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SIMULATING AND DESIGNING THE STOCHASTIC INBOUND INVENTORY ROUTING PROBLEM

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ABSTRACT

We consider the situation where parts from suppliers have to be picked up by a capacitated vehicle for an assembly plant that faces stochastic demand. We propose a two-phase heuristic to calculate "good" base stock policies for the stochastic inbound inventory routing problem. In this paper, we calculate multiple performance measures for the results obtained from the analytical model. The results show that the maximum average utilization of the vehicle is a significant factor in determining our fill rate. We consider several test cases to substantiate the results. The interactions between the performance measures for different situations are also discussed.

1 INTRODUCTION

Coordinating inventory and transportation decisions is a major issue in many assembly plants. Managing the inbound inventory across multiple suppliers and pickup routes is a key issue in the supply process when the demands for the different products are stochastic. In such a scenario, the stochastic inbound inventory routing problem strikes a balance between the transportation and inventory costs. As a result, the overall system efficiency improves and the service level to the assembly plant increases.

The objective of our problem is to calculate reorder intervals and order up-to levels for parts from each supplier such that the total cost is minimized. The total cost includes the inventory carrying cost, transportation cost and backlog costs. The inventory routing problem is a complex problem. It involves solving the NP-hard vehicle routing problem and optimizing a constrained, nonlinear problem. The routing component of the proposed heuristic is based on the location based heuristic of Bramel and Simchi-Levi 1995 for the inventory routing problem.

A delivery vehicle leaves the assembly plant and reaches a supplier. There it picks up a certain quantity of parts from the supplier. Once the delivery vehicle has picked up parts, the vehicle may visit other suppliers or it may return back to the assembly plant depending on the route the vehicle is traversing. Hence, a solution of the stochastic inventory routing problem is a set of routes, each assigned to one or more delivery vehicles, the cycle time for the routes and the order-up-to levels for parts from each supplier. The delivery vehicles travel those routes to pick up parts from all the suppliers that are used to satisfy the demand for the parts at the assembly plant.

In this paper, we present a simulation model for the stochastic inbound inventory routing problem. The purpose of the model is to assess the effectiveness of the heuristic solution procedure developed to obtain "good" solutions to the problem. The solution from the heuristic provides a set of routes. For the sake of simplicity in the simulation model, we consider only one route with three suppliers on it. The model is simulated with reorder intervals and order-up-to levels obtained from the heuristic solution. The different performance measures are calculated and the efficiency of the algorithm is established.

2 PROBLEM DEFINITION

We consider an assembly plant that has suppliers close by the assembly plant and others at distant locations. Delivery vehicles are sent out from the assembly plant to pick up parts from the suppliers. The delivery vehicles constitute a homogenous fleet of vehicles with limited capacity. The pick up routes are fixed and are run at fixed intervals or cycle times. When stockouts occur they are completely backlogged, and there are an ample number of vehicles to cover these routes. The problem is solved to find a trade-off between transportation costs, cycle inventory and safety inventory carrying costs at the assembly plant and stock-out costs at the assembly plant. In our model, we assume stock-outs are completely backlogged. The inventory control in this system uses base-stock (R, S) policies.

The uncertainty in the demand at the assembly plant and the fixed cycle times cause the pickup quantities to vary. Before trucks leave the assembly plant, the inventory position of the parts from a supplier is checked and an order is placed. The orders on a route are constrained by the capacity of the vehicle that is assigned to that route. The pickup quantity from each supplier is calculated such that the inventory position is raised as close as possible to the order-up-to level.

3 SOLUTION PROCEDURE

In our problem, we make the routing decisions first assuming an initial set of inventory decisions. Then, we update the inventory decisions based upon the routing decisions that are made. This iterative approach adds value to the integrated supply chain by achieving a balance between inventory costs and transportation costs.

We solve the first phase of the problem as a routing problem with demands approximated by their average deterministic values. We follow a fixed partition policy where suppliers are grouped using an extension of the location based heuristic of Bramel and Simchi-Levi 1995. We model the stochastic nature of the demand and the supply process as a finite stage Markov chain. At this stage we base our solution procedure on a modified newsvendor problem. The result of this procedure is a set of base stock policies for the parts from each supplier.

