

**A LOSS DEFAULT SIMULATION MODEL OF
THE FEDERAL BANK DEPOSIT INSURANCE FUNDS**

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ABSTRACT

This paper discusses a simulation model that is used in a martingale valuation approach to measure and value the risk of the FDIC deposit insurance funds. The FDIC insurance funds capitalize a portfolio of insurance policies, each issued to depositors of an individual commercial bank. To evaluate this portfolio, our methodology evaluates the insurance policies for depositors at each individual bank and aggregates to obtain the risk of the entire portfolio. To adequately model the risks associated with credit, interest rate, deposit growth, and loss rate, a multi-dimensional system is formulated. The risk measurement and valuation results are based on Monte Carlo simulation of the system risks.

1 INTRODUCTION

A rigorous approach to risk measurement and valuation of the risks of the FDIC insurance guarantees is essential for effective risk management. This paper develops such an approach, providing a valuable tool for evaluating the risks posed to the deposit insurance funds from potential bank failures. The approach is based on a martingale valuation methodology in Duffie et al. (2003), and is carried out using a Monte Carlo simulation model that incorporates the relevant risk factors. To evaluate the potential losses on the FDIC deposit insurance fund, our methodology evaluates each individual bank's potential failure and aggregates to obtain the risk of the entire fund. The four relevant risks of these insurance guarantees are included: interest rate,

credit, deposit growth, and loss severity. To adequately model these four correlated risks, a multi-dimensional system is formulated. The resulting risk management tool is constructed to be flexible and easily modified to incorporate extensions and generalizations.

Of the four risks, we model interest rate risk using a four-factor Heath-Jarrow-Morton (1992) (HJM) model. Credit risk is modeled using the reduced form methodology introduced by Jarrow and Turnbull (1992, 1995). Following the recent insights of Duffie and Lando (2001) and Cetin et al. (2004) in this regard, an intensity process is used, because regulators and the market have less information than do bank management. Less information can generate a totally inaccessible default time for the regulators and the market, whereas it may be a predictable stopping time for bank management (Protter 1990). The structural approach to credit risk of Merton (1974) is more appropriate when valuing these claims from the bank management's perspective. Deposit growth is modeled using various bank-specific, local- and macro-economic variables in a time series regression. The loss (or equivalent, recovery rate) process depends on the historical loss rates and the asset and liability structure of the bank.

The results of the Monte Carlo simulation provide a complete characterization of the risks faced by the FDIC's deposit insurance fund. Over a one-, five-, and ten-year horizon, we compute various quantiles and summary statistics for the number of bank failures, the total deposits in the failed banks, and the current values of the potential losses to the FDIC. From these computations, one obtains various risk measures and market valuations. For example,

the value at risk (VaR) measure over a one-year horizon using a 99% probability is the 99th percentile loss over a one-year horizon, or \$2.2 billion. Analogous VaRs for the five- and ten-year horizons are \$27 billion and \$109 billion, respectively. The market value of these losses to the FDIC insurance fund over the various horizons, valued as if traded on public capital markets, are given by the mean of the loss distribution. For a one-year horizon, the market value of the FDIC's losses are computed to be \$218 million. Analogous market values over the five- and ten-year horizons are \$2 billion and \$21 billion, respectively.

The remainder of this paper is organized as follows. Section 2 presents the martingale valuation methodology. This section characterizes the four risks present in FDIC insurance guarantees, and it presents both the risk measure construction and valuation technology. The simulation model is the content of section 3. Section 4 presents the parameter estimation used in the simulation, whose results are discussed in section 5.

2 VALUATION METHODOLOGY

This section introduces the notation and economic logic underlying the martingale valuation model for the FDIC deposit insurance funds. The valuation methodology is based on Duffie et al. (2003).

2.1 Model Structure

Let \mathcal{P} denote the statistical probability measure, and \mathcal{T} denote the time horizon of interest (one, five, or ten years in our setting). Default-free bonds of all maturities $T \in [0, \mathcal{T}]$ are traded with time t prices denoted $p(t, T)$, and various stock price indices introduced below. The spot rate of interest at time t is denoted r_t . We assume that markets are complete and arbitrage free, so that there exists a unique equivalent martingale probability measure \mathcal{Q} under which discounted prices are martingales. The discount factor at time t is $e^{\int_0^t r_s ds}$.

