A NEW METHODOLOGY TO EVALUATE THE REAL OPTIONS OF AN INVESTMENT USING BINOMIAL TREES AND MONTE CARLO SIMULATION

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ABSTRACT

This paper deals with a new methodology to evaluate the real operating options embedded in a manufacturing system investment. In a single product framework, the demand is assumed as the main source of uncertainty, therefore as a stochastic variable following a Geometric Brownian Motion (GBM). Then, focusing on the real option to expand the capacity at a certain time in the future, we have developed a new approach for the option payoff, looking forward in the time interval from the expansion date to the end of the planning horizon. The payoff function is the expected Net Present Value (NPV), at the expansion date, of the additional investment to increase the capacity, and it is calculated using Monte Carlo simulation. The option value is computed with a binomial tree algorithm. A numerical example and a sensitivity analysis of the option value as a function of some parameters are finally presented.

1 INTRODUCTION

When dealing with investment decisions, the traditional method is a simple NPV calculation of the different cash flows, in order to select the investment that has the highest positive NPV and discard the projects with negative NPV. In recent years, however, many researchers have shown that conventional economic analysis based on Discounted Cash Flow (DCF) techniques often undervalue projects with real operating options and other strategic interactions (Miller and Park, 2002). In particular, DCF analysis ignores the "operating flexibility" that gives project managers options to revise decisions in response to changing exogenous economic conditions (Copeland, 2002). The importance of such operating options becomes critical when the environment is highly volatile (extreme variations in market demand and product prices) and the technology is flexible (CNC machines or Reconfigurable Machine Tools - RMTs), thus allowing managerial intervention at a reasonable cost.

Real Option Analysis (ROA), by explicitly capturing the flexibility and its effects on uncertainty, provides for a consistent treatment of risk in the valuation of investment in production systems (Schwarz, 2001). An option is the right, but not the obligation, to take an action in the future. Options are most valuable when there is uncertainty; this is one of the most important shifts in thinking from the real options approach: uncertainty creates opportunities (Amram and Kulatilaka, 1999).

The most common real options are: the waiting-to-invest option, the option to alter the operating scale (option to expand, to contract, to shut down and restart), the option to abandon, and the option to switch (option to change the product mix or processes). For instance, in the valuation of an investment in a Reconfigurable Manufacturing System (RMS), the classical NPV approach might not take correctly into account the value of the real options embedded in the project. Since RMS allow functionality expanding and production capacity upgrading exactly when needed and with a reasonable investment (Koren *et al.*, 1999), the

value of real options for capacity or mix changes may be critical in the investment decision.

The focus of this paper is the development of an investment evaluation methodology improving the traditional DCF approach when dealing with projects having real operating options in an uncertain world. To accomplish this task, we have developed and tested an advanced decision support tool, able to compute the value of the real options embedded in an investment project.

The paper is organized as follows. Section 2 describes the real option framework, compared to the financial framework and other real option approaches in literature. Section 3 focuses on the particular payoff function developed for this new approach. Section 4 deals with the numerical implementation issues, and Section 5 shows and analyzes the results of a numerical example. Finally, in Section 6 the conclusions and future developments are drawn.

2 REAL OPTION FRAMEWORK

The primary objective here is to develop a comparison between the financial option framework and a real option scenario for a production system investment. Let us focus only on the counterpart of the call option, which is the option to expand, i.e. to increase the system capacity by purchase of additional equipment. Let us also assume that the system is producing a single product.

The first step is to find the equivalent of the financial option framework in the production system scenario. This issue has been addressed in various approaches presented in the literature; the most common is the one shown in Table 1 (Trigeorgis, 1991).

Table 1: Financial and Manufacturing Frameworks

Financial framework	*Manufacturing framework*
stock price: S	project DCF without the initial investment I_0: DCF
stock price volatility: σ	volatility of DCF: σ
exercise time: T	expansion time: T
exercise price: X	investment outlay at time T to increase the capacity: I_1
risk-free rate: r	risk-free rate: r
payoff function: max $\{S(T) - X, 0\}$	payoff function: max $\{e \cdot DCF(T) - I_1, 0\}$

The advantage of this approach is that it even enables use of Black-Scholes model, since it matches perfectly the financial framework: it is enough to substitute the variables. Nevertheless, it also has several weak points and drawbacks:

- the main source of uncertainty is the demand, more than the DCF of the project;
- the payoff of the option, for instance, to expand the capacity, cannot be estimated using only the

information at the exercise date, because it depends on the cash flows generated by the new investment in the future, until the end of the planning horizon;
- the exercise price is set a priori and the expansion value is just a percentage $e \cdot 100\%$ of the asset value at the expansion time, so there is no connection with the capacity change due to the demand increase;
- there is a time delay between the investment time and the date of the additional capacity installation and the system readiness to produce at the new throughput level, and this issue is not considered.

