Proceedings of the 2005 Winter Simulation Conference
M. E. Kuhl, N. M. Steiger, F. B. Armstrong, and J. A. Joines, eds.

SIMULATION-BASED OPTIMIZATION FOR REPAIRABLE SYSTEMS USING PARTICLE SWARM ALGORITHM

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ABSTRACT

We describe an approach based on particle swarm optimization (PSO) for determining the optimal allocation of spares as well as repair resources while satisfying a desired availability constraint. The proposed method expands the original PSO algorithm to handle stochastic constraints and discrete decision variables. Computational results show that the proposed approach is efficient for determining the optimal choice of spares and repair channels for multi-echelon repairable-item inventory systems.

1 INTRODUCTION

There is considerable interest in the design and performance of repairable item inventory systems. The general problem to be investigated is the determination of the optimal spare levels and repair capacities in a repairable item inventory systems in which a finite number of items is desired to be operational at any given time, and in which queueing may occur at the repair facilities when all channels – finite in number – are busy. Consider for example, the classical machine repair problem. The situation modeled has a population consisting of M+Y_s machines; we desire at all times to have M machines operational, an additional Y_s machines are spares that support the system (i.e. there are Y_S machines as cold standby). There are Y_r parallel repair channels. If more than Y_r machines require repair, a queue forms at the repair facility. Operating times, until failure, are exponentially distributed random variables with the mean time to failure of any machine denoted by $1/\lambda$. When an operating machine fails, it is instantaneously sent to the repair facility and a spare, if available, replaces the failed machine. Service times for repairs are generally distributed with the mean time to repair denoted by $1/\mu$.

A more complicated system consists of a single base with a central repair facility as well as a local repair capability. When an item fails, there is a known probability, α, that it can be repaired at the base [$(1 - \alpha)$ is the probability that it can be sent directly to depot repair]. Of those that are sent to base repair, a further fraction β (after undergoing service) cannot be fixed and are sent to depot repair

Finding the exact solution of the above models is very difficult because of the interdependence of the bases due to their joint utilization of the depot repair service. More specifically, the lead time between the placement of an order and the return of the repaired machine to the base is a function of the total number of bases and the number of backorders at each base. Thus, the interdependence between the bases results in a very complex lead-time distribution which is very difficult, if not impossible to determine. Due to the nondeterministic repair and failure rate and the two echelon system assumed in the model, elementary queuing theory results cannot be used to solve the stationary properties as has been done in similar research.

The optimization problem presented in this paper can be formulated as follows. Let $\mathbf{y} \in \Omega$ be a $(1 \times \mathrm{d})$ vector of decision parameters over which the optimization is to occur; the set $\Omega \in R^d$ is the discrete admissible set of decision parameters. In our case **y** represent number of spares and number of repair channels at the bases and at depots. Let f(**y**) be the objective function representing the total cost (T.cost) of running the system and let $g_i(\mathbf{y}, \omega)$ denote the ith performance measure of the system, where ω is the outcome of the random process. Let $g_i(\mathbf{y}) = E[g_i(\mathbf{y}, \omega)]$ be the ith expected performance of the system. The general mathematical formulation of the problem is given below.

$$\begin{aligned} &\textit{Minimize} \quad \text{T.cost} = \mathrm{f}(\mathbf{y}) \qquad (1) \\ &\text{Subject to} \quad \mathrm{g_i}(\mathbf{y}) = E[g_i(\mathbf{y}, \omega)] \geq \delta_i \qquad 1 \leq \mathrm{i} \leq \mathrm{m} \\ &\mathbf{y}, \text{ are non-zero, integer variables.} \end{aligned}$$

where δ is the minimum desired level of system performance. We are interested in those systems whose $g_i(\mathbf{y})$ can not be obtained through analytical means and therefore must be estimated from sample paths, e.g., via stochastic simulation. In the next section we propose an algorithm based on particles swarm optimization PSO to solve the above problem.