Let $i = 1, \dots, I$ represent the banks insured by the FDIC. Let $Y^i(t) \equiv \{Y_j^i(t), j = 1, \dots, N_y\}$ be a collection of characteristics of bank i at time t , e.g., the loan-to-deposit ratio of bank i at time t . These variables are known to the banks and the regulators, but perhaps not all of them, such as examination ratings, are available to the market.

2.2 Interest Rate Model

Since we will be using a simulation to evaluate the future losses to the FDIC deposit insurance fund, we employ a multi-factor HJM model to govern the term structure evolution, which allows maximum modeling generality and flexibility.

2.2.1 Forward Rate Process

We specify the evolution of the term structure using forward rates under the martingale measure \mathcal{Q} . Let $f(t, T)$ be the instantaneous (continuously compounded) forward rate at time t for the future date T . We use the following K factor model:

$$df(t, T) = \alpha(t, T)dt + \sum_{j=1}^K \sigma_j(t, T)dW_j(t), \quad (1)$$

$$\alpha(t, T) = \sum_{j=1}^K \sigma_j(t, T) \int_t^T \sigma_j(t, u)du,$$

$$\sigma_j(t, T) \equiv \min[\sigma_{rj}(T)f(t, T), M],$$

where M is a large positive constant, $\sigma_{rj}(T), j = 1, \dots, K$, are deterministic functions of T , and $W_j(t), j = 1, \dots, K$ are uncorrelated standard Brownian motions. Forward rates are approximately lognormally distributed under expression (1).

2.2.2 Spot Rate Process

With $r_t \equiv f(t, t)$, the spot rate process can be deduced from the forward rate evolution via

$$dr_t = \left. \frac{\partial f(t, T)}{\partial T} \right|_{T=t} dt + \alpha(t, t)dt + \sum_{j=1}^K \sigma_j(t, t)dW_j(t). \quad (2)$$

But $\alpha(t, t) = \sum_{j=1}^K \sigma_j(t, t) \int_t^t \sigma_j(t, t)du = 0$, so

$$dr_t = \left. \frac{\partial f(t, T)}{\partial T} \right|_{T=t} dt + \sum_{j=1}^K \min[\sigma_{rj}(t)r(t), M]dW_j(t) \quad (3)$$

under the martingale measure \mathcal{Q} .

2.3 State Variable Processes

We have two sets of state variables. Let $V(t) \equiv \{V_j(t), j = 1, \dots, N_v\}$ be a collection of macro-variables that are independent of a particular bank. These state variables are intended to capture the health of the economy at time t . Second, let $X(t) \equiv \{X_j(t), j = 1, \dots, N_x\}$ represent another collection of state variables also characterizing the state of the economy at time t . The difference between these two sets of state variables is that X_t represents the prices of traded assets, while $V_j(t)$ need not. We assume that these state variables give equivalent characterizations of the state of the economy.

For the subsequent analysis, we do not need to specify the evolution of $V_j(t)$, but we do need to do so for the traded assets. We assume that the traded state variables

follow a diffusion process under the martingale measure \mathcal{Q} :

$$dX_j(t) = r_t X_j(t)dt + \sigma_{xj} X_j(t) dZ_j(t), \quad (4)$$

where σ_{xj} is a constant, $Z_j(t)$ are correlated standard Brownian motions with $dZ_i(t)dZ_j(t) = \rho_{ij}dt$, and with respect to the term structure of interest rates, $dW_i(t)dZ_j(t) = \eta_{ij}dt$. Because the state variables represent traded prices, the drift of this process is the spot rate of interest r_t .

By Girsanov's theorem, under the statistical measure \mathcal{P} , the evolution of these state variables is

$$\frac{dX_j(t)}{X_j(t)} = \mu_j(t)dt + \sigma_{xj} d\tilde{Z}_j(t), \quad (5)$$

$$\text{where } d\tilde{Z}_j(t) \equiv \left(\frac{r(t) - \mu_j(t)}{\sigma_{xj}} \right) dt + dZ_j(t).$$

For the subsequent analysis, we define the *detrended state variables* $x_j(t)$ as

$$\frac{dx_j(t)}{x_j(t)} \equiv \frac{dX_j(t)}{X_j(t)} - \mu_j(t)dt = \sigma_{xj} d\tilde{Z}_j(t). \quad (6)$$

Under the martingale measure \mathcal{Q} , it evolves as

$$\frac{dx_j(t)}{x_j(t)} = [r(t) - \mu_j(t)]dt + \sigma_{xj} dZ_j(t). \quad (7)$$

