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A SIMULATION-BASED ALGORITHM FOR SUPPLY CHAIN OPTIMIZATION

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ABSTRACT

In a supply chain, there are wide variety of problems, such as transportation scheduling problems and warehouse location problems. These problems are independently defined as optimization problems, and algorithms have been proposed for each problem. It is difficult, however, to design an algorithm for optimizing a supply chain simultaneously because the problem is much more complex than the individual problems. We present a simulation-based optimization algorithm that optimizes a supply chain, exploiting both simulation and optimization techniques. This system leverages two existing algorithms, and will optimize a supply chain by executing simulations while changing the boundary conditions between the two algorithms. Experimental results show that a better solution to a supply chain can be found through a series of optimization simulations. A logistics consultant was satisfied with the solution. This system will be used in actual logistics consulting services.

1 INTRODUCTION

In a supply chain, there are wide variety of problems, such as transportation scheduling problems and warehouse location problems. These problems are independently defined as optimization problems, and algorithms have been proposed for each problem. For example, one problem that decides on the warehouse locations that minimize the transportation cost and the fixed cost of the warehouses is called the warehouse location problem (WLP) or facility location problem, and has been studied for many years (Beasley 1993, Hidaka and Okano 2003, Shmoys, Tardos, and Aardal 1997). A problem that makes a multi-modal transportation schedule is called the modal-shift transportation problem (MSTP), and a steepest descent algorithm has been proposed (Amano, Yoshizumi, and Okano 2003).

On the other hand, since it is becoming easier to access a wide variety of data due to the spread of IT infrastructures such as supply chain management systems, a basis for wide range optimizations is becoming a real possibility. For example, combining the WLP and the MSTP mentioned above, their coverage becomes a supply chain of a distribution network that includes the decisions for the number of warehouses and their locations, transportation schedules from the factories to the warehouses, and the transportation costs between the warehouses and the stores (see Figure 1). In this case, however, both problems are closely interdependent, since the warehouse locations of the WLP's output are also a part of the input for the MSTP. In general, we cannot always get a feasible solution by applying two algorithms independently and combining the two solutions. Even if we can get feasible solutions, we have no guarantee on the quality of those solutions. In addition, when handling an actual distribution network, we should consider the inventory cost and the ordering interval in addition to the transportation cost and the fixed cost for the warehouses. As shown above, the problem for a supply chain is much more complex than the individual problems. It is also much more difficult to design an algorithm to simultaneously optimize all parts of a supply chain. In actual logistic consultations, consultants manually calculate the logistics costs based on their intuition and experience.

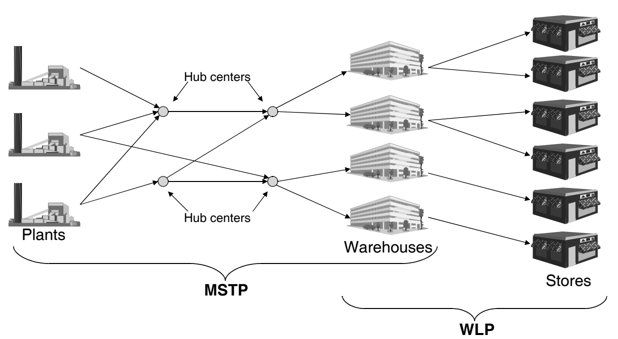

Figure 1: The distribution network and coverages of each problem.

We present a simulation-based optimization algorithm that exploits simulation and optimization techniques, which can optimize the supply chain of a distribution network. This system leverages two existing algorithms, and executes a series of optimization simulations while changing the boundary conditions, or warehouse locations, that connect the two algorithms. This algorithm gradually optimizes a supply chain through a series of simulations.

A prototype system that we developed will be used for logistics consultation services. Some logistics costs, which consultants manually calculated based on their intuition and experiences, can be computed automatically by this system. As a result, we believe that the productivity of consultants can be greatly improved.

2 MODELING SUPPLY CHAIN

We consider the distribution network as an example of a supply chain. We suppose that the distribution network covers the following: goods produced in factories are transported to warehouses by a multi-modal transportation system, are stocked in the warehouses for the stores, and are finally transported to the stores.