Karsak and Ozogul (2002) value expansion flexibility using American exchange options. Their model has two stochastic variables: the return and the cost of the expansion investment follow two correlated GBM. However, the payoff still depends on the difference between the two assets at the exercise time.

Feinstein and Lander (2002) value real options discounting expected cash flows at a weighted average of the risk-free interest rate and the project cost of capital. It is very close to the traditional NPV approach, but it is limited to one period binomial options.

Our approach differs from the ones presented in literature so far, because its focus is on the demand. Starting from the stochastic process of the demand, a real options framework is built in order to estimate the value of the option to expand the capacity in the future. The focus on a particular real option, for example the option to expand, is not restrictive at all, because all of the following theory can be easily applied to any real option.

The scenario is based on a production system able to manufacture a single product. The initial capacity C_0 is based on the initial investment I_0, therefore we are able to produce C_0 items per year of a single product and to sell them with a contribution margin m. We assume that the contribution margin does not change with passing of time.

Primary uncertainty lies in the market demand of the product, which is a stochastic variable $D(t)$. If the demand is D items per year, then we can sell min$\{D, C_0\}$ items and gain the net income $NI(D) = m \cdot \min\{D, C_0\}$. The previous expression is a very simple net income function, but any more complex function can be used, as long as it is a function of the demand.

Let $f(t, D(t))$ be the pricing function of a real option, depending on the time t and the demand $D(t)$. We assume the demand follows the GBM:

$$dD(t) = \mu \cdot D(t) \cdot dt + \sigma \cdot D(t) \cdot dW(t) \qquad (1)$$

where $dW(t)$ is a Wiener process, μ is the expected growth rate (drift) of the demand and σ is the volatility of the demand. These two parameters can be easily estimated from historical data.

The aim is now to find the price *f* for a derivative depending only on demand and time. The payoff of this derivative at the expiration date *T* is given by the function $\Phi(D(T))$, which depends on the derivative we are considering and it can be any kind of function. The pricing function is (Bjork, 1998):

$$f(t,D) = e^{-r \cdot (T-t)} \cdot E_{t,D}^{Q} \left[\Phi(D(T)) \right]. \qquad (2)$$

The previous expression is the well-known result on pricing a derivative discounting at the risk-free interest rate the expected payoff from the option at its maturity in a risk neutral world. Taking the expectation in a risk neutral world means that we have to use the risk neutral probability measure *Q*. Under this new probability measure, the demand follows a different process, with expected growth rate $\alpha = \mu - \lambda \cdot \sigma$ (risk neutral drift) instead of μ:

$$dD = (\mu - \lambda \cdot \sigma) \cdot D \cdot dt + \sigma \cdot D \cdot dW^{Q} \qquad (3)$$

where dW^{Q} denotes a *Q*-Wiener process, and λ is the market price of risk, which can be estimated using the Capital Asset Pricing Model (CAPM) (Hull, 2002).

Using the binomial tree approximation with time interval Δt_1, the parameters to be used are:

- up movement coefficient: $u = e^{\sigma \sqrt{\Delta t_1}}$,
- down movement coefficient: $d = e^{-\sigma \sqrt{\Delta t_1}} = 1/u$,
- risk neutral probability of an up movement: $q = \left(e^{(\mu - \lambda \cdot \sigma) \cdot \Delta t_1} - d \right) / (u - d)$.

Obviously, the smaller Δt_1 the better the approximation; we actually choose the number of steps $n_1 = T/\Delta t_1$.

In order to have a stable numeric scheme, the probability *q* must be limited between 0 and 1, meaning the following condition must hold:

$$\left| \mu/\sigma - \lambda \right| \cdot \sqrt{\Delta t} < 1 \iff n_1 > T \cdot (\mu - \lambda \cdot \sigma)^2 / \sigma^2 . \quad (4)$$

Therefore, it is always possible to choose a number of steps, n_1, that is big enough to satisfy the previous constraint.

3 THE PAYOFF FUNCTION

Now, we need to define a payoff function for the option value at the expiration date *T*, which is less or equal than the planning horizon T_{end}. At the expiration date, *T*, it is possible to increase the production capacity from C_0 to C_1 ($C_1 > C_0$) with the investment outlay I_1.

Hence, the real option we are considering is the counterpart of an European call option with exercise price I_1. In addition, we assume we know the Net Income of the production, which is a function *NI* (*D*) of the demand.