3.1 Outline of Solution Procedure

1. Phase I: Calculating reorder intervals
   (a) Group suppliers on routes
   (b) Sequence suppliers on routes
   (c) Calculate route frequencies and define cycle times for each supplier
2. Phase II: Calculating order-up-to levels
   (a) Compute order-up-to levels for the capacitated base-stock policies

3.2 Calculating Reorder Intervals

To calculate the reorder intervals, we base our solution procedure on the Location Based Heuristic (LBH) proposed by Bramel and Simchi-Levi 1995. The LBH is formulated as a capacitated facility location problem, solved, and then transformed back to give a solution for the routing problem. We calculate the cost of connecting the suppliers directly to the assembly plant

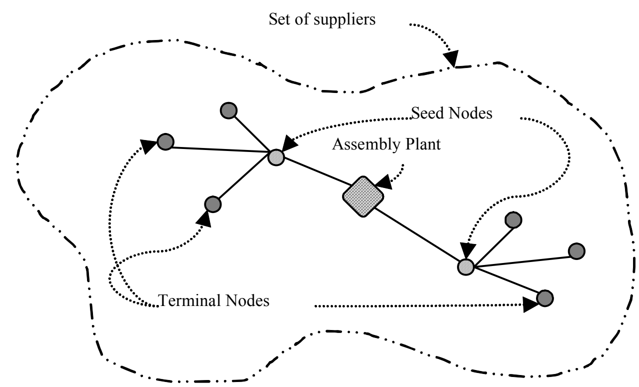

Figure 1: Calculating Costs

The cost includes the transportation cost and the cycle inventory carrying cost. We then calculate the additional cost of adding a supplier to another route. With these costs, we solve a binary, integer program to obtain the best grouping of suppliers.

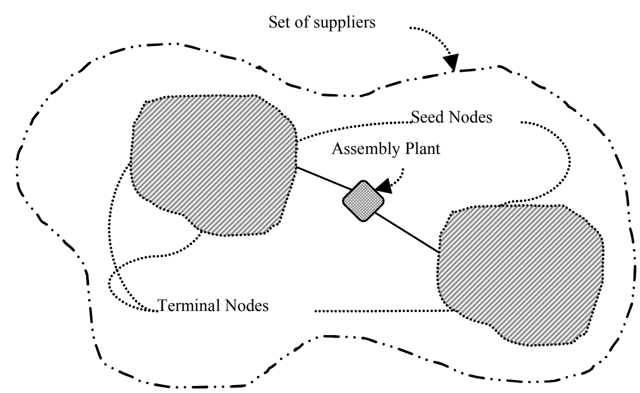

Figure 2: Grouping Suppliers

Once the suppliers are grouped into their respective routes, we solve a traveling salesman problem for each route. The traveling salesman problem is used to find the sequencing of the suppliers on each route. With the current grouping of suppliers on each route, we update the cycle time of each route and recalculate the transportation and inventory costs associated with suppliers on each route. The cycle time of each route gives us the reorder interval for suppliers on the route. Thus, the reorder interval for each supplier is calculated taking into consideration the transportation and the cycle inventory carrying costs associated with its route.

