral stock transshipments in a two-location inventory system with fixed and joint replenishment costs. *Naval Research Logistics* 46:525–547.
- Herer, Y. T., M. Tzur, and M. E. Yucesan. 2004. The multi-location transshipment problem. *IIE Transactions*, forthcoming.
- Herer, Y. T., and M. Tzur. 2001. The dynamic transshipment problem. *Naval Research Logistics* 48:386–408.

- Hoadley, H., and D. P. Heyman. 1977. A two-echelon inventory model with purchases, dispositions, shipments, returns and transshipments. *Naval Research Logistics* 24:1–19.
- Jonsson, H., and E. A. Silver. 1987. Analysis of a two-echelon inventory control system with complete redistribution. *Management Science* 33:215–227.
- Kleijnen, J. P. C., and J. Wan. 2007. Optimization of simulated systems: OptQuest and alternatives. *Simulation Modelling Practice and Theory* 15:354–362.
- Krishnan, K. S., and V. R. K. Rao. 1965. Inventory control in N warehouses. *Journal of Industrial Engineering* 16:212–215.
- Lee, H. 2002. Aligning supply chain strategies with product uncertainties. *California Management Review* 44:105–119.
- Ozdemir D., E. Yucesan, and Y. T. Herer. 2006. Multi-location transshipment problem with capacitated transportation. *European Journal of Operational Research* 175:602–621.
- Robinson, L.W. 1990. Optimal and approximate policies in multiperiod multilocation inventory models with transshipments. *Operations Research* 38:278–295.
- Tagaras, G., and M. Cohen. 1992. Pooling in two-location inventory systems with non-negligible replenishment lead times. *Management Science* 38:1067–1083.
- Tagaras, G. 1989. Effects of pooling in two-location inventory systems. *IIE Transactions* 21:250–257.
- Tagaras, G. 1999. Pooling in multi-location periodic inventory distribution systems. *Omega International Journal of Management Science* 27:39–59.

Mysore), MBA at University of Saskatchewan, Saskatoon, Canada and Ph.D. in Industrial Engineering at University of Manitoba, Winnipeg, Canada. His current research interests are in supply chain management, design of next generation factory layouts, intelligent agent modeling of automated warehouse systems, application of RFID technology to improve intra-plant and inter-plant logistics, and integration of design and planning activities in advanced logistical systems. His email address is [<s.heragu@louisville.edu>](mailto:s.heragu@louisville.edu)

AUTHOR BIOGRAPHIES

BANU Y. EKREN is a Ph.D. student in the Department of Industrial Engineering at University of Louisville in USA. She received his B.S. Degree in Industrial Engineering from Dokuz Eylul University, Turkey, in 2001, and M.Sc. Degree in Industrial Engineering from Pamukkale University, Turkey, in 2004. Her research interests include discrete-event-simulation, simulation based optimization, logistics, and facility design problems. Her current research focuses on analytical models for Autonomous Vehicle Storage and Retrieval Systems. Her e-mail address is [<b0ekre01@louisville.edu>](mailto:b0ekre01@louisville.edu)

SUNDERESH S. HERAGU is Professor and the Mary Lee and George F. Duthie Chair in Engineering Logistics in the Industrial Engineering department at the University of Louisville. He is also Director of the Logistics and Distribution Institute (LoDI). He completed his B.S. in Mechanical Engineering at Malnad College of Engineering, Hassan, India (affiliated to University of