To calculate the payoff we need to look forward to the additional Net Incomes due to the new capacity C_1 from *T* to the end of the planning horizon T_{end}, and compare their values, discounted at time *T*, with the additional investment I_1. So the payoff is basically the net present value of the additional investment at time *T*. This expected NPV might be computed using another binomial tree, starting from each terminal node of the first one. The expected value would be the weighted sum of the NPVs of all the possible paths in the tree, being the weights the probabilities of following each path. To implement this payoff calculation we need to generate and evaluate all the possible paths in the tree. If the time step in the time interval [*T*, T_{end}] is Δt_2 (which can be different than the time step Δt_1 in the time interval [0, *T*]) then there are $n_2 = (T_{end} - T)/\Delta t_2$ steps. The number of paths grows exponentially with n_2, since it is given by 2^{n_2}. Therefore, the execution of this algorithm would be very slow even with small values of n_2.

To evaluate the payoff function $\Phi(D(T))$, which actually does not depend only on the demand at the exercise date *T*, but on the demand on all the time interval [*T*, T_{end}], we can overcome the drawbacks of the algorithm described in the previous paragraph using the Monte Carlo simulation. Since the payoff is a function of the path in the interval [*T*, T_{end}], and we already know the parameters for a risk neutral simulation of the demand stochastic process, Monte Carlo simulation seems to be the best way to approach this problem.

Let us consider the node *h* at the end of the binomial tree (*h* goes from $-n_1$ to n_1 with step 2). The demand in this node is given by $D_h(T) = u^h \cdot D_0 \cdot \Delta t_1$, where D_0 is the initial demand. Since in the interval [*T*, T_{end}] we are using a different time step Δt_1, the demand has to be rescaled, so that the initial demand for the simulation starting at time $t_0 = T$ is finally $D(t_0) = D_h(T) \cdot \Delta t_2 / \Delta t_1 = u^h \cdot D_0 \cdot \Delta t_2$. The capacity must be rescaled as well, so $C_{0,2} = C_0 \cdot \Delta t_2$ and $C_{1,2} = C_1 \cdot \Delta t_2$.

Using the solution to the risk neutral demand process (3), we can easily generate the demand path in the time interval [*T*, T_{end}]:

$$D(t_j) = D(t_{j-1}) \cdot \exp\left(\left(\alpha - \frac{\sigma^2}{2} \right) \cdot \Delta t_2 + \sigma \cdot \sqrt{\Delta t_2} \cdot \varepsilon_j \right) \quad (5)$$

where $j = 1,\dots n_2$ and $\varepsilon_j \sim N(0,1)$ are n_2 independent samples drawn from a standardized normal distribution. At each time t_j, the income differential due to the increase of capacity is given by:

$$\Delta NI_j = NI\left(D(t_j), C_{0,2} \right) - NI\left(D(t_j), C_{0,1} \right). \quad (6)$$

To discount the net incomes at time *T*, the rate to be used is a "risk adjusted rate" of the project, *R*, which is the company cost of capital if the expansion investment has the same risk of the existing business. As a general rule, if the project risk is different from the one of the existing assets, the project should be evaluated at its own opportunity cost of capital. Both the company and the project cost of capital can be estimated using the CAPM (Brealey and Myers, 1996).

Discounting at the rate *R*, the *DCF* at time *T* is:

$$DCF_i = \sum_{j=0}^{n_2} \exp\left(-R \cdot j \cdot \Delta t_2\right) \cdot \Delta NI_j \qquad (7)$$

where the subscript *i* stands for the *i*th replication. Hence, if we switch the node index from *h* to $k = 0,..n_1$, with $h = 2 \cdot k - n_1$, then the *i*th payoff at the node *k* is:

$$\Phi_{k,i} = \max\left\{DCF_i - I_1, 0\right\}. \qquad (8)$$

Denoting with *n* the number of replications, the expected payoff at the node *k* is the average over the *n* replications $\overline{\Phi}_k = n^{-1} \cdot \sum_{i=1}^{n} \Phi_{k,i}$.

Once the payoffs at all the terminal nodes are known, the option value V_t at time $t < T$, can be computed using the expression (9), where p_k is the probability density function of the binomial distribution with parameters n_1 and *q*, reported in the equation (10):

$$V_t = \exp\left(-r \cdot (T - t)\right) \cdot \sum_{k=0}^{n_1} p_k \cdot \overline{\Phi}_k \qquad (9)$$

$$p_k = \binom{n_1}{k} \cdot q^k \cdot (1-q)^{n_1-k}. \qquad (10)$$

Figure 1 shows the combined methodologies, binomial tree and Monte Carlo simulation.