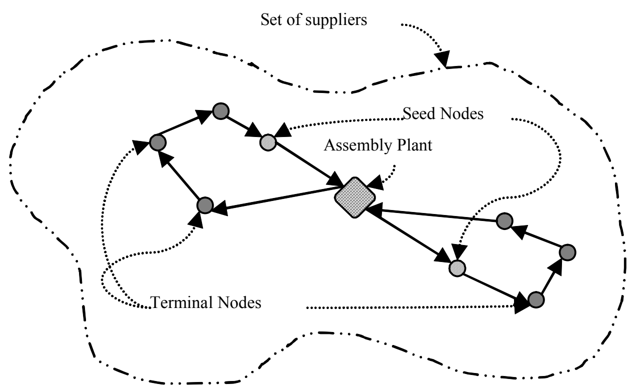

Figure 3: Sequencing Suppliers on a Route

3.3 Calculating Order-Up-To Levels

The calculation of optimal order-up to levels for the suppliers is based on Moran's dam model. The optimal order-up-to levels are calculated by finding a tradeoff between the safety inventory carrying cost and the stock-out cost. First, a shortfall distribution is calculated for each supplier. The shortfall is the quantity that we are not able to pick up from the supplier due to limited vehicle capacity. The shortfalls are modeled as a truncated Markov chain and the stationary distribution of shortfalls is calculated. We calculate the convolution of the demand distribution and the shortfall distribution. The critical ratio is calculated from the inventory carrying rate and the stock-out cost rate. The value of the convolution distribution that corresponds to the critical value gives the optimal order. The critical ratios and shortfall distributions are different for each supplier and determine the optimal order-up-to level for each supplier.

4 SIMULATION MODEL

The simulation package ProModel is used to develop a discrete event simulation model of the inbound logistics for the assembly plant. The model gives information on number of stock-out occurrences, fill rates, vehicle utilization, inventory stored and the costs associated with the logistics.

The model simulates multiple one day scenarios where there are three suppliers. The assumptions in the model make the demand process independent of the supply process. The stock-outs due to parts from one supplier do not affect the demand process and hence the other suppliers. Thus, the simulation model can look at each route independently. The following paragraphs detail the logic of the simulation model.

4.1 Data Input

The simulation model uses the following input data:

1. Demand distribution at the assembly plant,
2. Reorder interval for each supplier,
3. Order-up-to levels for parts from each supplier,
4. Inventory carrying cost of parts from each supplier,
5. Stockout cost for parts from each supplier,
6. Capacity of the delivery vehicles,
7. Allocated vehicle capacity for parts from each supplier, and
8. Fixed cost for a vehicle traveling a route.

The demand at the assembly plant is assumed to follow a Poisson process. Hence the interarrival time between demands follows an exponential distribution. The loading and unloading time at the suppliers is included in the travel time to the supplier.

4.2 Inventory Processing

The logic used to updated inventory when a vehicle delivers parts and when demand occurs are shown in Figures 4 and 5, respectively.

S_i = Order-up-to level for parts from supplier i
IP_i = Inventory position of parts from supplier i
NI_i = Net Inventory of parts from supplier i
OH_i = On-hand Inventory of parts from supplier i
Q_i = Allocated vehicle capacity for parts from supplier i
r_i = Numbers of parts from supplier i used in one unit of the end product
sk_i = Number of parts from supplier i that are backordered at the assembly plant

