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INFORMATION SYNCHRONIZATION EFFECTS ON THE STABILITY OF COLLABORATIVE SUPPLY CHAIN

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ABSTRACT

In this paper, the dynamics of a collaborative supply chain have been analyzed using transform techniques. The general conditions for stability of the supply chains are derived and the effects of inter-player sampling intervals are analyzed. System dynamics simulation models of the different members of the supply chain are developed. *Z*-transform techniques are employed to derive the general stability conditions (settings of control parameters that produce stable response). The variation in the supply chain's stability in response to the information synchronization frequency is examined by relating the update frequency to the sampling interval of the underlying difference equations. Existence of instability due to improper parameter selection and improper sampling interval selection is thus confirmed, and guidance for the selection of appropriate parameters to guarantee system stability is presented. Simulations are used to confirm our analysis and help demonstrate the stable or unstable behavior of the supply chain.

1 INTRODUCTION

In current dynamic markets, supply chains as a whole need to respond quickly to changes in customer demand and adapt quickly to changes in technology. This necessitates modeling and analysis of the dynamic behavior of supply chains (Ortega and Lin 2004). Analysis of supply chain dynamics helps reveal periods of inventory build-ups, stock-outs, overtime production and production shutdown, which costs the companies in terms of profits, market position and customer satisfaction. The dynamic behavior or dynamic complexity is said to arise from the interaction between the various system components over time (Sterman 2000). Dynamic systems, characterized by delays, feedbacks and nonlinearities, cannot be adequately captured using mathematical programming techniques such as linear/ non-linear/ stochastic programming. A natural choice to examine the supply chain dynamics is the application of control theoretic and system dynamics simulation techniques. This often involves capturing of the supply chain system using feedback-based structures (Forrester 1961, Edghill and Towill 1990, Sterman 2000) and analysis of the system through the application of control theoretic tools such as block diagram algebra, Bode plots, and functional transformations (John *et al*. 1994, Grubbström and Wikner 1996, Disney and Towill 2002, Disney *et al*. 2003). In this research work, modified causal loop diagrams are used to capture (model) the supply system, which are then analyzed by applying *z*-transform technique (a type of function transformation technique). Function transformation technique maps the system from the time domain to the frequency domain; the advantages of which are summarized below:

- Efficient tool to examine the critical design parameters and identify ranges of parameter values that give good transient response (Ortega and Lin 2004),
- Standard techniques exist to analyze the system performance. Also, the calculations can be computationally very simple (Bissell 1996),
- Closed loop transfer functions of the system can be obtained that enables to gain insight into the stability of the system,
- Appropriate integration of transfer functions with simulation enables additional system analysis (Disney and Towill 2002),
- Transforms can be used to capture the stochastic properties by serving as moment generating functions (Grubbström 1998).

A comprehensive literature review on the use of control theoretic concepts for the dynamic analysis of supply chain systems can be found in Ortega and Lin (2004) and in Disney and Towill (2002). John *et al*. (1994) demonstrated the stabilizing effect of including a supply line component into an inventory and order based production control system, using block diagrams and Laplace transform. White (1999) has showed that simple inventory management systems are analogous to the proportional control in conventional control theory, and has demonstrated the use proportional, integrative and derivative (PID) control algorithms can result in saving of up to 80%. Disney and Towill (2002) studied and presented the conditions for stability of a Vendor Managed Inventory (VMI) supply chain and recommended parameter settings for two feedback loops identified to avoid instability.

2 MODELING THE SUPPLY CHAIN

In this research, a two echelon collaborative supply chain system is considered. The supply chain consists of a Manufacturer and Retailers. In a collaborative supply chain configuration, the supply chain members cooperate with each other, sharing resources and capabilities, and together plan and execute supply chain operations (Lambert *et al*. 1998, Lejeune and Yakova 2005). The supply chain members develop a common set of objectives for a particular business function, the popular one being the inventory function. This is mainly to curb the increased fluctuations in inventory levels and order quantities caused by the bullwhip effect. Vendor managed inventory, a type of collaborative configuration (Lee and Whang 2000) is modeled in this paper. Retailers send the inventory levels and end customer sales data to the Manufacturer. The Manufacturer uses a MIN-MAX inventory policy to determine the quantity of goods to be dispatched.

The models developed in this research improve over the past works (John *et al*. 1994, Disney and Towill 2002) by the explicit representation of the frequency of information update, through the use of a sampling interval, to study its effect on the system stability.

In the following sections, the collaborative inventory management simulation model is defined and then translated into difference equation models which can be readily simulated. Causal Loop Diagrams (CLDs) are used to represent the conceptual feedback structure of systems (Sterman 2000). The CLDs are interpreted as follows:

- In each causal link, the variable at the tail of the arrow is called as the independent variable and the variable at the head of the arrow is called as the dependent variable.
- A positive (+) causal link means that when the independent variable increases (decreases), the dependent variable increases above (decreases below) what would have been if the independent variable did not change.
- A negative (–) causal link means that when the independent variable increases (decreases), the dependent variable decreases below (increases above) what would have been if the independent variable did not change.