4 NUMERICAL IMPLEMENTATION

Since we are using a discrete time algorithm, to obtain a result that is a good approximation of the real solution (which is unknown), the number of steps in the binomial tree n_1 should be appropriately high. On the other hand, the number of steps in the simulation equals the number of cash flows to be taken into account in the time interval [*T*, T_{end}] when comparing the alternative investments "increase the capacity" and "do nothing". Therefore, the time step Δt_2 depends on what time basis the sales are surveyed and the cash flows are computed in the company. We assumed $\Delta t_2 = 0.1$ years.

The code was written and executed with MATLAB®. This software tool provides fast implementation, high precision in numerical computations, and quick execution when the code is properly vectorized. In addition, there are tools that allow data import/export with a Microsoft Excel® worksheet. The Monte Carlo simulation was implemented using MATLAB® internal normal random generator; it is a table lookup algorithm developed by Marsaglia, which uses a step function to approximate the normal probability density function ("ziggurat algorithm"). The resulting distribution is exactly normal, the generation is as fast as for the uniformly distributed numbers, and the generator period is 2^{64} (Moler, 2001).

As far as the binomial algorithm is concerned, using MATLAB® it is very easy and fast to implement the sum of expression (9), with the statistical function that efficiently returns the binomial probability density function (10), even when n_1 is a big number.
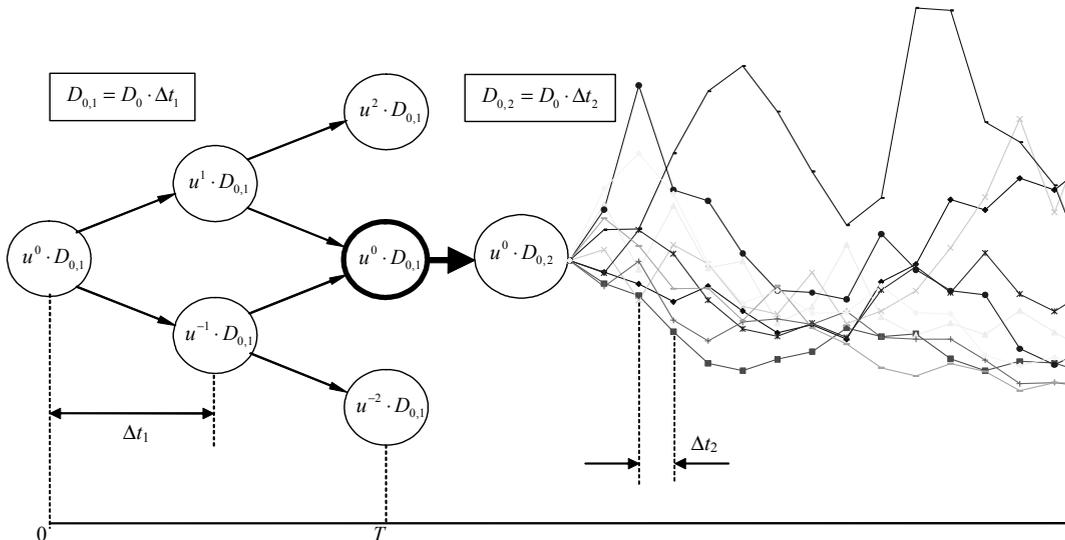
Figure 1: Binomial Tree and Monte Carlo Simulation

Another critical parameter that deeply affects the goodness of the result is the number of replications *n*, run during the Monte Carlo simulation of the demand paths. Since the number of replications is set to be at least 50 and, above all, the samples Φ_i (in the following we omitted the subscript *k* because we are considering the payoff for a generic node) are independent, then it is possible to calculate the number of replications in order to control the relative error of the estimate. To perform this task, one should choose *n* such that the following criterion is met:

$$\left(z_{1-\alpha/2} \cdot \sqrt{s^2/n}\right)\Big/\overline{\Phi} \le \gamma/(1-\gamma) \qquad (11)$$

where $z_{1-\alpha/2}$ is the inverse cumulative standard distribution function, s^2 is the variance of the samples Φ_i, and γ is the maximum relative error allowed with probability $1 - \alpha$ (Brandimarte, 2001). We assumed $1 - \alpha = 95\%$.