Since the number of warehouses and their locations have large impacts on the logistic costs in a supply chain, affecting the transportation cost and the fixed cost of warehouses, it is very important from a strategic viewpoint to decide on appropriate number and locations. The problem of deciding on warehouse locations that minimize the transportation cost and the fixed cost for the warehouses is called the warehouse location problem (WLP) or facility location problem, and has been studied for many years (Beasley 1993, Hidaka and Okano 2003, Shmoys, Tardos, and Aardal 1997). In the WLP, however, the ordering interval from the factories to the warehouses and the inventory cost are not considered. Therefore this is not an adequate model for a practical supply chain.

At the same time, the transportation from the factories to the warehouses is done by multi-modal carriers, such as trucks, trains, and ships, possibly via hub centers where goods are transshipped between carriers. Such a transportation problem, where the operating time windows are not assumed, is called the less-than-truckload (LTL) problem or the transportation network design problem, and has often been studied (Crainic and Roy 1992, Katayama and Yurimoto 2002, Powell and Sheffi 1989). As for modeling a practical supply chain, it is more desirable to consider the operating time windows. We have proposed the MSTP (Amano, Yoshizumi, and Okano 2003), which is an extension of LTL, that can deal with operation time windows.

In this paper, we model a supply chain as a combination of the multi-modal transportation system of the MSTP and deciding warehouse locations of the WLP, which is shown in Figure 1. In following subsections, we will review previous work on the WLP and the MSTP, and then we will introduce our supply chain model.

2.1 Previous Works

2.1.1 The Warehouse Location Problem

The (uncapacitated) WLP is a problem to minimize the sum of the transportation cost and the fixed cost of warehouses. Let C be the set of stores, W be the set of candidate locations for warehouses, f_j be the fixed cost for opening a warehouse $j \in W$, and c_{ij} be the cost to supply store i from warehouse j. The WLP is defined as follows:

$$\min_{x,y} \quad WLP(x,y) = \sum_{i\in C}\sum_{j\in W} c_{ij}x_{ij} + \sum_{j\in W} f_j y_j$$

$$\begin{aligned} \text{subject to} \quad & \textstyle\sum_{j\in W} x_{ij} = 1 && \text{for each } i \in C \\ & 0 \le x_{ij} \le y_j \le 1 && \text{for each } i \in C, j \in W \\ & x_{ij}, y_j \in \{0,1\} && \text{for each } i \in C, j \in W \end{aligned}$$

where x and y are decision variables. $x_{ij} = 1$ decides if store i is supplied from warehouse j. $y_j = 1$ decides if warehouse j is open. The first term of $WLP(x,y)$ represents the transportation cost between warehouses and stores and the second one represents the fixed cost of the warehouses. The first constraint means that each store must be supplied by only one warehouse. The second constraint means that the stores must be supplied by open warehouses. The last constraint means that the variables are zero-one. For this problem, Beasley proposed a Lagrangian relaxation algorithm which can find optimal or near optimal solutions quickly (Beasley 1993).

2.1.2 The Modal-Shift Transportation Problem

In the MSTP, we are given a transportation network consisting of nodes as points representing plants, warehouses, and hub centers, arcs as legs with which carriers are associated, and delivery orders between points. The problem is to find the optimal schedule of carriers for each leg and the optimal routing of delivery orders between points so that the total carrier cost is minimized and the tardiness of delivery orders is also minimized. The name "modal shift" comes from the fact that the problem can model a multi-modal transportation including trucks, trains, ships, etc., in which carriers may be associated with operating time windows or schedules.

The following notations are used:

- the set of points: N,
- the set of delivery orders: D.

Each delivery order $i \in D$ has the following properties:

- the set of candidate routes: $C_i = \{R_{i1}, \ldots, R_{i|C_i|}\}$,
- the set of legs along each route: $R_{ik} = \{a_{ik1}, \ldots, a_{ik|R_{ik}|}\}$, $1 \leq k \leq |C_i|$,
- the penalty cost when an order cannot be delivered: p_i,
- the due date: d_i,
- the earliest starting time, and
- the weight.

Each carrier $j \in V_a$ defined for leg $a \in N \times N$ has the following properties:

- the travel time: t_{ja},
- the transportation cost: c_{ja},
- the earliest starting time,
- the latest starting time, and
- the capacity.

The decision variables in the MSTP are the following:

- The choice of a candidate route for delivery order i: λ_i,
- The departure time of the carrier j defined for leg a: τ_{ja},
- The binary variable for delivery order i to be carried by carrier j for leg a: κ_{ija}.

Using the above notations, the MSTP is defined as in Figure 2.