2.1 Notation

The notations used for describing the system dynamic simulations models are presented below, where *i* represents the index of the product {1...*N*}, *r* represents the index of the retailer {1...*R*} and t represents the index of the time period {1...*T*}.

	Manufacturer Terms
ψ^M_{ir}	fractional adjustment rate of Goods-In-Transit
φ^M_{ir}	fractional adjustment rate of inventory
FDR^M_{irt}	Forecasted Demand
ρ^M_{ir}	exponential smoothing constant
$GITR^M_{irt}$	Goods-In-Transit
MIN^M_{irt}	MIN level or reorder point
MAX^M_{irt}	MAX level or order-up-to point
τ^M_{ir}	time to change the MIN-MAX levels
$DISR^M_{irt}$	DISpatch order rate
$DRATER^M_{irt}$	product Delivery RATE
LR^M_{irt}	product delivery Leadtime
	Retailer Terms
$RINV^R_{it}$	Retailer INVentory
$RSALES^R_{it}$	end customer Retailer SALES rate

2.2 Description of the Manufacturer Model

The Manufacturer manages the Retailer's inventory as part of their collaborative configuration using vendor managed inventory strategy (Figure 1a). The Manufacturer obtains the current inventory levels (*RINV*) and the end customer sales data (*RSALES*) from the Retailers; and uses a MIN-MAX inventory control policy to determine the dispatch quantities to the Retailer. The underlying equations of the models, relating the different variables shown in Figure 1, are described as follows.

The Manufacturer is assumed to forecast demand (*FDR*) of its products based on a first order exponential smoothing of the customer sales rate (*RSALES*), with a smoothing constant ρ (Figure 1a, top right). For the purpose of this research, it is desirable to minimize the impact of forecasting methods on the system dynamics. Based on pilot experiments by Venkateswaran and Son (2004), it is seen that exponential smoothing exhibits the least oscillations and has the desirable property of low mean absolute deviation as opposed to moving average method.

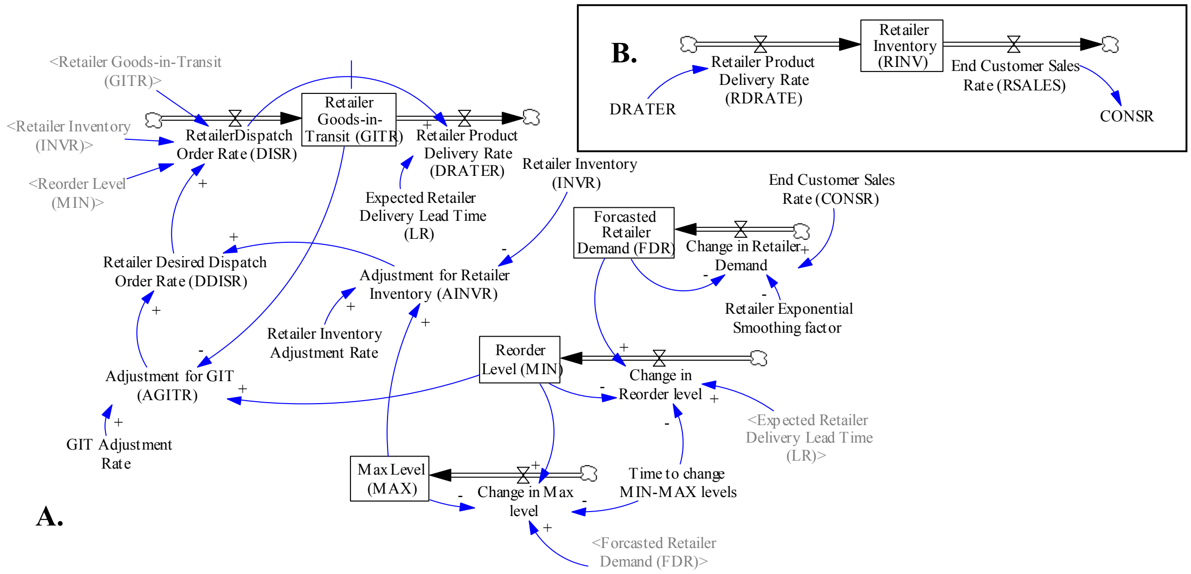

Figure 1: CLD of collaborative management of Retailer's inventory (a) Manufacturer model, (b) Retailer model

$$d[FDR^M_{ir}]/dt = (RSALES^R_{it-1} - FDR^M_{irt-1}) \cdot \rho^M_{ir} \quad (1)$$

The re-order level (*MIN*) is modeled to cover for the forecasted demand (*FDR*) during the expected delivery lead time (*LR*). The order-up-to level (*MAX*) accounts for the forecasted demand for the next period. Both *MIN* and *MAX* dynamical change in response to the forecasted demand and the delivery lead time.

$$d[MIN^M_{ir}]/dt = (LR^M_{irt} \cdot FDR^M_{irt-1} - MIN^M_{irt-1})/\tau^M_{ir} \quad (2)$$

$$d[MAX^M_{ir}]/dt = (FDR^M_{irt-1} + MIN^M_{irt-1} - MAX^M_{irt-1})/\tau^M_{ir} \quad (3)$$