The only additional computation required is the estimation of the mean $\overline{\Phi}$ and the variance s^2 for a first trial of 50 pilot replications; then, solving *n* from the previous expression, we can find the minimum number of replications to run. After running *n* replications, the algorithm checks whether the condition (11) still holds with the new estimates of $\overline{\Phi}$ and s^2; if not, more replications are added solving again the expression (11). Actually, the condition is checked also at some values before *n*, to avoid waiting for useless replications when the initial estimates are not very good. Furthermore, no more replications are added if the condition (11) is not met yet after a running time of 5 minutes since the beginning of that simulation.

In order to reduce the number of replications we implemented the antithetic sampling as a variance reduction technique. Therefore, two demand paths are generated for each replication, using two opposite random sequences ε_j and $-\varepsilon_j$. After the payoffs of these sequences are computed, the average of them is the sample Φ_i to be considered. This methodology actually works only if the negative correlation in the input random numbers yields a negative correlation in the output payoffs as well. This actually happens, because of the monotonic relationship between the random input and the payoff, through the composition of the demand function (5), the net income function (6), the *DCF* function (7) and the payoff function (8). As a result of this variance reduction technique, the number of replications to run in order to get an estimate with a given relative error is, on average, at least 60% less.

5   NUMERICAL EXAMPLE

The numerical data assumed to test the algorithm are reported in Table 2. In the following example we did not consider any historical data, because it is not a real case study. Therefore, the values of the parameters μ, σ, and λ, and all the other input values as well, are just arbitrary numbers.

Table 2: Numerical Data

risk free rate	$r = 5\%$ per year
planning horizon	$T_{end} = 4$ years
exercise time	$T = 2$ years
initial demand	$D_0 = 125{,}000$ products per year
drift of demand	$\mu = 0.9$ per year
volatility of demand	$\sigma = 0.5$ per year
market price of risk	$\lambda = 0.8$ per year
initial capacity	$C_0 = 160{,}000$ products per year
contribution margin	$m = \$\ 15.00$ per product
expansion investment	$I_1 = \$\ 1{,}000{,}000$
new capacity	$C_1 = 250{,}000$ products per year
risk adjusted rate	$R = 15\%$ per year

We are interested in the option value V_0 at time $t = 0$; hence, everywhere in the following, we will denote it simply as *V* or "option value."

A first analysis focused on the tuning of the parameters n_1, number of steps in the binomial tree, and γ, maximum relative error allowed for the payoffs estimates. They both affect the precision of the option value calculation.

Figure 2 shows the option values when n_1 increases from 10 to 100, and γ is 10%, 5% and 1%. The precision improvement costs, on average, a running time 3.5 times greater when γ decreases from 10% to 5%. If we want γ to decrease from 5% to 1%, then the running times grow by a factor of 16.5. The convergence when n_1 increases is almost the same when $\gamma = 5\%$ and 1%; the curve with $\gamma = 1\%$ is just a little smoother. Furthermore, there is no observable improvement tied to n_1 increase, when the values are already within the precision allowed by γ. Hence we assumed $n_1 = 50$ and $\gamma = 5\%$. The average running time was about 1.2 minutes. Of course, with better computational power available and higher precision required, these parameters should have greater values.
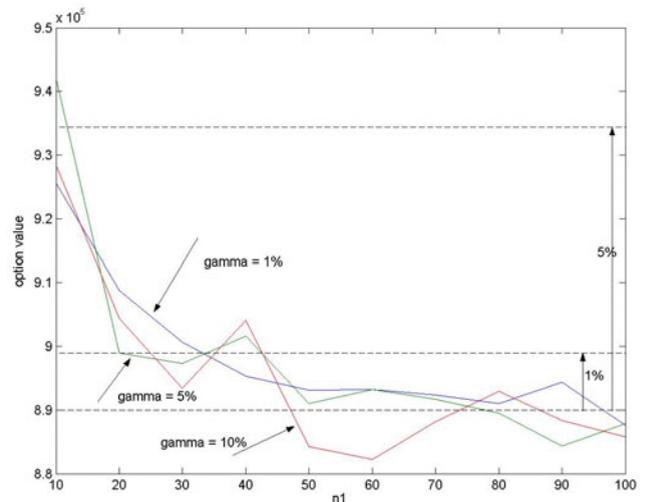
Figure 2: Convergence and Error

After the algorithm has been tested and its parameters properly tuned, it is very interesting to understand how the option value changes when the main input variables change. The option values have then been computed for different values of the parameters μ, σ, and λ, according to the plan reported in Table 3, featuring 7*7*5 = 245 points.

Table 3: Experimental Plan μ, σ, λ

	min	max	step	points
μ	− 0.75	0.75	0.25	7
σ	0.2	1.1	0.15	7
λ	0	2	0.5	5

Figure 3 shows the contour plot for the function $V(\mu, \sigma)$ when $\lambda = 0.5$. When the market price of risk λ increases the result is similar, only with much lower option values.