The objective function to be minimized is the sum of the cost for the required carriers and the penalty for delivery orders that violate the due date. Let c_{ija} be the apportioned cost of c_{ja} to delivery order i and p_{ia} be the apportioned penalty of delivery order i to leg a. The objective function of the MSTP can be rewritten as:

$$\sum_{i \in D} \min_{\lambda_i} \left(\sum_{a = a_{i\lambda_i l}, 1 \leq l \leq |R_{i\lambda_i}|} \sum_{j \in V_a} (c_{ija} + p_{ia}) \kappa_{ija} \right),$$

which can be minimized by finding the lowest cost route for each order if c_{ija} and p_{ia} are assumed to be fixed. The difficulty, however, is that c_{ija} and p_{ia} are functions of λ. We have proposed in (Amano, Yoshizumi, and Okano 2003) an algorithm based on a steepest descent method and a greedy heuristic, in which c_{ija} and p_{ia} are calculated in the previous run of the greedy heuristic. The steepest descent method determines a routing for each delivery order, and the greedy heuristic assigns the delivery orders to carriers and determines their departure times. For more details, the readers are referred to (Amano, Yoshizumi, and Okano 2003).

2.2 A Supply Chain Model

In this paper, we model a supply chain model as the combination of the MSTP and the WLP, and call it *the warehouse location and transportation problem* (WLTP). Since the WLTP can handle the inventory cost and the ordering interval in addition to the other costs handled by the WLP and the MSTP, it is more suitable model for a practical supply chain.

In the WLTP, we are given a set of plants, a set of candidate locations of warehouses, a set of stores with which demand is associated, a transportation network, and carriers between the plants and warehouse candidates. The demand of a store is defined as an average quantity of goods that need to be supplied from a nearby warehouse. The problem is to choose warehouse locations where goods should be stocked for the stores, taking into account the following objectives:

- The transportation cost from warehouses to stores,
- The fixed cost of the warehouses,
- The inventory cost at the warehouses,
- The transportation cost from the plants to the warehouses.

Note that the standard WLPs only consider the first two objectives, whereas in the actual logistics in the manufacturing or distribution industries, which the WLTP is based on, the last two objectives are also important.

The transportation cost from warehouse j to store i is defined as c_{ij}, which is the same as in $WLP(x,y)$. The fixed cost of warehouse j is also defined as f_j. The inventory cost is assumed to be proportional to the amount of goods stocked at the warehouses. The inventory at a warehouse depends on the total demand of the stores that are supplied from the warehouse and also on the replenishment cycle, or equivalently, the *ordering interval* denoted by t. When the ordering interval t is three days, for example, delivery orders from plants to warehouses amounting to the demand for three days are generated every three days. This means that goods are transported to the warehouses every three days, and the inventory level at each warehouse forms a triangular-shaped curve with a period of three days. Let μ be the demand per day and I be the ordering interval, so the average inventory per day is $I\mu/2$ (see Figure 3). Both demand and inventory are treated as weights. In the WLTP, the inventory cost (per day) is calculated as $bI\mu/2$, where the coefficient b is a unit inventory cost per weight.

Each store $i \in C$ is associated with a plant (supplier) $s_i \in N$ and the demand per day μ_i. Stores are assumed to be supplied from the nearest warehouses that are open and to which goods are supplied from s_i. This means that a warehouse may need to be supplied with goods from multiple plants. This is the same transportation problem as

$$\begin{array}{lllr}
\min_{\lambda,\tau,\kappa} & MSTP(\lambda,\tau,\kappa) = \sum_{a\in N\times N}\sum_{j\in V_a} c_{ja}z_{ja} + \sum_{i\in D}\ p_i\cdot \text{tardiness}(i) & & \\
\text{subject to} & \lambda_i \in \{1,2,\ldots,|C_i|\} & \text{for each } i\in D, & \text{[candidate]}\\
 & \kappa_{ija}\in\{0,1\} & \text{for each } i\in D,\ j\in V_a \text{ for each leg } a, & \text{[zero-one]}\\
 & \sum_{j\in V_a}\kappa_{ija}=1 & \text{for each } i\in D,\ j\in V_a,\ a\in R_{i\lambda_i}, & \text{[non-divisible]}\\
 & z_{ja} = \{1 \text{ if } \sum_{i\in D}\kappa_{ija}>0,\ 0 \text{ otherwise}\} & & \text{[carrier usage]}
\end{array}$$

z_{ja} and κ_{ija} are feasible with respect to the time windows and the load capacity of carriers j for leg a.
$tardiness(i)$ is defined as $max_{j\in V_a}\{0,\kappa_{ija}(\tau_{ja}+t_{ja})-d_i\}$.