2.4 Deposit Growth Model

For each bank, the FDIC insurance guarantee covers the insured bank deposits. Depositors are insured up to \$100,000 of their deposits per ownership category. For example, suppose a bank customer has an account in their name of \$105,000 and a joint account with a spouse with a balance of \$280,000. The individual account is insured up to \$100,000 and the individual's portion of the joint account is insured up to \$100,000. Uninsured deposits for this individual would be \$5,000 of the individual account and \$40,000 from the joint account. For more information on deposit insurance coverage, see www.fdic.gov. If the insured bank defaults, the FDIC pays the insured depositors and stands in their place as a claimant in the receivership. Here we have provided the most conceptually simple example, a payout, to describe the resolution of a bank. The FDIC resolves banks in the least costly manner, which typically involves selling some of the assets and liabilities to an acquirer, otherwise known as a purchase and assumption transaction. Although the FDIC covers insured deposits, we chose to model the evolution of total deposits, primarily due to data limitations. Insured deposits were not reported quarterly for much of our sample period, and the numbers that are reported are estimates of insured deposits. We

assume the total deposits of bank i at time t are given by

$$D_t^i \equiv D^i(t, Y^i(t), V(t)), \quad (8)$$

where $D_0^i = D^i(0, Y_j^i(0), V_k(0))$ are the observed balances at time 0. The deposit balance evolution depends on the variables $Y^i(t), V(t)$, which are known to the banks and the regulators, but perhaps not to the market.

2.5 Bank Default Model

Let τ_i be the random default time at bank i , and denote its point process by $N_i(t) \equiv \mathbf{1}\{\tau_i \leq t\}$.

2.5.1 Default Intensity Process

Following Lando (1998), we assume that the default point process for bank i , $N_i(t)$, follows a nonhomogeneous (nonstationary) Poisson process with intensity $\lambda_t^i = \lambda^i(t, Y^i(t), V(t))$ under the statistical measure \mathcal{P} . We assume that the default processes are independent across banks.

In general, this intensity process is different under the martingale measure \mathcal{Q} . Under an equivalent change of measure, this default intensity can be written as

$$\kappa_t \cdot \lambda^i(t, Y^i(t), V(t)),$$

where κ_t is a suitably bounded and integrable stochastic process, adapted to the filtration generated by $Y^i(t), V(t)$ (see Jarrow, Lando, Yu 2005). This is the intensity process used for valuation. If, after conditioning upon the state variables $Y^i(t), V(t)$, default risk is idiosyncratic, then Jarrow, Lando, Yu (2005) show that $\kappa_t = 1$.

2.5.2 The Loss Rate Process

If default occurs, the loss to the insurance fund as a percent of the banks deposits is assumed to be given by

$$\delta_t^i \equiv \delta^i(t, Y^i(t), V(t))$$

at the time of default t . Note that this loss rate process depends on the same set of state variables as the default intensity process and the deposit growth model.

2.6 Risk Measures and Valuation

This section discusses risk measures and valuation of the FDIC insurance guarantees.

For bank i , the loss faced by the FDIC at some future date T is given by

$$\delta_{\tau_i}^i D_{\tau_i}^i e^{+\int_{\tau_i}^T r_s ds} \mathbf{1}\{\tau_i \leq T\}. \quad (9)$$

If default occurs before time T , then the FDIC incurs the losses $\delta_{\tau_i}^i D_{\tau_i}^i$, future valued to time T using the spot rate of interest. If default does not occur, then this is zero. The entire FDIC insurance funds losses are the aggregate losses across all banks:

$$L_T = \sum_{i=1}^I \delta_{\tau_i}^i D_{\tau_i}^i e^{+\int_{\tau_i}^T r_s ds} \mathbf{1}\{\tau_i \leq T\}. \quad (10)$$

Given the stochastic processes for the forward rate process and state variables, the distribution for the losses L_T is completely determined by expression (10). Due to the dimension of the problem, a Monte Carlo simulation will be used to compute various risk measures and values.