In the MIN-MAX inventory control policy employed, the dispatch order is generated when the inventory level falls below a set re-order level. This dispatch ordering rule also accounts for the orders in the pipeline, that is, dispatch orders placed but not yet delivered to the Retailers.

$$DISR^M_{irt} = \begin{cases} DDISR^M_{irt}, if\,(GITR^M_{irt} + RINV^R_{it} \le MIN^M_{irt}) \\ 0, otherwise \end{cases} \quad (4)$$

$$DDISR^M_{irt} = \psi^M_{ir} \cdot (MIN^M_{irt-1} - GITR^M_{irt-1}) + \varphi^M_{ir} \cdot (MAX^M_{irt-1} - RINV^R_{it-1}) \quad (5)$$

The fractional adjustment rates of product goods-in-transit (ψ) and product inventory (φ) describe the rate at which the shortfall in the desired and current levels of product goods-in-transit and product inventory are corrected, respectively. For instance, ψ=0 completely ignores the delivery line, and ψ=1 completely accounts for the delivery line. Hence, selection of the adjustment rates reflects the ordering nature of the firms.

The goods-in-transit level accumulates the difference in the product dispatch rates (*DISR*) and the product delivery rates (*DRATER*).

$$d[GITR^M_{ir}]/dt = DISR^M_{irt} - DRATER^M_{irt} \quad (6)$$

Also, the product delivery rate is modeled as a pipeline delay of the product dispatch ordering rate with a lead time (*LR*).

$$DRATER^M_{irt} = DISR^M_{ir,t-LR} \quad (7)$$

2.3 Description of the Retailers' Model

Now, since the decision-making involved in the management of stock at Retailer has been handled by the Manufacturer, the Retailer model is simplified as shown in Figure 1b. It is assumed that the orders not fulfilled by the Retailer are immediately lost, which is a reasonable assumption for the Retailer sales and other delivery-sensitive sales function.

$$d[RINV^R_i]/dt = DRATER^M_{irt-2} - RSALES^R_{it} \quad (8)$$

3 SYSTEM ANALYSIS USING *Z*-TRANSFORM TECHNIQUE

In this paper the *z*-transform technique is used to obtain the generalized transfer functions and later the stability conditions in terms of the following system parameters:

- Fractional adjustment rate of goods-in-transit to Retailer ψ_{ir}^{M} ,
- Fractional adjustment rate of Retailer inventory φ_{ir}^{M} ,
- Exponential smoothing constant for forecast ρ_{ir}^{M} ,
- Time to change min-max levels τ_{ir}^{M} , and
- Product delivery lead time LR_{ir}^{M} .

The first step prior to the transfer technique analysis is the discretization of the differential equations presented in Section 2. In sampled (discretized) systems, a continuous function *f*(*s*) is represented by a sequence of values f_t: $f_t = f(s)\big|_{s=t,\delta}$ where $t = 0, 1, 2, \ldots$ and $\delta > 0$ is the discretization step or the sampling interval. The Manufacturer's model of the collaborative inventory management system (Section 2.2) is discretized with a sampling interval of δ and the Retailer's part (Section 2.3) is discretized with a sampling interval of Δ. It is noted that all the stocks in the system model are the only equations defined as the differential equations, to reflect the accumulation of the stock over time. The discretized form of Equations (1)-(3), (6) and (8) are shown as follows:

$$FDR_{irt}^{M} = FDR_{irt-1}^{M} + \delta \cdot (RSALES_{it-1}^{R} - FDR_{irt-1}^{M}) \cdot \rho_{ir}^{M} \quad (9)$$

$$MIN_{irt}^{M} = MIN_{irt-1}^{M} + \delta\left(LR_{ir}^{M} FDR_{irt-1}^{M} - MIN_{irt-1}^{M}\right) / \tau_{ir}^{M} \quad (10)$$

$$MAX_{irt}^{M} = MAX_{irt-1}^{M} + \delta \cdot \begin{pmatrix} FDR_{irt-1}^{M} + \\ MIN_{irt-1}^{M} - MAX_{irt-1}^{M} \end{pmatrix} \cdot \tau_{ir}^{M} \quad (11)$$

$$GIT_{irt}^{M} = GIT_{irt-1}^{M} + \delta \cdot (DISR_{irt}^{M} - DRATER_{irt}^{M}) \quad (12)$$

$$RINV_{it}^{R} = RINV_{it-1}^{R} + \Delta \cdot (DRATER_{irt-2}^{M} - RSALES_{it}^{R}) \quad (13)$$

The sampling intervals δ and Δ are said to correspond with the frequency at which the information is updated internal and external to the player, respectively. Typically, in the past research works (Grubbström 1998, Disney and Towill 2002), the sampling interval δ is implicitly assumed to be equal to Δ and both set at 1, indicating a weekly update of the ordering rule. In the current research work, the impact of the frequency of information update on the dynamics of the supply chain system is explicitly measured.