Figure 2: The formulation of MSTP.

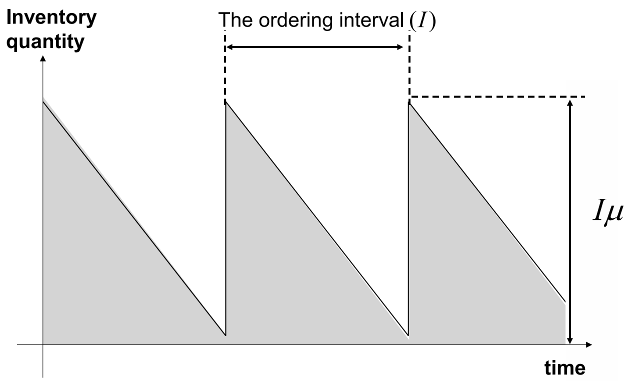

Figure 3: An inventory curve at a warehouse.

discussed in the MSTP. As we will see in Section 3.1, the MSTP can be extended to handle the inventory cost for use in the WLTP.

The WLTP is defined as:

$$\min_{x,y,\lambda,\tau,\kappa,t} \quad WLTP(x,y,\lambda,\tau,\kappa,t) = WLP(x,y) + MSTP_{x,y}(\lambda,\tau,\kappa,t)$$

subject to

The constraints of the WLP are observed for the warehouses and stores,
The constraints of the MSTP are observed for the transportation between the plants and warehouses,
The inventories at the warehouses are feasible with respect to the demands of the stores,

where λ,τ,κ, and t, for each value of x and y, are solutions of $MSTP_{x,y}(\lambda,\tau,\kappa,t)$, which is an extension of the MSTP in which the optimal ordering interval t is determined taking into account the tradeoff between the transportation cost and the inventory cost. The details of $MSTP_{x,y}(\lambda,\tau,\kappa,t)$ will be described in the next section. Note that the input data of the MSTP depends on the solution of the WLP. For example, the set of points in the MSTP should be the warehouses that are selected for use in the WLP in addition to the plants and hub centers. The MSTP, therefore, needs x and y as input data that are decision variables of the WLP. This form of a mathematical program–consisting of two levels of problems such as WLP and $MSTP$–is called a bilevel program (Vicente and Calamai 1994).

Since the WLP cannot optimize λ,τ,κ, and t, and $MSTP_{x,y}$ needs to input the solution of WLP, x and y, an optimal solution cannot be obtained by solving WLP and $MSTP$ one after another even if each solver can find optimal solutions. Therefore, in the next section, we will transform the WLP to incorporate the output of the MSTP, and propose a simulation-based optimization algorithm.

3 A SIMULATION-BASED ALGORITHM

We propose a simulation-based optimization algorithm for the WLTP, which leverages two existing algorithms, and executes a series of optimization simulations while changing the boundary conditions, or warehouse locations, that connect the two algorithms. This algorithm gradually optimizes a supply chain through a series of simulations.

First, we introduce a new decision variable for the ordering interval to extend the MSTP. The extended MSTP can handle the inventory cost as well as the ordering interval. Then we show the WLTP can be transformed to the form of the WLP by approximating the MSTP. This means that an algorithm for the WLP can solve the approximated WLTP where the MSTP is approximated.

3.1 Extending MSTP

We extend the MSTP to handle the inventory cost and the ordering interval. The role of the extended MSTP is to create a transportation schedule from the plants to the warehouses and provide some parameters for the reformulated WLTP, which is described in Section 3.2. We use the following notations:

- the set of plants, warehouses opened by the reformulated WLTP, and hub centers: N,
- the set of demand: D,

where D is a set of feasible triplets of plant p, warehouse j, and a set of stores S, Each demand $i \in D$ has the following properties:

- the set of candidate routes: $C_i = \{R_{i1}, \ldots, R_{i|C_i|}\}$,
- the set of legs along each route k: $R_{ik} = \{a_{ik1}, \ldots, a_{ik|R_{ik}|}\}$, $1 \leq k \leq |C_i|$,
- the penalty cost for tardiness: p_i,
- the inventory cost per weight: b_i,
- the set of delivery orders: $D_o^i = \{o_{i1}, o_{i2}, \ldots, o_{i|D_o^i|}\}$,

where D_o^i is generated depending on the solution of the reformulated WLTP, x and y, and the tentative value t for the ordering interval. For the pair of plant p and warehouse j that corresponds to demand i, let μ_{pj} denote the total demand of the corresponding stores. The $\mu_{pj} \times t$ delivery orders of weight 1 are generated every t days within a scheduling horizon of H days. This means that $|D_o^i| = \mu_{pj}H$. Each delivery order $l \in D_o^i$ is also associated with:

- the due date of the delivery order o_{il}: d_{il},
- the release date: $r_{il} = 1 + t \times \lfloor \frac{l-1}{H} \rfloor$,

where t is the tentative value of the ordering interval I. Each carrier $j \in V_a$ defined for leg $a \in N \times N$ has the following properties:

- the travel time: t_{ja},
- the transportation cost: c_{ja},
- the earliest starting time,
- the latest starting time, and
- the capacity.

Using the above notations, we define the extended MSTP of the WLTP as in Figure 4.

Note that the set of delivery orders for demand $i \in D$, D_o^i, is changed depending on the value of t. In addition to the objective function of the MSTP described in Section 2.1.2, the inventory cost at the warehouses is added in the formulation shown in Figure 4. Since it is the sum of the inventory cost for all of the demand, we can apportion this cost to the route cost, and the objective function can be defined as sum of the route costs. Let v_{ia} denote the apportioned cost of the inventory cost to leg a of delivery order i. The objective function can be rewritten as follows:

$$\sum_{i \in D} \min_{\lambda_i} \left(\sum_{a = a_{i\lambda_i l}, 1 \leq l \leq |R_{i\lambda_i}|} \sum_{j \in V_a} (c_{ija} + p_{ia} + v_{ia}) \kappa_{ija} \right).$$

When the ordering interval, t, is given, the objective function becomes the same form as that in the original MSTP. Therefore, we can use the algorithm from (Amano, Yoshizumi, and Okano 2003), which is based on a steepest descent method and greedy heuristics. Usually, the unit of the ordering interval is a day, and its range is from 1 day to 7 days. Therefore, we can obtain solutions of the $MSTP_{x,y}(\lambda, \tau, \kappa, t)$ by simulating all of the ordering intervals.

3.2 Reformulating WLTP

Once all of the decision variables of the extended MSTP are determined, we can calculate the transportation and penalty costs, and can apportion those costs to each warehouse. For each warehouse, the sum of the transportation and penalty costs per weight can be calculated. Let $T_{j,x,y}$ be the transportation and penalty costs per weight for warehouse j with given x and y. We regard the objective function value of the extended MSTP as the sum of the transportation and the penalty costs of all of the warehouses as follows:

$$\min_{\lambda,\tau,\kappa,t} MSTP_{x,y}(\lambda, \tau, \kappa, t) = \sum_{j \in W} T_{j,x,y} \sum_{i \in C} x_{ij}\mu_i + \sum_{j \in W} \frac{b_j t}{2} \sum_{i \in C} x_{ij}\mu_i.$$

Suppose that we can approximately regard $T_{j,x,y}$ as a constant value, T_j, independent of x and y. This means $T_j \approx T_{j,x,y}$. Let I be the optimal value of t. Suppose also that T_j and I can be regarded as constant values around the optimal solutions of the extended MSTP. When x, y, T, and I are given, the objective function of the extended MSTP can be rewritten as a linear function of x_{ij} as follows:

$$\begin{aligned} & MSTP_{x,y}(\lambda, \tau, \kappa, t) \\ & \approx \sum_{j \in W} T_j \sum_{i \in C} x_{ij}\mu_i + \sum_{j \in W} \frac{b_j I}{2} \sum_{i \in C} x_{ij}\mu_i \\ & = \overline{MSTP_{T,I}}(x,y). \end{aligned}$$