It can be readily noticed in Figure 3, that the option value grows with the increasing demand drift μ, while the dependency on the demand volatility σ is more complex.

Looking at the down half of the contour plot, the option value seems to grow with the volatility, exactly like it happens in the financial framework.

However, if we look at the top half of the plot, then the option value decreases when the volatility increases. Furthermore, if we look closer at the area around the half of the plot, the relationship between the option value and the volatility might not even be monotonic.

This result deserves a further analysis, so we have reported the graphs of $V(\mu)$ for the 7 values of σ, and $V(\sigma)$ for the 7 values of μ. These curves are shown respectively in Figure 4 and Figure 5.
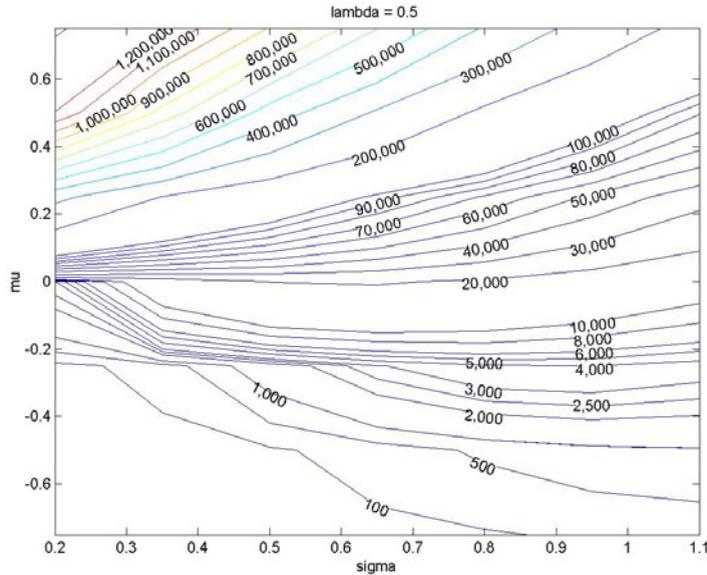
Figure 3: $V(\sigma, \mu)$ when $\lambda = 0.5$
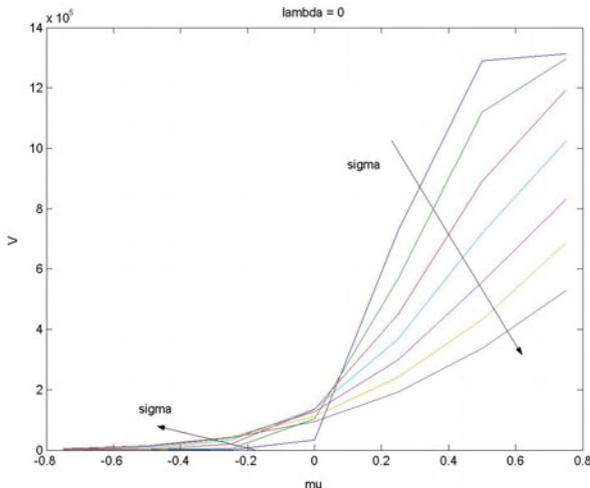
Figure 4: $V(\mu)$ when $\lambda = 0$
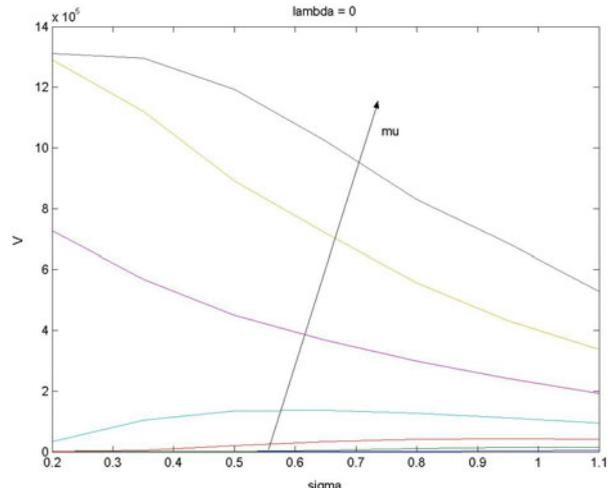
Figure 5: $V(\sigma)$ when $\lambda = 0$