The second step prior to the *z*-transform analysis is the linearization of the non-linear functions present in the system models, which in this case is the Equation (4). Linearization is important as the exact solution using z-transform analysis can be obtained only for linear systems, which serves as the approximate solution to the non-linear functions. It is seen that the *DISR* is a discontinuous piecewise function, where the condition for choosing the dispatch order quantity $(GITR_{irt}^{M} + INVR_{irt}^{M} \leq MIN_{irt}^{M})$ changes dynamically over time. Hence, to enable analysis using transformation techniques, it is assumed that $DISR_{irt}^{M} = DDISR_{irt}^{M}$ at all times.

3.1 *Z*-Transform Description

The *z*-transforms of the discretized collaborative inventory management system, described by Equations (9)-(13), (4), (5), (7), are given as follows:

$$FDR_{ir}^{M}[z] = \frac{\delta \cdot \rho \cdot RSALES_{i}^{R}[z]}{z - 1 + \delta \cdot \rho} \quad (14)$$

$$MIN_{ir}^{M}[z] = \frac{\delta \cdot LR_{ir}^{M} \cdot RSALES_{i}^{M}[z]}{z \cdot \tau_{ir}^{M} - \tau_{ir}^{M} + \delta} \quad (15)$$

$$MAX_{ir}^{M}[z] = \frac{\delta \cdot RSALES_{i}^{M}[z] + z \cdot \delta \cdot MIN_{ir}^{M}[z]}{z \cdot \tau_{ir}^{M} - \tau_{ir}^{M} + \delta} \quad (16)$$

$$GIT_{ir}^{M}[z] = \frac{z \cdot \delta(DISR_{ir}^{M}[z] - DRATER_{ir}^{M}[z])}{z - 1} \quad (17)$$

$$RINV_{i}^{R}[z] = \frac{\Delta(DRATER_{ir}^{M}[z] - z^{2} RSALES_{i}^{R}[z])}{z(z-1)} \quad (18)$$

$$DISR_{ir}^{M}[z] = \frac{\begin{array}{c}\psi_{ir}^{M} \cdot (MIN_{ir}^{M}[z] - GITR_{ir}^{M}[z]) \\ +\varphi_{ir}^{M} \cdot (MAX_{ir}^{M}[z] - INVR_{ir}^{M}[z])\end{array}}{z} \quad (19)$$

$$DRATER_{ir}^{M}[z] = z^{-LR} \cdot DISR_{ir}^{M}[z] \quad (20)$$

The *z*-transform of the exponentially smoothing demand forecasting function (Equation 9) is as shown in Equation (14). Equations (15) and (16) represent the *z*-transforms of the MIN and MAX functions shown in Equations (10) and (11), respectively. The goods-in-transit for the Retailer and the Retailer's Inventory policies, shown in Equations (12) and (13) are converted into the z-domain (Equations 17 and 18) using the Heaviside Step Function or the integration term $1/(1\text{-}z^{-1})$ (Disney and Towill 2002). It is noted that the sampling interval for the Retailer's inventory (Equation 13) is Δ and the delivery order is updated in the Retailer's inventory two sampling periods later. The z-transform of Equation (4)-(5) is shown in Equation (19). The z-domain conversion of pipeline delay LR^{M}_{ir} policy for the delivery rate is shown in Equation (20).

Using algebra, the general transfer function for *DISR*/*RSALES* has been obtained by solving Equations (14) – (20) simultaneously (shown in Appendix). For the remainder of this analysis, the exponential smoothing parameter ρ is fixed at an arbitrary value of 0.2; the time to change min-max levels parameter τ is fixed at 1 week. In order to avoid solving a transcendental function, the value of the product delivery lead time (*LR*) is set at 2 weeks. Substituting the above parameter values, the transfer function *DISR*/*RSALES* is reduced as follows:

$$\frac{DISR^{M}_{ir}[z]}{RSALES^{R}_{i}[z]} = \frac{z^{3}\begin{pmatrix} 5(z-1)^{3}z\Delta\varphi + 11(z-1)^{2}z\delta\Delta\varphi \\ +\delta^{3}\left(\left(-1+2z^{2}+z(\Delta-1)\right)\varphi + 2(z-1)\psi\right) \\ +(z-1)\delta^{2}\left(\left(-1+z+7z\Delta\right)\varphi + 2(z-1)\psi\right)\end{pmatrix}}{\left(\delta+z-1\right)^{2}\cdot\left(-5+5z+\delta\right)\cdot\left(z^{5}+\Delta\varphi - z^{2}\delta\psi + z^{4}(-1+\delta\psi)\right)} \quad (21)$$

4 STABILITY ANALYSIS

It is important to understand how the supply chain system responds to any change in its input (i.e. sales rate), especially under a fluctuating market. Does the response result in increasing amplitude oscillations and chaos in general, or does the response appear controllable and damped? Thus it becomes essential to know under what conditions the system is stable or unstable.

Now, a system given by its closed form transfer function is said to be stable if all the roots (poles) of the transfer function's denominator polynomial is within the unit circle in the complex plane (Jury 1964). Systems with poles that are outside the unit circle or with repeated poles on the unit circle are said to be *unstable*, as they expand. Systems with non-repeating poles on the unit circle are termed as *critically/marginally stable*, as they neither converge nor expand. In general, complex pair of poles inside (outside) the unit circle indicate an oscillatory damping (growth) in the system output; and real poles inside (outside) the unit circle indicate an exponential damping (growth) in the system output, which maybe oscillatory. Also it can be observed that, the further inside the unit circle the poles are, the faster the damping and, hence higher the stability. The roots of the numerator polynomial (zeros) represent the roots of the feed forward part of the transfer function of a system. There is no restriction on the values of zeros other than that required to obtain a desired frequency or impulse response.