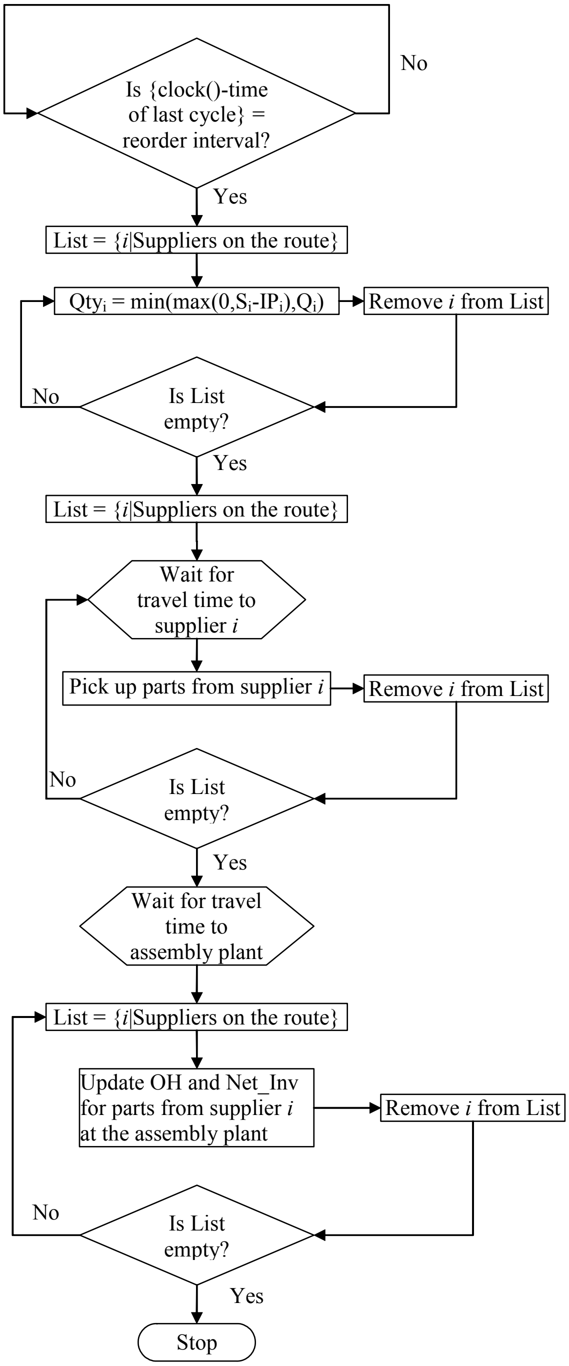

Figure 4: Order Processing Logic

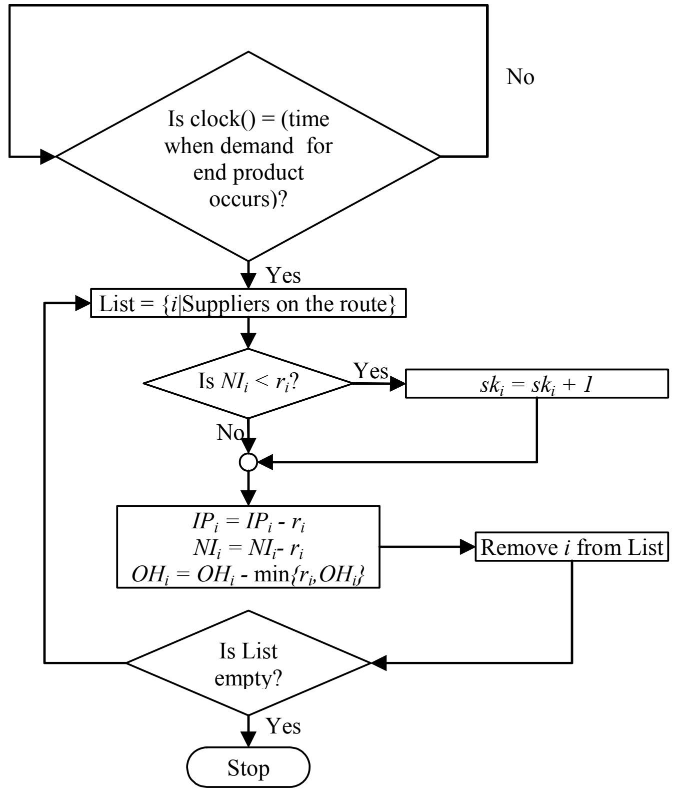

Figure 5: Demand Processing Logic

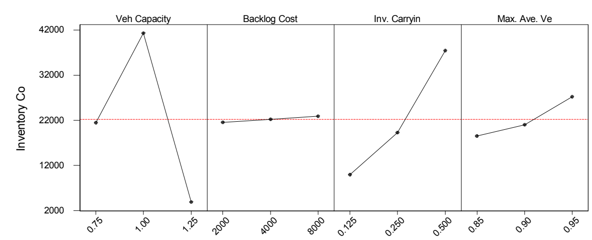

Figure 9: Main Effects Plot for Daily Inventory Carrying Cost

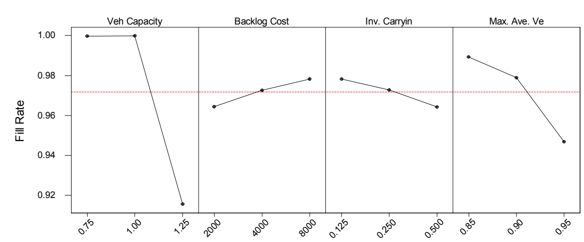

Figure 6: Main Effects Plot for Fill Rate

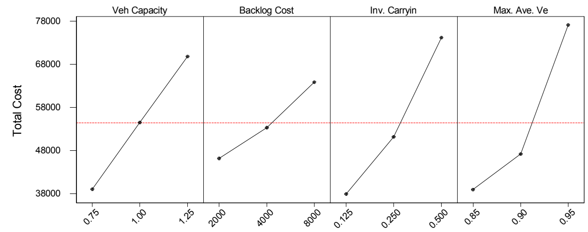

Figure 7: Main Effects Plot for Daily Total Cost

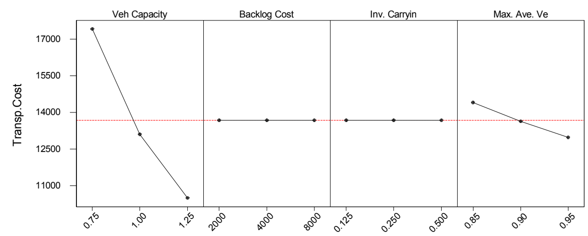

Figure 8: Main Effects Plot for Daily Transportation Cost

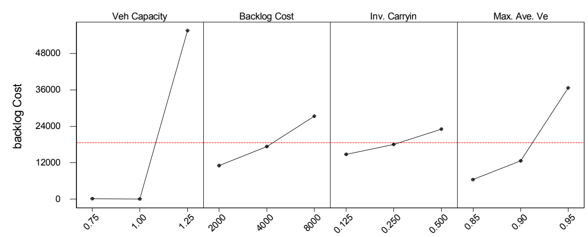

Figure 10: Main Effects Plot for Daily Backlog Cost

5 RESULT ANALYSIS AND COMPARISONS

The model is executed with different values of inventory carrying cost, backlog cost, vehicle capacity and maximum average vehicle utilization. Maximum average vehicle utilization is the limit placed on the average vehicle utilization for the route. The excess vehicle capacity is a sort of safety capacity that is needed when demand at the plant is high. We conducted 81 experiments using three different levels for each of the 4 factors: inventory carrying cost (12.5%, 25% and 50%), backlog cost per unit of assembled product ($2000, $4000 and $8000), vehicle capacity in truckloads (0.75, 1 and 1.25) and maximum average vehicle utilization (85%, 90% and 95%). We conducted 100 runs where each run was 200 days long with an appropriate warm up period. The summary of the results are shown by the main effect plots in Figures 6-10.

In Figure 6, we see that the fill rate increases with increasing backlog cost. This is because we carry more inventory to avoid backlogs. The fill rate also decreases as the inventory carrying cost increases. This occurs because it becomes more expensive to carry inventory and the less inventory we have, the more likely that backlogs will occur. From Figure 7 we see that the daily total cost increases as the