2.6.1 Loss Risk Measures

Given the losses as quantified in expression (10), we can compute the loss distribution for the FDIC fund at any time T , i.e.,

$$\mathcal{P}(L_T \leq k) \text{ for any } k \geq 0. \quad (11)$$

One might also be interested in the α -quantile of this distribution, defined as

$$k_\alpha = \inf\{k : \mathcal{P}(L_T \leq k) \geq \alpha\}. \quad (12)$$

2.6.2 Present Value of Losses

The present value of this loss due to bank i is

$$E_t^{\mathcal{Q}} \left[\delta_{\tau_i}^i D_{\tau_i}^i e^{-\int_t^{\tau_i} r_s ds} \mathbf{1}\{\tau_i \leq T\} \right], \quad (13)$$

where $E_t^{\mathcal{Q}}[\cdot]$ corresponds to conditional expectation under the martingale measure \mathcal{Q} using the information up to time t . This is the cost of the FDIC insurance guarantee for bank i . Analogously, one can compute the present value of the insurance proceeds to determine whether or not FDIC insurance is properly priced; see Duffie, Jarrow, Purnanandam, and Yang (2003).

The present value (PV) of the losses to the entire FDIC insurance fund over the time interval $[0, T]$ is given by

$$PV(L_T) = \sum_{i=1}^I E_t^{\mathcal{Q}} \left[\delta_{\tau_i}^i D_{\tau_i}^i e^{-\int_t^{\tau_i} r_s ds} \mathbf{1}\{\tau_i \leq T\} \right]. \quad (14)$$

We estimate expression (14) using Monte Carlo simulation.

3 SIMULATION MODEL

This section presents the simulation model used to evaluate losses to the FDIC insurance funds. To compute the risk

measures and present values of the FDIC insurance losses using Monte Carlo simulation, we must first simulate the time series processes for $Y_j^i(t)$, $V_k(t)$ for all j, k, i . Unfortunately, this direct simulation has two problems. One, the complexity of the default, loss, and deposit growth models (see the subsequent sections) makes the direct simulation of these models problematic. Second, even if this were not the case, the dimension of a direct simulation would be too large given that the number of banks (I) is approximately 9,000 (the dimension of the problem is $(N_y \times N_v \times I)$). To make the simulation tractable, we need to reduce the dimension of this problem. In addition, to avoid the need to estimate the market prices of risk associated with the state variable processes $V_k(t)$, we will use the traded state variables $X_k(t)$ instead of the local- and macro-variable indices $V_k(t)$.

3.1 Projection to Lower Dimensions

For simulation, we use the traded asset prices $X_j(t)$, and only a *static* subset of the bank's characteristics, denoted by

$$\bar{Y}^i \equiv \{\bar{Y}_j^i, j = 1, \dots, \bar{N}_y\},$$

which includes characteristics like the bank's geographical location.

The simulated insured deposit growth process is given by its conditional expectation, given the reduced information set, i.e.,

$$\widehat{D}_t^i \equiv E^{\mathcal{P}} \left[D^i(t, Y^i(t), V(t)) \middle| \bar{Y}^i, X(t) \right], \quad (15)$$

with $\widehat{D}_0^i = D_0^i$. Using the strong Markov property of a diffusion process, we can write this as

$$\widehat{D}_t^i \equiv \widehat{D}^i(t, \bar{Y}^i, X(t)), \quad (16)$$

where $X_j(t)$ follows the process in (5) under the statistical measure \mathcal{P} , and where $X_j(t)$ follows the process in (4) under the martingale measure \mathcal{Q} . We compute valuation using expression (16) under the martingale measure \mathcal{Q} .

Analogous to the deposit growth model, we use the following intensity process in the simulation:

$$\widehat{\lambda}_t^i \equiv E^{\mathcal{P}} \left[\lambda^i(t, Y^i(t), V(t)) \middle| \bar{Y}^i, X(t) \right] \quad (17)$$

$$= \widehat{\lambda}^i(t, \bar{Y}^i, X(t)).$$

This is the intensity process used for valuation. Under the martingale measure \mathcal{Q} , as previously discussed, this default

intensity can be written as

$$\kappa_t \cdot \widehat{\lambda}^i(t, \bar{Y}^i, X(t)), \tag{18}$$

where κ_t is a suitably bounded and integrable stochastic process, adapted to the filtration generated by $X_k(t)$ for all k . This is the intensity process used for valuation. In the simulation, we set $\kappa_t = \kappa$, a constant. Furthermore, we assume that default risk is conditionally diversifiable, as in Jarrow, Lando, Yu (2005), and set $\kappa = 1$. In subsequent research, we will explore the impact of utilizing market determined estimates for $\kappa \neq 1$.