However, the denominator polynomial of the transfer functions is often of a higher order, and the algebraic solution involves complex mathematical calculations. In such cases, it is desirable to test the location of the roots on the complex z-plane, without explicitly solving for the roots. In this research, Jury's Test (Jury 1964) is employed to determine the location of the roots. Though this method enables a solution, it still involves tedious calculations, which are hence performed by the authors by using Mathematica®.

For a given characteristic polynomial (denominator polynomial of the transfer function), $c(z) = a_0 z^{n} + a_1 z^{n-1} + \cdots + a_n \ (a_0 > 0)$, Jury's Table is constructed as shown in Figure 2. For stability, all a^{k}_{0}s must be positive.

$$\begin{array}{ccccc|l}
a_0 & a_1 & \cdots & a_{n-1} & a_n & \\
a_n & a_{n-1} & \cdots & a_1 & a_0 & b_n = a_n / a_0 \\
\hline
a^{n-1}_{0} & a^{n-1}_{1} & \cdots & a^{n-1}_{n-1} & & \\
a^{n-1}_{n-1} & a^{n-1}_{n-2} & \cdots & a^{n-1}_{0} & & b_{n-1} = a^{n-1}_{n-1} / a^{n-1}_{0} \\
\hline
\vdots & & & & & \\
\hline
a^{0}_{0} & & & & &
\end{array}$$

where $a^{k-1}_{i} = a^{k}_{i} - b_n a^{k}_{k-1}$ and $b_k = a^{k}_{k} / a^{k}_{0}$

Figure 2: General Jury's Table for n^{th} Order Polynomial

4.1 Stability Conditions and Dynamic System Response

In this section, the conditions for stability for particular settings of sampling intervals are derived, and sample simulation results are presented to illustrate the system response. The sampling intervals Δ and δ are set equal to 1 week. Substituting the values of Δ and δ into Equation (21),

$$\frac{DISR^{M}_{ir}[z]}{RSALES^{R}_{i}[z]} = \frac{z^{2}\left(\left(-3+3z-4z^{2}+5z^{3}\right)\varphi + 2(z-1)\psi\right)}{\left(-4+5z\right)\left(z^{5}+\varphi - z^{2}\psi + z^{4}(-1+\psi)\right)} \quad (22)$$

The denominator polynomial of the *DISR* transfer function in Equation (22) is expanded to reveal a polynomial in the 6th degree. The coefficients of z are used to construct the Jury's Table (not shown). For stability, all a_0^k s (with $k = 1\ldots5$) must be positive. In the Jury table, the a_0^k s in their natural form are very lengthy mathematical expressions, which are not shown in this paper for the sake of brevity. Now, the stability conditions are derived in terms of α and β by solving for the roots of a_0^0.

The stability conditions are illustrated on the parameter plane, as shown in Figure 3. Each curve in Figure 3 illustrates a stability condition. The stable region is below the dotted lines and above the solid line, to the left of the intersection of the curves. The system is guaranteed to be stable when the values of ψ and φ are restricted to the stable region.

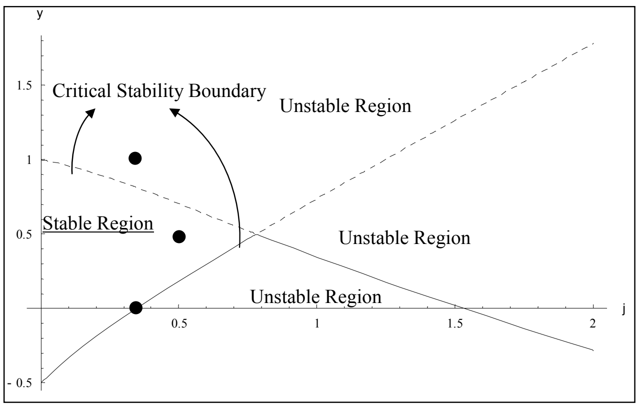

Figure 3: Stability Regions on the ψ-φ Parameter Plane

Three sampled data points (shown as black dots in Figure 3) are used to illustrate the system response *DISR*, as shown in Figures 4, 5, and 6. The results have been obtained by simulating the system dynamic models of the Manufacturer and the Retailer with different values of ψ and φ, for a unit change in the demand input. The system dynamic models of the Manufacturer and the Retailer have been implemented using Powersim® as two separate models. The models are then synchronized using the 'Co-Model' feature provided by Powersim®. This feature allows the multiple models to be run together, each with their own sampling intervals. Also, by using the 'Chain Objects' feature of Powersim® the data inventory (*RINV*) and end customer sales (*RSALES*) at the Retailer are transferred to the Manufacturer, and the product delivery order (*DRATER*) data is transferred from the Manufacturer to the Retailer. Figures 4, 5, and 6 illustrate the case of stable system response (ψ = 0.5 and φ = 0.5), the case of critical stability (ψ = 0 and φ = 0.35), and the case of unstable system response (ψ = 1 and φ = 0.35), respectively.