Most multi-echelon repairable item work has concentrated on steady state solution and revolves around the METRIC (Multi-Echelon Technique for Recoverable Item Control) type of model , which assumes an infinite population of items that can fail and infinite repair capacity, so that no queue ever forms at the repair facility [Sherbrooke [1,2]). Clearly, in many practical situations, this may not be the case when there are only a small number of machines and/or repair facilities. A comprehensive review of the relevant literature on METRIC and its extensions can be found in [3,4].

Gross et. al. [5] present a closed queuing network model for multi-echelon repairable-item inventory model with Markovian repair and failure rates. The Markovian property enabled the authors to use the closed queuing network theory to model the stochastic process and an implicit enumeration algorithm is used to solve the optimization problem

Ahmed et. al.[6] present a heuristic integrated approach of simulated annealing with simulation to determine the design parameters of multi-echelon repairable-item inventory system.

Recently, a new evolutionary computation technique, called particle swarm optimization (PSO), has been proposed and introduced [7,8,9,10]. This technique combines social psychology principles in socio-cognition human agents and evolutionary computations. PSO has been motivated by the behavior of organisms such as fish schooling and bird flocking. Generally, PSO is characterized as simple in concept, easy to implement, and computationally efficient. Unlike the other heuristic techniques, PSO has a flexible and well-balanced mechanism to enhance and adapt to the global and local exploration abilities.

In this paper, a novel PSO based approach is proposed to optimize the repairable-item inventory system with state-dependent repair and failure rates. The proposed method expands the original PSO algorithm to handle stochastic constraints and discrete decision variables. Computational results show that the proposed approach is efficient in determining the optimal choice of spares and repair channels for multi-echelons repairable inventory systems algorithm to handle stochastic constraints and variables integrality requirements.

2 PARTICLE SWARM OPTIMIZATION (PSO)

The PSO algorithm is an evolutionary technique consists of a swarm of particles each represents a solution point in a multidimensional, real valued search space of possible problem solution. The particles evaluate their positions relative to a goal (fitness) at each iteration, and particles in a local neighborhood share memories of their best positions, then use those memories to adjust their own velocities, and thus subsequent positions. So by letting information about good solutions spread out through the swarm, the particles will tend to move to good solution in the search space.

Suppose that the search space is d-dimensional (d is the number of optimized parameters), then at iteration n the jth particle of the swarm $\mathbf{Y}_j(n)$ can be represented by a d-dimensional vector, $\mathbf{Y}_j(n) = [y_{j,1}(n),..., y_{j,d}(n)]$, where y's are the optimized parameters and $y_{j,k}(n)$ is the position of the jth particle with respect to the kth dimension at iteration n . The velocity of the flying particles (position change) at iteration n can be represented by another d-dimensional vector, $\mathbf{V}_j(n) = [v_{j,1}(n),..., v_{j,d}(n)]$ where $v_{j,k}(n)$ is the velocity component of the jth particle with respect to the kth dimension. As particle moves through the search space it keeps track of the best visited position so far. For the jth particle, the best previously visited position is denoted as $\mathbf{P}_j(n) = [p_{j,1}(n),..., p_{j,d}(n)]$. Defining g as the best index of the best particle in the swarm (i.e., the g-th particle is the best). Hence, $\mathbf{P}_g(n) = [p_{g,1}(n),..., p_{g,d}(n)]$ represent the best position in the swarm up to iteration n. At each iteration the $\mathbf{P}_j$ vector and the $\mathbf{P}_g$ vector are combined to adjust the velocity along each dimension, and that velocity is then used to compute a new position for the particle. The swarm is manipulated according to the following two equations [7,8]:

<u>Velocity updating</u>

$$v_{j,k}(n+1) = \psi(n)v_{j,k}(n) + W_1 R_1 (p_{j,k}(n) - y_{j,k}(n)) + W_2 R_2 (p_{g,k}(n) - y_{j,k}(n)) \qquad (2)$$

Where

$\psi(n)$ = inertia weight at iteration n.