Following a similar line of reasoning, the loss rate process is

$$\begin{aligned} \widehat{\delta}_t^i &\equiv E^{\mathcal{P}} \left[\delta^i(t, Y^i(t), V(t)) \middle| \bar{Y}^i, X(t) \right] \\ &= \widehat{\delta}^i(t, \bar{Y}^i, X(t)), \end{aligned} \tag{19}$$

where $\widehat{\delta}_0^i = \delta_0^i$ is the observed loss rate on deposits defaulting at time 0, $X_j(t)$ follows the process in (5) under the statistical measure \mathcal{P} , and $X_j(t)$ follows the process (4) under the martingale measure \mathcal{Q} .

Although the lower-dimensional projection described above is used to facilitate computation, this projection has an economic interpretation. This formulation is consistent with only bank management and regulators observing the bank specific characteristics $Y_j^i(t)$ for all i, j , perhaps because this is proprietary information. In contrast, the market sees only a static subset of these bank characteristics represented by the variables $\bar{Y}_j^i, j = 1, \dots, \bar{N}_y$. Consequently, given the market's reduced information set, the deposit growth and bankruptcy processes are given by expressions (16), (17), and (19). Under this interpretation, these processes are the correct ones to use for market valuation of the FDIC insurance guarantees (see Duffie and Lando 2001; Cetin et al. 2004).

3.2 Simulation Model Algorithm

To compute the loss distribution to the FDIC insurance fund (11) or its present value (14), we need to be able to simulate the forward rate process, the traded state variables, and the bankruptcy processes. The simulation algorithm is now described under the martingale measure \mathcal{Q} . The analogous simulation can take place under the statistical measure, with the appropriate change in drift.

- Step 1. Discretize the time interval $[0, T]$ as $t = 0, 1, 2, \dots, T$. Generate a sample path for $W_1(t), \dots, W_K(t), Z_1(t), \dots, Z_{N_x}(t)$, over this discretization, called a scenario. Note that these vari-

ables are multivariate normally distributed with covariance matrix given in Figure 1.

- Step 2. Given a sample path for $W_1(t), \dots, W_K(t), Z_1(t), \dots, Z_{N_x}(t)$, use expressions (1) and (4) to obtain a sample path for the forward rates $f(t, T)$ and traded state variables $X_j(t)$. In the subsequent simulation, we actually use the detrended trade state variables $x_j(t)$ and add the spot rate of interest r_t .
- Step 3. Use the sample paths for the forward rates and state variables to obtain realizations of the deposits $\widehat{D}^i(t, \bar{Y}^i, X(t))$, the intensity process $\kappa \cdot \widehat{\lambda}^i(t, \bar{Y}^i, X(t))$, and the loss rate $\widehat{\delta}^i(t, \bar{Y}^i, X(t))$.
- Step 4. For each bank and the given scenario, determine if default occurs during the horizon, and if so, the actual default time, i.e., compute $\mathbf{1}\{\tau_i \leq T\}$, and then τ_i if the indicator function is 1. We describe how to carry out this default process simulation in detail in the next subsection. Given a failure, the loss rate process $\widehat{\delta}^i(t, \bar{Y}^i, X(t))$ then applies to the deposits $\widehat{D}^i(t, \bar{Y}^i, X(t))$ to determine the loss to the insurance funds.
- Step 5. For this scenario, compute L_T in (10).
- Step 6. Repeat steps 1–5 for M replications. From this collection of scenarios, the risk measures and values can be computed. For example, expression (14) is estimated by its sample mean

$$\overline{PV}_M(L_T) \equiv \frac{1}{M} \sum_{\omega=1}^M L_T(\omega) e^{-\int_0^T r_s(\omega) ds}, \tag{20}$$

where ω is a particular replication (sample path).