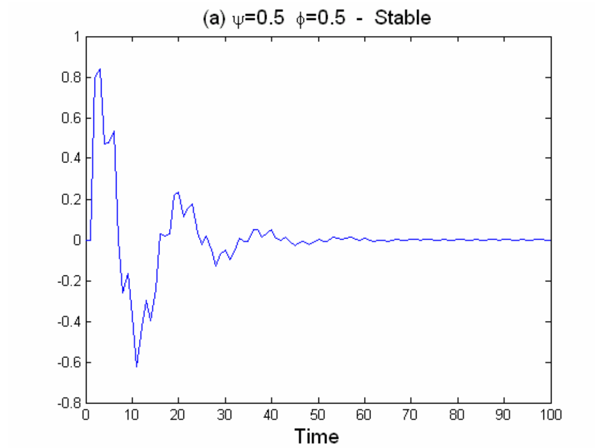

Figure 4: Stable Dynamic Response (*DISR*)

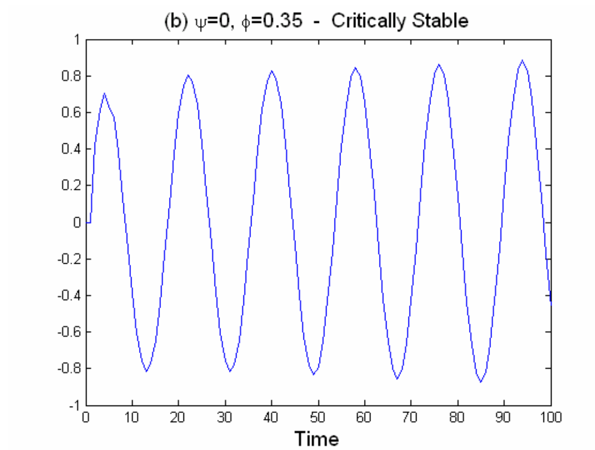

Figure 5: Critically Stable Dynamic Response (*DISR*)

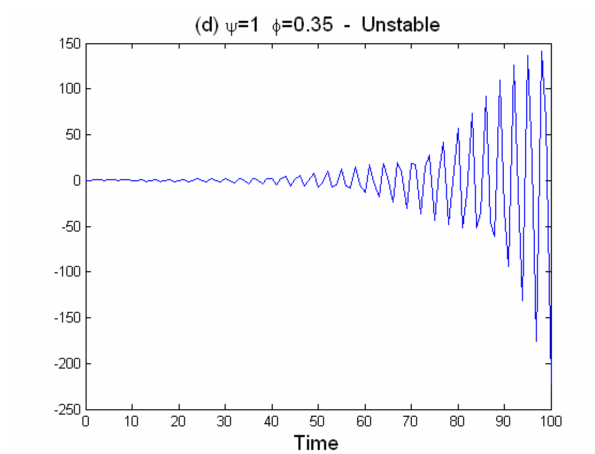

Figure 6: Unstable Dynamic Response (*DISR*)

5 EFFECT OF INFORMATION SYNCHRONIZATION

In this work, the impact of the frequency of information update on the dynamics of the supply chain system is explicitly measured. Typically in the past research works, the frequency at which the Retailers send their demand and inventory information to the Manufacturer is the same as the frequency at which the Manufacturer makes decisions (e.g. Disney and Towill 2002). That is, if the frequency or sampling interval for making decisions at the Manufacturer is every week then the Retailers send updated data every week. In this section, the possible differences in the frequency of information update at the different players and their effect on overall system stability is analyzed. That is, what happens when the Manufacturer updates its information every day but the Retailers send their updated information every week? How should the decision parameters at the Manufacturer be selected so that system operation accounts for the difference in the update frequency and continues to be stable?

The effect of different settings of the sampling intervals δ and Δ on the stability conditions in terms of ψ and φ are analyzed in the following sub-sections.

5.1 Case I: δ = Δ

First, the effect of setting the sampling intervals such that the ratio $\Delta/\ \delta = 1$ is analyzed. That is, frequency at which the Retailers send their demand and inventory information to the Manufacturer is the same as the frequency at which the Manufacturer makes decisions. The stability conditions in terms of ψ and φ are obtained (similar to Section 4.1) for each of the following cases: $\delta = \Delta = 1$, $\delta = \Delta = 1/2$, $\delta = \Delta = 1/7$. The sampling interval 1/2 corresponds to an update of information twice a week, and an interval of 1/7 corresponds to update of information every day.

The stability conditions for the different settings of the sampling interval are plotted in the $\psi - \varphi$ parameter plane, as shown in Figure 7. In Figure 7, curves of the same color represent the stability conditions for a particular setting of the sampling interval. For each setting, the stable region is below the dotted lines and above the solid line, and to the left of the intersection of the curves.