Using the above approximation, the WLTP can be transformed as follows:

$$\begin{aligned} & WLTP(x,y,\lambda,\tau,\kappa,t) \\ & = WLP(x,y) + MSTP_{x,y}(\lambda,\tau,\kappa,t) \\ & \approx WLP(x,y) + \overline{MSTP_{T,I}}(x,y) \\ & = WLP(x,y) + \sum_{j \in W} T_j \sum_{i \in C} x_{ij}\mu_i \\ & \quad + \sum_{j \in W} \frac{b_j I}{2} \sum_{i \in C} x_{ij}\mu_i \\ & = \sum_{i \in C} \sum_{j \in W} (c_{ij} + T_j\mu_i + \frac{b_j I}{2}\mu_i) x_{ij} + \sum_{j \in W} f_j y_j \\ & = \sum_{i \in C} \sum_{j \in W} c'_{ij} x_{ij} + \sum_{j \in W} f_j y_j, \\ & = WLP_{T,I}(x,y), \end{aligned}$$

$$\min_{\lambda,\tau,\kappa,t} \quad MSTP_{x,y}(\lambda,\tau,\kappa,t) \quad = \sum_{a\in N\times N}\sum_{j\in V_a} c_{ja}z_{ja} + \sum_{i\in D} Tb_i\frac{t\mu_i}{2} + \sum_{i\in D}\sum_{l\in D_o^i} p_i \cdot \mathrm{tardiness}(i,l)$$

subject to
$\lambda_i \in \{1,2,\ldots,|C_i|\}$ for each $i \in D$ [candidate]
$\kappa_{lja} \in \{0,1\}$ for each $l \in D_o^i,\ j \in V_a$ for each leg a and $i \in D$, [zero-one]
$z_{ja} = \{1 \text{ if } \sum_{i\in D}\kappa_{lja} > 0,\ 0 \text{ otherwise}\}$ [carrier usage]

z_{ja} and κ_{lja} are feasible with respect to the time windows and the load capacity of carriers j for leg a.
$tardiness(i,l)$ is defined as $max_{j\in V_a}\{0, \kappa_{lja}(\tau_{ja}+t_{ja})-d_i\}$, and x and y represent configuration of warehouse locations which determine N and D for this problem.

Figure 4: The formulation of the extended MSTP.

where $c_{ij} + T_j\mu_i + \frac{b_jI}{2}\mu_i$ is replaced by c'_{ij}.

This means that we can transform the WLTP to the form of the WLP by adding the inventory cost at warehouse and the transportation cost between plants and warehouses to the transportation cost between the warehouses and the stores. If we can obtain reasonable values of T and I, we can solve the WLTP using an algorithm for the WLP.

3.3 A Simulation-based Algorithm for the WLTP

We propose a simulation-based optimization algorithm for the WLTP, which improves the accuracy of the parameters of T and I through a series of simulations, using both the extended MSTP and the reformulated WLTP.

The algorithm is described in Figure 5, and Figure 6 shows its algorithmic framework. In Step 1, parameters, T and I, are initialized to 0 and 1 respectively. In Step 2, the reformulated WLTP is solved using an algorithm for the WLP. In Step 3, the extended MSTP is solved using an algorithm for the MSTP with x and y obtained in Step 2. In Step 4, the objective function value of WLTP is calculated using the solutions obtained in Steps 2 and 3. The terminal condition in Step 5 is arbitrary. For example, we can use a maximum number of iterations or some metric for checking convergence. If the terminal condition is not satisfied, we update the parameters, T and I, and then go back to Step 2. The method to update T and I is discussed in the next section.

3.4 Updating Parameters

The computation of T requires special care because, in the minimization of $MSTP$ in Step 3, only those T_j whose y_j were set to 1 in Step 2 are considered. The set of j whose y_i equals 1 is different from iteration to iteration, and T_j may differ greatly with different y. Therefore, it is important to obtain values of T that can be used for any configuration of y, not only for the specific configuration of y.

When $\overline{MSTP_{T,I}}(x,y)$ is less than or equal to $MSTP_{x,y}(\lambda,\tau,\kappa,t)$, $WLP_{T,I}$ is also less than or equal to $WLTP$ and $WLP_{T,I}$ gives a lower bound of $WLTP$. A lower

1. Initialize T, I.
2. $x, y = \arg\min_{x,y} WLP_{T,I}(x,y)$.
3. $\lambda,\tau,\kappa,t = \arg\min_{\lambda,\tau,\kappa,t} MSTP_{x,y}(\lambda,\tau,\kappa,t)$.
4. Calculate $WLTP(x,y,\lambda,\tau,\kappa,t)$.
5. If the terminal condition is not met, update T and I based on the result from Steps 2 and 3, and go to Step 2.
6. Output the best solution found.

Figure 5: Proposed algorithm for WLTP.