In Figures 4 and 5 we set $\lambda = 0$, primarily because the option values are greater and it improves legibility of the graphs. The option value always increases with respect to the demand drift μ. On the other hand, when $\mu > 0$, the option value decreases with the volatility in the range [0.2, 1.1]. To better understand what happens when $\mu \leq 0$ we need to zoom in the lower part of Figure 5, because the option values are much smaller when $\mu \leq 0$. This is showed in the following Figure 6.
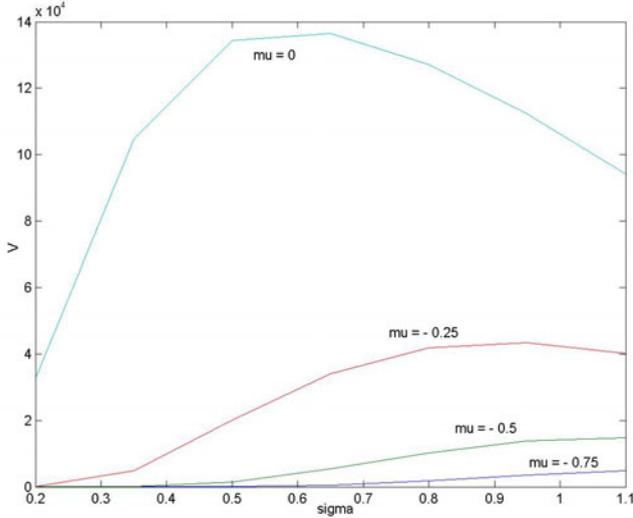
Figure 6: Enlargement of Figure 5 when $\mu \leq 0$

Looking at Figure 6 it is clear that when $\mu = 0$ the option value grows up to a maximum, and then it decreases. When $\mu < 0$ the maximum shifts to the right, so that within the interval of σ [0.2, 1.1] the option value looks like it is almost always increasing.

This quite surprising result can be explained looking at the payoff function, which is the main difference between this algorithm for a real European call and the financial European call. In particular, let us consider the demand process (5). The demand changes according to a deterministic term that depends on $\alpha' = \alpha - 0.5\ \sigma^2 = \mu - \lambda \cdot \sigma - 0.5\ \sigma^2$, and a stochastic term that depends on σ. When the volatility σ increases, α' decreases and becomes negative, covering the linear increase and the randomness of the stochastic term. Therefore, the payoff decreases with the volatility, and so does the option value. This effect is highlighted when the drift $\mu > 0$, whereas when $\mu < 0$, α' is always negative, and the effect of the stochastic term is greater.

Another parameter worthwhile taking into account, is the initial value of the demand D_0. It is actually better to consider the new parameter $\beta = D_0 / C_0$, which is related to the extra capacity existing in the production system at the beginning (when $\beta < 1$).

So far we have assumed $\beta = 0.78$; now let it go from 0.5 to 1.5 with step 0.25. Assuming the market price of risk $\lambda = 0.5$, the new experimental plan is reported in Table 4.

Table 4: Experimental Plan μ, σ, β

	min	max	step	points
μ	-0.75	0.75	0.25	7
σ	0.2	1.1	0.15	7
β	0.5	1.5	0.25	5

Figure 7 shows the contour plot for the function $V(\sigma, \beta)$ when $\mu = 0.5$. Predictably, the option value grows with β, whereas it again decreases with the increase of volatility σ. Like in the previous analysis, this trend reverses when $\mu < 0$. Figure 8 shows the same plot when $\mu = -0.5$, and the option value clearly increases with the volatility σ, at least when $\beta \leq 1$.
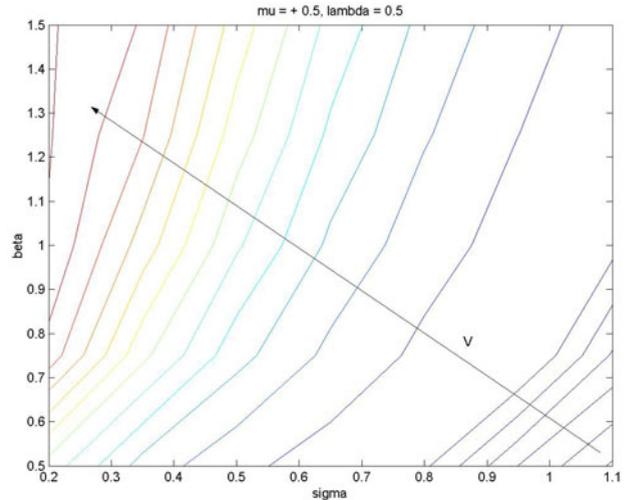
Figure 7: $V(\sigma, \beta)$ when $\mu = 0.5$
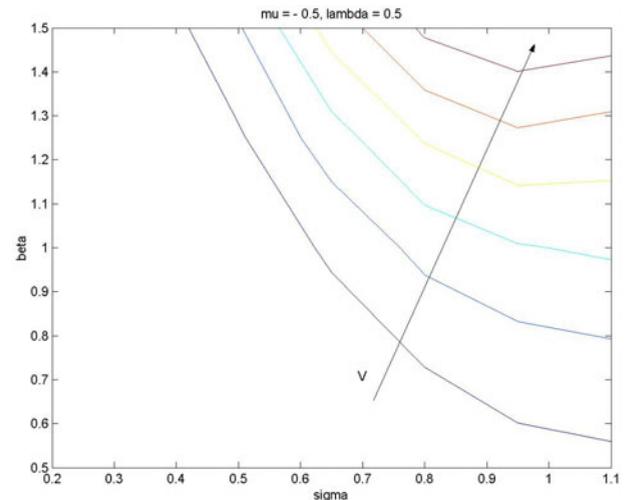
Figure 8: $V(\sigma, \beta)$ when $\mu = -0.5$