It is observed from Figure 7 that, the region of stability for sampling interval equal to 1 week (black color curves) is enclosed by the region of stability for sampling interval equal to 1/2 (red color curves), which in turn, is enclosed by the region of stability for sampling interval equal to 1/7 (blue color curves). Now, higher values of ψ and φ indicate an aggressive ordering policy that aims to rectify the inventory and good-in-transit discrepancies faster. The sampling interval settings of 1/7 (daily updates) presents the largest stability region, thus allowing for a more aggressive ordering policy. Consequently, the sampling interval settings of 1 (weekly updates) allows for a very cautious ordering policy. This ascertains a partly intuitive result that for aggressive firms it is desirable to have frequent updates of information albeit other factors such as the costs of information are trivial.

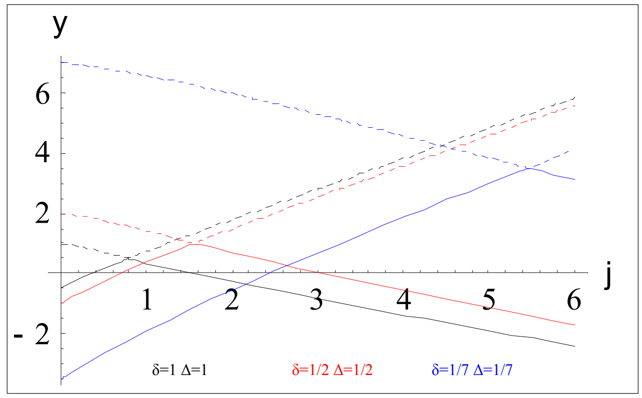

Figure 7: Stability Regions in the $\psi - \varphi$ Plane for Different Sampling Intervals ($\delta = \Delta$)

5.2 Case II: δ ≠ Δ

In this section, the effect of settings the sampling intervals such that $\Delta < \delta$ or $\Delta > \delta$ are examined. For the case of $\Delta < \delta$, the Retailers update their information more frequently than the Manufacturer. Intuitively this setting ($\Delta < \delta$) must perform better (i.e. larger stability region) as the Manufacturer uses more accurate data for its decision making. For the case of $\Delta > \delta$, the Retailers update their information less frequently than the Manufacturer. Intuitively this setting ($\Delta > \delta$) must have a smaller stability region as the Manufacturer uses not-so-accurate data for its decision making.

The stability conditions in terms of ψ and φ are obtained (similar to Section 4.1) for the following four settings: ($\delta = 1$, $\Delta = 1/2$), ($\delta = 1$, $\Delta = 1/7$), ($\delta = 1/2$, $\Delta = 1$), ($\delta = 1/7$, $\Delta = 1$). The stability conditions for the first two settings of the sampling interval ($\Delta < \delta$) and ($\delta = 1$, $\Delta = 1$) are plotted in the $\psi - \varphi$ parameter plane, as shown in Figure 8. In Figure 8, curves of the same color represent the stability conditions for a particular setting of the sampling interval. For each setting, the stable region is below the dotted lines and above the solid line, and to the left of the intersection of the curves.

It is observed from Figure 8 that, the region of stability for setting $\delta = 1$, $\Delta = 1$ (black color curves) is enclosed by the region of stability for setting $\delta = 1$, $\Delta = 1/2$ (blue color curves), which in turn, is enclosed by the region of stability for setting $\delta = 1$, $\Delta = 1/7$ (red color curves). This implies that frequent update of information on the side of the Retailer alone improves system wide performance (larger stability region). This result agrees with our intuition that as the Manufacturer uses more accurate data for its decision making, better the performance. It is also noted that frequent updates at the Retailer side ($\Delta < \delta$) forces the firm to give less weightage for goods-in-transit discrepancy and more importance to the Retailer inventory discrepancy. As a result, even if the firm fully accounts for the goods-in-transit levels (ψ=1), the system becomes unstable for any values of φ.

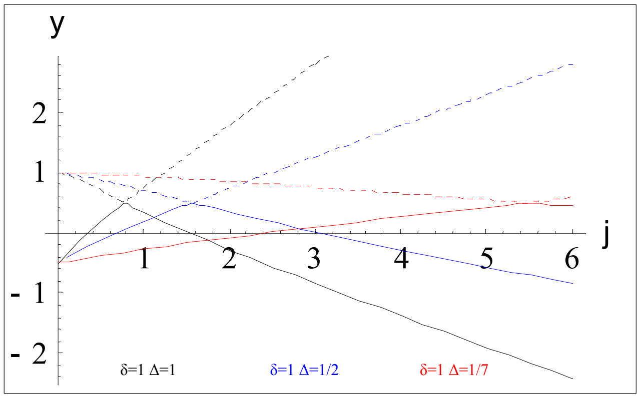

Figure 8: Stability Regions in the $\psi - \varphi$ Plane for Different Sampling Intervals (Δ < δ)

The stability conditions for the case (Δ > δ), for the settings (δ = 1, Δ = 1) (δ = 1/2, Δ = 1) and (δ = 1/7, Δ = 1) are plotted in the $\psi - \varphi$ parameter plane, as shown in Figure 9. Now, it is observed that the stability conditions for all the three information update settings overlap with each other. That is, there is no change in the stability region. This indicates that the Manufacturer gains no advantage by making frequent decisions based on less accurate or even obsolete information from the Retailer. Hence it is desirable and could be cost effective for the Manufacturer to pace their decisions equal or slower than the rate at which the Retailers can update the information.