inventory carrying rate, backlog cost, vehicle capacity and maximum average vehicle utilization increase. The transportation cost plots shown in Figure 8 show that transportation cost goes down when the vehicle capacity increases due to the reduced number of trips made. The transportation cost also decreases when the maximum average vehicle utilization increases.

The interesting plots of inventory carrying costs and backlog costs are shown in Figures 9 and 10. We see that the inventory carrying cost increases at first with an increase in vehicle capacity but starts decreasing once the vehicle capacity is increased further. The backlog costs are reversed in behavior. With a slight increase in capacity the backlog cost stays at almost zero but when vehicle capacity is increased further, the backlog costs increase drastically. This behavior of the inventory costs is difficult to understand. Further research is needed to understand the cause of these effects. The inventory cost and the backlog cost both increase when the vehicle utilization approaches 100%. This is due to the fact the system cannot clear the outstanding backorders when the vehicle capacity approaches 100%.

6 CONCLUSION

When the same solution from the analytical model is used with a demand distribution that has less variation than an exponential distribution, the results are very comparable. The high variation of the exponential distribution affects the results very much. In terms of future work, we are looking to account for this variation better.

There are a number of obvious extensions of this work. Simulation can be used to determine the order-up-to levels, and these order-up-to levels can be compared with the results from the analytical model. The analytical model must also be modified to account more precisely for the system behavior when demand variation is high.

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