To better understand the effect of β, Figure 9 shows the graphs of $V(\sigma)$ for the 5 values of β. The option value increases with σ when $\beta \leq 1$, whereas it has a maximum when $\beta > 1$.
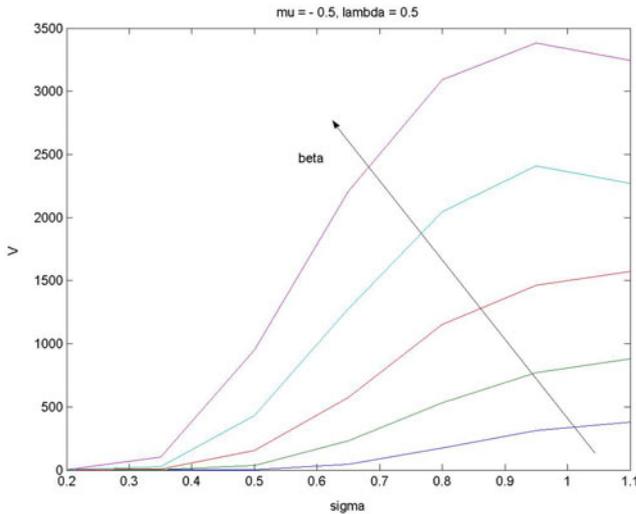
Figure 9: $V(\sigma)$ when $\mu = -0.5$

Like 0 for the drift μ, 1 seems to be a critical value for β, impacting on the option value as a function of the volatility σ.

6 CONCLUSIONS

In this paper, we have shown a new approach to evaluate real operating options of a production system making a single product. The originality of the methodology is:

- the stochastic variable is the product demand, therefore the market price of risk λ has been introduced in the model;
- the payoff function depends on the demand between the expansion date and the end of the planning horizon, and is estimated using Monte Carlo simulation.

These two points are addressing some of the drawbacks of most of the approaches found in the literature, as highlighted in Section 2. The other shortcomings will be addressed in the future developments of this research.

We have focused on an expansion option, and we have developed a tool able to compute the option value for a wide range of input parameters.

A sensitivity analysis has been performed on the option value as a function of the following parameters:

- μ the drift of the demand,
- σ the volatility of the demand,
- λ the market price of risk,
- β the ratio between initial demand and capacity.

The surprising result is that the option value is not always increasing with the volatility, such as it happens with financial options. We showed that the option value is always increasing with the volatility only when $\mu < 0$ and $\beta \leq 1$. When $\mu > 0$, it is always decreasing, while for other values it can have a maximum.

Davis (2002) found a very similar result, even though with a different real option model. He shows that growth option value is very likely to be declining in volatility for at- or in-the-money options, whereas it is likely to increase for out-of-the-money growth options. This is not different from our conclusions; actually, when the option value is always increasing with the volatility, the option is almost out-of-the-money, as the reader can notice from the low values on the vertical axes of Figures 6 and 9 (the order of magnitude is 10^4 and 10^3). On the other hand, when the option value is always decreasing with the volatility, the option is deeply in-the-money, as the reader can notice from the high values on the vertical axes of Figure 5 (the order of magnitude is 10^5 and 10^6).

Davis (2002) states that firms holding at- or in-the-money growth options are justified in avoiding increased market volatility. According to the model presented in this paper, we can extend this result to manufacturing systems holding expansion options.

Nevertheless, the presented results are only the first step in the development of a decision support tool able to evaluate real options embedded in investment in production systems. Future research will focus on different kinds of real options, a varying exercise time (American option), multiple options models, and different kinds of stochastic processes other than the GBM.

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