3.3 Simulating the Default Processes

We provide the details as to how to carry out step 4. The nonhomogeneous (nonstationary) Poisson process with rate (intensity) function $\lambda(t)$ can be simulated in two main ways (cf. Law and Kelton 2000):

1. Inverse Transform Method. In general, this method requires integration of the rate function $\lambda(t)$, and then performing an inversion. In our setting, the general form is as follows:
Generate I independent unit exponentially distributed random variables $E^i, i = 1, \dots, I$.
Compute

$$\begin{aligned} \tau_i &\equiv \inf\{s \in [0, T] : \\ &\int_0^s \kappa \cdot \widehat{\lambda}^i(t, \bar{Y}^i, X(t)) dt \geq E^i\}. \end{aligned} \tag{21}$$

$$\begin{matrix}
 dW_1(t) \\
 \vdots \\
 \vdots \\
 dW_K(t) \\
 dZ_1(t) \\
 \vdots \\
 \vdots \\
 dZ_{N_x}(t)
 \end{matrix}
 \begin{bmatrix}
 dt & 0 & \dots & 0 & \eta_{11}dt & \eta_{12}dt & \dots & \eta_{1N_x}dt \\
 0 & dt & \ddots & \vdots & \eta_{21}dt & \ddots & & \vdots \\
 \vdots & \vdots & \ddots & 0 & \vdots & & \ddots & \vdots \\
 0 & \dots & 0 & dt & \eta_{K1}dt & \dots & \dots & \eta_{KN_x}dt \\
 \eta_{11}dt & \eta_{12}dt & \dots & \eta_{1N_x}dt & dt & \rho_{12}dt & \dots & \rho_{1N_x}dt \\
 \eta_{21}dt & \ddots & & \vdots & \rho_{21}dt & dt & & \\
 \vdots & \vdots & & \vdots & \vdots & & \ddots & \\
 \eta_{K1}dt & \dots & \dots & \eta_{KN_x}dt & \rho_{1N_x}dt & \dots & \dots & dt
 \end{bmatrix}$$

Figure 1: Covariance Matrix for $W(t), Z(t)$

2. Acceptance-Rejection Method. This method is also known as “thinning,” and requires simulation of a homogeneous (stationary) Poisson process of sufficiently large rate $\lambda^* \geq \lambda(t) \forall t$. Event epochs on a sample path τ_1, τ_2, \dots are “accepted” (kept) or “rejected” (deleted) according to a Bernoulli coin flip with probability $\lambda(\tau_i)/\lambda^*$.

The advantages of the Acceptance-Rejection Method are its simplicity and easy application for complicated rate functions, because it avoids the integration and inversion operations. Aside from being potentially non-trivial, these operations may also require additional storage. The chief disadvantage of the Acceptance-Rejection Method is that in general it will require a larger number of random numbers to be generated.

A special case of the Acceptance-Rejection Method is a discrete-time Bernoulli (coin flipping) where the discrete time increments are sufficiently small compared to the expected number of Poisson events in the increment (i.e., the expected number should be well under 1). In other words, it exploits the defining properties that the nonhomogeneous Poisson process still retains:

- (i) Poisson distribution;
- (ii) independent increments.

This special case of the Acceptance-Rejection Method is extremely simple, but a disadvantage of is that one random number (for each failure process simulated) must be generated every period.

For the case that we consider, where the intensity process is constant between quarters, the simulation methods simplify. Assume that over periods of length Δ , denoted by I_i , that $\lambda(t)$ is constant with rate λ_i , and that $P(N(I_i) > 1) \approx 0 \forall i$, where $N(I)$ is the number of Poisson events over the interval I . The Poisson assumption then implies $P(N(I_i) = 1) \approx \lambda_i \Delta = P_i$. This P_i is the conditional failure probability in a quarter that is provided by the empirical estimation of λ_i^j . The estimation of λ_i^j is described above in section 4.0.3. Thus, the failure process can be simulated as a discrete-time

Bernoulli process where failure occurs in the period with probability λ_i^j . The actual failure time can be generated randomly (i.e. from a uniform distribution) over the period.

Note that the assumption of $P(N(I) > 1) \approx 0$ is necessary only to use the Bernoulli random variable. More generally, one could use a Poisson random variable with parameter $\lambda \Delta$ (or $\gamma(t) = \int_0^\Delta \lambda(t) dt$ for the non-constant case). Under the assumption that the rate is constant over the period, the Poisson event (unordered) epochs would then be randomly distributed over the period using the property of a homogeneous Poisson process that given $N(I) = n$, the distribution of unordered epochs are independently and identically distributed $U(0, \Delta)$ (replace by cumulative density function, $\gamma(\cdot)/\gamma(\Delta)$ for the non-constant case). This case would not apply in the context of modeling bank failures since we assume that a bank can only fail once over the horizon (or at least in a quarter).