W_1 , W_2 = two positive constants.

The role of the inertia weight ψ(n), in Equation (2), is considered critical for PSO convergence behavior. A large inertia weight facilitates global exploration (searching new areas), while a small one tends to facilitate local exploration, i.e., fine-tuning the current search area. A suitable value for the inertia weight usually provides balance between global and local exploration abilities and consequently results in a reduction of the number of iterations required to locate the optimum solution [9]. Parameters W1 and W2 are the cognitive and social learning rates, respectively. These two rates control the relative influence of the memory of the neighborhood to the memory of the individual. Recent work reports that choosing larger cognitive parameter , W1, than social parameter, W2, but with $W1 + W2 \leq 4$ produce better performance [9,11].

Position updating

Based on the update velocities, each particle changes its position according to the following equation

$$y_{j,k}(n+1) = v_{j,k}(n) + y_{j,k}(n) \qquad (3)$$

3 ENHANCEMENTS TO THE BASIC PSO ALGORITHM

The basic PSO algorithm presented in (2) and (3) has been used in the literature for unconstraint continuous optimization problems. This section presents an enhancement to the basic PSO algorithm to handle stochastic constraints and variables integrality requirements.

3.1 Constrained Optimization

Since we are considering a stochastically constrained optimization problem in this paper, the PSO algorithm must be modified to reflect the feasibility conditions of the system as well as the random behavior of the simulation output. To consider the constraints of the system, one can either searches the whole space but only keeps records of feasible solutions or accept infeasible moves with a penalty. In the later case, the constraints of the problem are relaxed and incorporated into the objective function with associated penalty. The penalty approach has proven to be inappropriate to our test problems since it stops at local minima, hence we use the PSO algorithm which keeps records of feasible moves only.

Let $g_i(\mathbf{y}) = E[g_i(\mathbf{y},\omega)] \geq \delta_i$; be the ith expected performance constraint of the system where **y** is the vector of decision parameters and ω is the output from the random process. Letting $g_i(\mathbf{y},\omega)$ denote the estimate of $g(\mathbf{y})$ on the ith replication and running the simulation n times, the estimate for $g(\mathbf{y})$ can be determined over the n replications as $\hat{g}(\mathbf{y}) = 1/n\sum_{i=1}^{n} g(\mathbf{y},\omega_i)$. The variance of $\hat{g}(\mathbf{y})$ is given by $\mathrm{var}[g(\mathbf{y},\omega_i)]/n$, where $\mathrm{var}[g(\mathbf{y},\omega_i)]$ is estimated in the usual fashion as $\frac{\sum_{i=1}^{n}(g(\mathbf{y},\omega_i)-\hat{g}(\mathbf{y}))^2}{n-1}$. Thus, assuming the classical assumption of hypothesis testing to hold, we wish to test $H_o : g(\mathbf{y}) \geq \delta$ against the alternative hypothesis $H_1 : g(\mathbf{y}) < \delta$. We accept H_o if $g(\mathbf{y}) > \delta - t_{(n-1),(1-\alpha)}\hat{\sigma}_{\hat{g}(\mathbf{y})}$ where $t_{(n-1),(1-\alpha)}$ is the upper $(1-\alpha)$ critical point for the t distribution with (n-1) degrees of freedom and $\hat{\sigma}_{\hat{g}(\mathbf{y})}$ denotes an unbiased estimator of the standard deviation of $\hat{g}(\mathbf{y})$ .Using this criterion, the original constraints can be transformed into a manageable form as follows:

$$UB = \hat{g}(\mathbf{y}) + t_{n-1,1-\alpha}\hat{\sigma}_{\hat{g}(\mathbf{y})} > \delta$$

where UB is the upper confidence limit calculated for the response $g(\mathbf{y})$ at the $(1-\alpha)$ level.