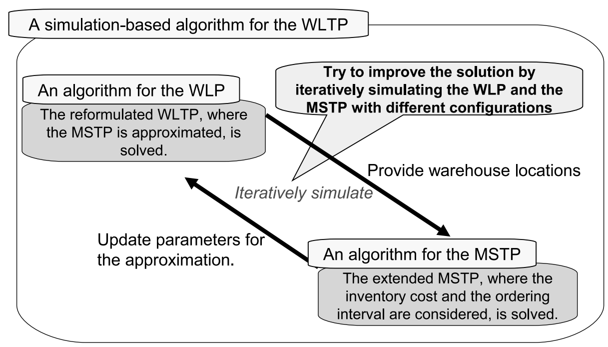

Figure 6: The algorithm framework.

bound is an important index when evaluating the quality of solutions. It is desirable to use T and I that satisfy $\overline{MSTP_{T,I}}(x,y) \leq MSTP_{x,y}(\lambda,\tau,\kappa,t)$. We call this inequality the lower bound inequality.

Since $\overline{MSTP_{T,I}}(x,y)$ is a monotonic increasing function with respect to both T and I, the initial values of T and I should be 0 and 1, respectively, which are the minimum values of each parameter. Those values should be updated to give a tighter lower bound as the search proceeds. However, it is difficult to satisfy this lower bound inequality for an arbitrary search space. Therefore, it is desirable for the updating method to satisfy the lower bound inequality as much as possible.

Let T_j^τ be T_j at the τ-th iteration of the WLTP algorithm. Our updating method is $T_j^\tau = \alpha T_j^\tau + (1-\alpha)T_j^{\tau-1}$, where α is the update ratio (whose range is $0 < \alpha \leq 1$). This updating method considers the value in the previous iterations to some

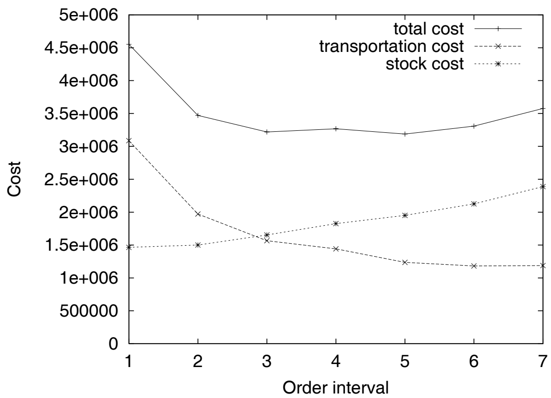

Figure 7: Costs with several ordering intervals.

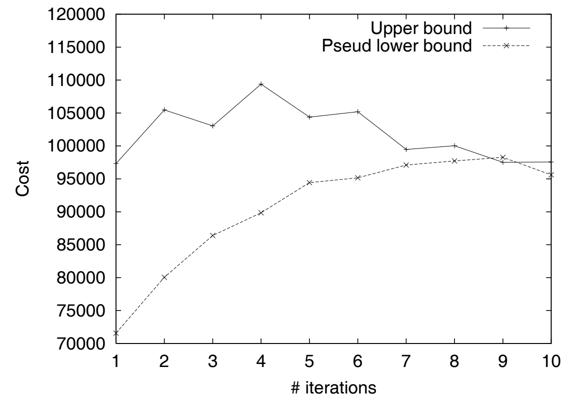

Figure 8: The objective value for each iteration ($\alpha = 1.0$).

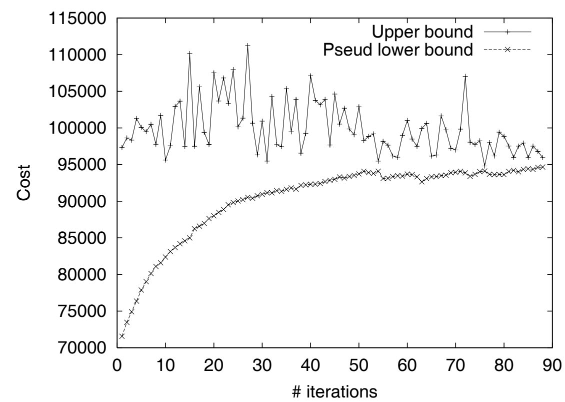

Figure 9: The objective value for each iteration ($\alpha = 0.1$).

degree. The aim of this method is to prevent the value of T_j from being changed too greatly and to keep satisfying the lower bound inequality. When $\alpha = 1$, T_j is completely replaced by the newly calculated value. When α is close to 0, T_j is updated smoothly and it is possible for the lower bound inequality to continue to be satisfied. The ordering interval, I, can be updated by the same algorithm.