However, it turns out that for the constant-over-an-interval case, the expression (21) representing the Inverse Transform Method also simplifies considerably, since integrals become rectangular areas. Again, let P_i denote the conditional failure probability in period i as above. Let $E \sim \text{exp}(1)$. Then,

$$Q = \text{failure period} = \min \left\{ q : \sum_{i=1}^q P_i \geq E \right\},$$

$$\tau = \text{failure time} = (Q - 1)\Delta + \left(X - \sum_{i=1}^{Q-1} P_i \right) \frac{\Delta}{P_Q}.$$

The corresponding algorithm is as follows:

- Set $Sum := 0$ and $Q := 1$.
- Generate a random variable $E \sim \text{exp}(1)$.
- Loop until $Q > \#$ quarters to simulate:
If $Sum + P_Q \geq E$, then return failure time

$$\tau = (Q - 1 + (X - Sum)/P_Q)\Delta;$$

else $Sum := Sum + P_Q; Q := Q + 1$.

This method requires only a single random number per failure process simulated, and there is no complicated integration or inversion required, nor extra storage of sample path quantities, just one simple counter sum (corresponding to the cumulative integral).

In numerical test cases with 8,532 banks, 19 state variables, a 4-factor HJM interest rate model and a 10-year horizon, we found that the difference in using the two different failure processes is indistinguishable for a single replication with Δt of 1 week (1/52), because the forward rate process simulation dominates the failure rate process in terms of computational burden. However, for 200 replications with Δt of a quarter (1/4), we found a reduction in computation time of nearly 50%, as in this case, the forward rate process no longer dominates the simulation nearly as much.

4 PARAMETER ESTIMATION

This section presents the parameter estimation procedures and results for the underlying stochastic processes, including the forward rates, the stock price indices, and the deposit growth and loss rate models.

To estimate the forward rate process given in expression (1), we employ a principal component analysis as discussed in Jarrow (2002). Given a time series of discretized forward rate curves $\{f(t, T_1), f(t, T_2), \dots, f(t, T_{N_r})\}_{t=1}^m$, where N_r is the number of discrete forward rates observed, the interval between sequential time observations is Δ , and m is the number of observations. Then, percentage changes are computed $\left\{\frac{f(t+\Delta, T_1)-f(t, T_1)}{f(t, T_1)}, \dots, \frac{f(t+\Delta, T_{N_r})-f(t, T_{N_r})}{f(t, T_{N_r})}\right\}_{t=1}^m$. From the percentage changes, the $N_r \times N_r$ covariance matrix (from the different maturity forward rates) is computed, and its eigenvalue/eigenvector decomposition calculated. The normalized eigenvectors give the discretized volatility vectors

$$\{\sigma_{rj}(T_1)\sqrt{\Delta}, \dots, \sigma_{rj}(T_{N_r})\sqrt{\Delta}\}, j = 1, \dots, N_r.$$

To compute the parameters of expression (4), we use the quadratic variation, which is invariant under a change of equivalent probability measures. Given is a time series of $\{X_i(t)\}_{t=1}^m$ where the interval between sequential time observations is Δ , a quarter, and m is the number of observations. Define $\Delta X_i(t) \equiv [X_i(t + \Delta) - X_i(t)]$. This could be done using log differences instead of returns or using the detrended variables $x_j(t)$ instead of $X_j(t)$. We compute

$$\frac{1}{m} \sum_{t=1}^m \left(\frac{\Delta X_i(t)}{X_i(t)} \right)^2, \tag{22}$$

giving an estimate of $\sigma_{xi}^2 \Delta$. Next we calculate

$$\frac{1}{m} \sum_{t=1}^m \left(\frac{\Delta X_j(t)}{X_j(t)} \frac{\Delta X_i(t)}{X_i(t)} \right), \tag{23}$$

giving an estimate of $\sigma_{xj} \sigma_{xi} \rho_{ji} \Delta$. To obtain the correlation between the forward rates, the house price index, and the bank stock price index η_{ji} for $j = 1, \dots, K$, we compute

$$\frac{1}{m} \sum_{t=1}^m \left(\frac{\Delta f(t, T_k)}{f(t, T_k)} \frac{\Delta X_i(t)}{X_i(t)} \right), \tag{24}$$

giving an estimate of

$$\sum_{j=1}^K \sigma_{rj}(T_k) \sigma_{xi} \eta_{ji} \Delta.$$

This is computed for $k = 1, \dots, K$ for distinct T_1, \dots, T_K , yielding K equations in K unknowns $\{\eta_{1i}, \dots, \eta_{Ki}\}$. The estimates of $\sigma_{rj}(T_k)$ come from the forward rate principal components analysis discussed in the previous section. Solving this system gives the estimates. This is done for all i . For this estimation, we set $K = 4$, and we use the four forward rate maturities $T_1 = 1/2, T_2 = 1, T_3 = 3, T_4 = 5$.