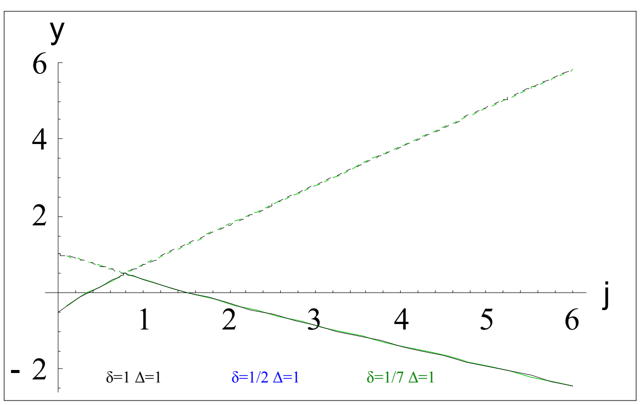

Figure 9: Stability Regions in the $\psi - \varphi$ Plane for Different Sampling Intervals (Δ > δ)

6 CONCLUSIONS

A collaborative supply chain configuration employing vendor managed inventory has been modeled and analyzed. Generalized stability conditions have been derived using *z*-transform technique with the system parameters including adjustment rate for goods-in-transit (ψ), adjustment rates for inventory levels at Retailers (φ), exponential smoothing constant for forecasting demand (ρ), time to change the MIN and MAX levels (τ), the product delivery lead time and the sampling intervals (δ and Δ). The system response measured is the dispatch order quantities (*DISR*). The Jury's Test has been employed to derive the stability conditions for the *DISR* transfer function whose characteristic polynomial is a higher order polynomial.

In this paper, the possible differences in the frequency of information update at the different players and their effect on overall system stability has been analyzed. The stability conditions in terms of ψ and φ are obtained by mapping the frequency of information update to the sampling intervals of Manufacturer (δ) and Retailer (Δ). For the case in which δ = Δ, it is found that the sampling interval settings of 1/7 (daily updates) presents the largest stability region, thus allowing for a more aggressive ordering policy, and the sampling interval settings of 1 (weekly updates) allows for a very cautious ordering policy. It is also found that frequent updates of information on the side of the Retailer (Δ < δ) alone improves system wide performance (larger stability region). Also, even when the Manufacturer makes updates more frequent than the Retailers (Δ > δ) they gain no advantage as their decisions are based on obsolete information from the Retailer. Hence it is desirable and could be cost effective for the Manufacturer to pace their decisions equal or slower than the rate at which the Retailers can update the information.

The natural progression of this paper would involve the future extension of this work in two directions. One direction is a horizontal extension to supply chain settings, where the stability effects of information update frequency between members of the supply chain are analyzed in relation to the strategy of the supply chain (viz. lean, agile, traditional) employed. The other direction is a vertical extension within an enterprise to analyze the dynamic interactions between hierarchical planning and scheduling systems resulting in stable or unstable behavioral patterns, especially in the presence of disturbances.

APPENDIX: GENERAL TRANSFER FUNCTION

The general transfer function of *DISR*/*RSALES* in terms of all the system parameters: adjustment rate for goods-in-transit (ψ), adjustment rates for inventory levels at Retailers (φ), exponential smoothing constant for forecasting demand (ρ), time to change the MIN and MAX levels (τ), the product delivery lead time and the sampling intervals (δ and Δ), is shown below:

$$\frac{DISR_{ir}^{M}[z]}{RSALES_{i}^{R}[z]} = \frac{\left(z^{LR+1}\left(\begin{array}{l} z^4\Delta\tau^2\varphi + z^3\Delta\tau\left(-3\tau+\delta\left(2+\rho\tau\right)\right)\varphi \\ +z^2\left(\begin{array}{l} LR\cdot\delta^3\rho\varphi + 3\Delta\tau^2\varphi - 2\delta\Delta\tau\left(2+\rho\tau\right)\varphi \\ +\delta^2\left(\Delta\left(\varphi+2\rho\tau\varphi\right)+\rho\tau\left(\varphi+LR\cdot\psi\right)\right)\end{array}\right) \\ +z\left(\begin{array}{l} -\Delta\tau^2\varphi+\delta\Delta\tau\left(2+\rho\tau\right)\varphi \\ +\delta^3\rho\left(\left(1-LR+\Delta\right)\varphi+LR\cdot\psi\right) \\ -\delta^2\left(\Delta\left(\varphi+2\rho\tau\varphi\right)+2\rho\tau\left(\varphi+LR\cdot\psi\right)\right)\end{array}\right) \\ -\delta^2\rho\left(\delta-\tau\right)\left(\varphi+LR\cdot\psi\right)\end{array}\right)\right)}{\left(\begin{array}{l}\left(-1+z+\delta\rho\right)\left(\delta+\left(z-1\right)\tau\right)^2 \\ \left(z^{3+LR}+\Delta\varphi-z^2\delta\psi+z^{2+LR}\left(-1+\delta\psi\right)\right)\end{array}\right)}$$

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