4 EXPERIMENTS AND A PROTOTYPE SYSTEM

The instance data for these experiments was generated for an imaginary manufacturing company that has stores all over Japan. This imaginary company has two factories, one in Tokyo and one in Osaka, 40 candidate locations for warehouses, and 500 stores. This company wants to reduce the total logistics cost by closing and merging some of the warehouses.

We adopted the Lagrangian relaxation algorithm by Beasley for the WLP (Beasley 1993), and the steepest descent method for the MSTP (Amano, Yoshizumi, and Okano 2003). The terminal condition is that the solution of the WLTP converges or the number of iterations reaches 100.

First, to assess the functionality of the extended MSTP, we open all of the warehouses and simulate inventory, transportation, and total costs with several ordering intervals. In Figure 7, the horizontal axis represents the ordering interval, and the vertical axis represents the cost. From Figure 7, we can see that the inventory cost increases as the ordering interval becomes longer. This is because the average inventory increases in proportion to the ordering interval. In contrast, the transportation cost decreases as the ordering interval becomes longer. When the ordering interval is long, the delivery order quantity becomes large and goods tend to be aggregated to full truckloads, which lowers the transportation cost. From the results of this experiment, we can see that there is a tradeoff between the inventory cost and the transportation cost.

Figures 8 and 9 show the results of the WLTP algorithm with α value of 0.1 and 1.0, respectively. The pseudo-lower bound represents the value of $WLP_{T,I}$ (Step 2 in Figure 5). Since T and I may be greater than the actual values, this pseudo-lower bound is not the true lower bound. Therefore, the pseudo-lower bound may be greater than the upper bound.

In both cases of $\alpha = 0.1$ and 1.0, the initial solutions are the same and the value was 97,326. The configuration of warehouse locations obtained in the initial solution is identical to that of the $WLP(x,y)$ without considering the transportation cost between plants and warehouses. For $\alpha = 1.0$, the initial solution was the best solution. On the other hand, for $\alpha = 0.1$, the best solution was 94,811, which is 2.6% smaller than the initial solution. These experimental results show that a better solution can be found by updating the parameters smoothly through a series of optimization simulations.

We developed a prototype system using this simulation-based optimization algorithm. Figure 10 shows a screenshots of our system. A logistics consultant was satisfied with these optimization results and this prototype system. This system will be used for logistics consultation services.

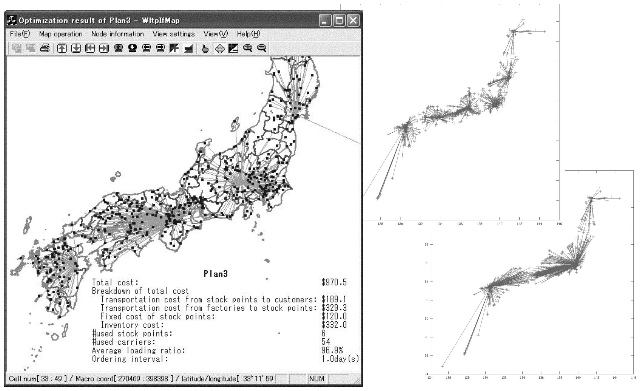

Figure 10: Screenshots of a prototype system.

5 CONCLUSION

We proposed a simulation-based optimization algorithm, which can optimize the supply chain of a distribution network. This algorithm leverages two existing algorithms, and executes a series of optimization simulations while changing the boundary conditions, or warehouse locations, that connect the two algorithms. This algorithm gradually optimizes a supply chain through a series of simulations.

Through an empirical study, we showed that there is a tradeoff between the transportation cost and the inventory cost. By adjusting the ordering interval appropriately, the total cost can be reduced. The experimental results showed that present solution converged to a near optimal solution compared with the pseudo-lower bound, and a better solution can be found by updating the parameters smoothly through a series of optimization simulations.

A simulation-based optimization system that we developed will be used for logistics consultation services. Some logistics costs, which consultants manually calculated based on their intuition and experiences, can be computed automatically by this system. As a result, we expect that the productivity of consultants can be greatly improved.

There are several remaining problems, such as improvement of the updating algorithm and development of a method to obtain appropriate values of α. However, the framework that unifies the two existing algorithms has the potential to solve large optimization problem more easily. It becomes possible to optimize a wider range of supply chains as IT infrastructures such as SCM are adopted more widely.

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