Empirically, $\{X_i(t)\}_{t=1}^m$ includes a series of house price indices and a series of bank price indices. House prices are measured for the nine census regions by the Office of Federal Housing Enterprise Oversight (OFHEO) indices, available quarterly since 1975. Comparable bank price indices were compiled from Center for Research in Security Prices (CRSP) data. (Among other things, CRSP compiles daily data on stock trades.) A total of 267 stocks was used. The banks were divided into ten groups: a set of money center banks and one set for each of the census regions. An equal weighted index was created for each group of banks.

For the empirical default intensity, we estimated a standard bank failure model. A pooled time series, cross-sectional model with a logistic specification was used. The sample included all banks and thrifts with the necessary data between December 1984 and December 2002. The model described above was used to generate estimates of the probability of failure for December 2002.

Deposit growth was measured on a year-over-year basis to eliminate seasonal effects. Data was taken from the period March 1986 to March 2003.

The estimate of loss for individual institutions is calculated using a model similar to that used to estimate losses for the Least Cost Test. This model described here is used only when the FDIC does not have enough time to enter the bank and value the assets on site. The model uses loss rates for six types of assets: consumer loans, commercial loans, securities, mortgages, owned real estate (ORE) and

other assets. The model assumes the following about losses on asset categories. First, the loss rates on cash and federal funds are zero. Second, there is a category of assets including intangible assets that experience 100% losses. Third, fixed assets such as bank premises will have the same loss rate as that experienced on ORE. The loss rates for the asset types are calculated using a sample of 369 failures from the 1990 to 2002 period. The loss rates are applied to the assets on the balance sheet, and, given the liability structure, we calculated a loss for each institution.

5 SIMULATION RESULTS

5.1 Numerical Results

Table 1 contains the simulation results for the one-, five-, and ten-year horizons. These results are based on 10,000 replications of the model with starting values given by March 2005 data. The table shows the distribution of the number of bank failures, the total deposits in the failed banks, and the current value of the losses to the FDIC. Total losses are discounted as in expression (20) from the estimated failure time to June 2005. Losses are discounted to June 2005 instead of March 2005, because the data for March are available with approximately a two month lag. Decisions based on the forecasted losses would be made at the end of June 2005. Discounted losses are used so they are comparable to the dollar value of the FDIC capitalization on June 2005.

The results for the one-year horizon indicate that the FDIC should expect approximately five failures between July 2005 and June 2006, and the expected total deposits for the failed banks should be on the order of \$5.2 billion. The expected loss to the insurance funds is \$218 million, and the median loss is approximately \$49 million.

The distribution presented in Table 1 can be used to construct various risk measures. For example, the value-at-risk measure (VaR) over a one-year horizon using a 99% probability is the 99th percentile loss, or \$2.3 billion. The market value of losses to the FDIC insurance fund over the one-year horizon as given in expression (20), valued as if traded on public capital markets, is given by the mean of the loss distribution, or \$218 million.

Table 1 provides the same estimates over longer horizons. For example, the 99% VaR is \$27 billion over five years, \$109 billion over ten years. The market value of the losses amount to \$2.2 billion over five years, and \$21 billion over ten years.

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Table 1: Simulation Results: March 2005

		Number of Failures	Total Deposits in Failed Banks	Total Losses to the FDIC
			(in millions of \$)	
1 Yr. Horizon	Mean	4.66	5,208	218
	Standard Deviation	2.36	38,088	1,036
	Median	4	390	49
	95th Percentile	9	8,394	652
	99th Percentile	12	135,854	2,274
5 Yr. Horizon	Mean	27.26	41,762	2,175
	Standard Deviation	19.74	139,009	4,873
	Median	23	5,584	991
	95th Percentile	52	230,242	6,111
	99th Percentile	88	776,767	27,228
10 Yr. Horizon	Mean	67.49	174,927	20,810
	Standard Deviation	67.79	445,331	505,559
	Median	48	36,093	3,920
	95th Percentile	160	953,306	40,240
	99th Percentile	383	2,407,438	108,719

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