3.2 Discrete/Integer Variables.

In this paper, we consider two different modifications to the basic PSO algorithm in order to accommodate integrality requirements as suggested by Gerhard and Sobieszczanski [12]. The first approach is straightforward. The position of each particle is modified to represent a discrete point, by rounding each position coordinate to its closest discrete value after applying position update (3).

In the second approach the position of each particle is modified to represent a discrete point, by considering a set of candidate discrete values about the continuous point, obtained after applying position update. The candidate discrete points are obtained by rounding each continuous position coordinate to its closest upper and lower discrete values. The discrete point to use as the new position for the particle is selected from the candidate set of discrete points as the point with the shortest perpendicular distance to the velocity vector. Experiments with both approaches indicate that there is no significant difference in the performance of the PSO algorithm when using the first as compared to the second approach. Accordingly, we adopt the first approach in this paper.

4 COMPUTATIONAL RESULTS

To obtain good parameters setting for the proposed PSO algorithm, several runs have been performed with different values of the PSO key parameters. In our implementation, the initial inertia weight $\psi(n)$ is set to 1.2 and gradually decreased to 0.2. The swarm's size was set equal to 5D for all test cases where D is the corresponding dimension of the problem. W_1 is set equal to 2.5 and W_2 is set equal to 1.5.

We consider a two-echelon repairable item inventory system . We will consider the system in steady state situation. In this system the decision parameters are spares and repair channels at base (Y_{sb} , Y_{rb}) and repair channels at the depot (Y_{rd}). In this system let A_1 represent the fraction of time that all M machines are operational, and let A_2 represent the fraction of time that at least 0.9 M machines are operational. If we let P_k to be the probability that k machines are operational, then the mathematical formulation can be presented as follows:

$$\textit{Minimize}\ \ Z = C_{sb}Y_{sb} + C_{rb}Y_{rb} + C_{rd}Y_{rd}$$

subject to

$$A_1 = \sum_{k=M}^{M+Y_{sb}} P_k \geq \delta = 0.9$$

$$A_2 = \sum_{k=0.9M}^{M+Y_{sb}} P_k \geq \delta = 0.98$$

Y_{sb} , Y_{rb} , Y_{sd} are non-negative integers.

Gross et. al. [5] solved a problem similar to that described in this section for a Markovian system in steady state. The Markovian property enabled the authors to use the closed queuing network theory to model the stochastic process and an implicit enumeration algorithm is used to solve the optimization problem. We used the proposed approach to find the optimal solution of the problem. The mean failure rate of each machine is 1.0. The mean base repair rate is 5.0. The mean depot repair rate is 5.0. The fraction of failures which are base repairable, α, is 0.5. The cost per spare C_{sb}, is 20. The cost per repair channel at the base, C_{rb}, is 8. The cost per repair channel at the depot, C_{rd} is 10. The fraction of items worked on at the base which require depot repair, β, is 0.5. We consider five test cases with M= 5, 10, 20, 30 and 40. The comparison between our approach and the analytical solution using the closed queuing network with the implicit enumeration is given in Table 4. The accuracy of our proposed approach can clearly be noticed from the results presented in Table 1.

5 CONCLUSIONS

In this paper, a novel PSO based approach is proposed to optimize the repairable-item inventory model with state-dependent repair and failure rates as well as steady-state environment. The proposed method expands the original PSO algorithm to handle stochastic constraints and discrete variables. Computational results show that the proposed approach is efficient in determining the optimal choice of spares and repair channels for the multi-echelons repairable inventory system.

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Table 1. Comparison between the proposed approach and the analytical solution.

	PSO approach				Analytical solution			
M	Y^*_{sb}	Y^*_{rb}	Y^*_{rd}	T.cost	Y^*_{sb}	Y^*_{rb}	Y^*_{rd}	T.cost
5	3	2	2	96	3	2	2	96
10	5	3	4	164	5	3	4	164
20	9	4	6	272	8	5	6	260
30	11	6	8	348	11	6	8	348
40	15	9	6	432	14	8	10	444

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