Proceedings of the 2002 Winter Simulation Conference
E. Yücesan, C.-H. Chen, J. L. Snowdon, and J. M. Charnes, eds.

MODELING CONSIDERATIONS FOR WIDE AREA SEARCH MUNITION EFFECTIVENESS ANALYSIS

David R. Jacques

Air Force Institute of Technology
2950 P Street, Suite 201C
WPAFB, OH 45433, U.S.A.

ABSTRACT

There are currently several wide area search munitions in the research and development phase within the Department of Defense. Progress on the individual technologies is promising, but there are insufficient analytical tools for evaluating the effectiveness of these concept munitions. This paper examines some of the modeling aspects of wide area search munitions with Autonomous Target Recognition (ATR) capability. The unique aspect of the munition problem is that a search agent is lost whenever an attack is executed. This significantly impacts the overall effectiveness in a multi-target/false target environment. ATR measures of performance will be introduced, and described in terms of a confusion matrix for the sensor. The single munition/single target and general multi-munition/multi-target cases will be discussed, and a simple application will be used to validate the modeling constructs.

1 INTRODUCTION

Several types of wide area search munitions are currently being investigated within the U.S. Department of Defense research labs. These munitions are being designed to autonomously search, detect, recognize and attack mobile and relocatable targets. Additional work at the basic research level is investigating the possibility of having these autonomous munitions share information and act in a cooperative fashion (see references for Gillen 2002 and Jacques 2002). While some of the research is promising, most of it is relying heavily on empirical algorithms and simulation to evaluate the performance of the multi-munition system. Analysis appears to be lacking with regards to the fundamental nature of the wide area search munition problem, to include identification of the critical munition and target environment parameters that must be adequately modeled for a valid simulation. Some classic work has been done in the area of optimal search (see reference for Koopman 1980), but this work does not address the multi-target/false target scenario where an engagement comes at the expense of a search agent. Further, this work needs to be extended for application in cooperative behavior algorithms. This paper will present some basic analytical results for simple search scenarios in order to provide a setting for the numerical simulation work.

2 GOVERNING EQUATIONS

2.1 Single Munition/Single Target Case

A formula describing the probability of mission success for the single munition/single target scenario is as follows:

$$P_{MS} = P_K \cdot P_{TR} \cdot P_{LOS} \cdot P_E \tag{1}$$

where

P_K	$\equiv$	probability of target kill given Target Report (TR)
P_{TR}	$\equiv$	prob. of correct Target Report given clear Line of Sight (LOS) to the target
P_{LOS}	$\equiv$	prob. of clear LOS given target is in the Field of Regard (FOR)
P_E	$\equiv$	prob. of encountering the target given the target is in the search area.

The expression in (1) is not the most general, but is easily shown to be equivalent to the more general equations. For example, P_K represents the product of guidance, hit, and kill probabilities. P_{TR} represents the product of detection and confirmation probabilities, where confirmation could be either classification or identification depending upon the level of discrimination being employed by the munition being considered. P_{LOS} can be included in P_{TR}, and that is the convention that will be followed for the remainder of the development.

With the exception of P_E, the other probabilities are expressed as single numerical values, or, in the case of

P_{TR}, a table of values sometimes referred to as a confusion matrix, to be discussed in the next section. The term confusion matrix stems from the fact that it represents the probability of both correct *and* incorrect target reports. P_E is a function of the area to be searched, the density function describing the probable target location, and the ordering of the search process. Consider an autonomous munition looking for a single target (see Figure 1). For now we shall assume a single target is uniformly distributed amongst a Poisson field of false targets in the area A_S. A false target is considered to be something that has the potential for fooling the Autonomous Target Recognition (ATR) algorithm (e.g., similar size, shape). Because we are considering single shot munitions, the probability of successfully engaging a target in the incremental area ΔA is conditioned on not engaging a false target prior to arriving at ΔA. The incremental probability of encountering a target in ΔA can be expressed as:

$$\Delta P_E \;=\; P_{\overline{FTA}}(A) \cdot \frac{\Delta A}{A_S} \tag{2}$$

where $P_{\overline{FTA}}(A)$ is the probability of having no false target attacks while searching the area A leading up to ΔA. A closed form expression $P_E(A_S)$ can be obtained as follows. Let

η	$\equiv$	false target probability density
$P_{FTA\|FT}$	$\equiv$	probability of false target attack given encounter
α	$\equiv$	False Target Attack Rate (FTAR)
$P_{FT_{j,A}}$	$\equiv$	false target attack probability distribution

and define

$$\alpha = \eta P_{FTA|FT}. \tag{3}$$

$P_{FT_{j,A}}$ represents the distribution of j, the expected number of false target attacks which would be reported by the seeker in a non-commit mode, as a function of the area searched, A. It is a Poisson distribution with parameter $\lambda_{false} = \alpha A$.

$$P_{FT_{j,A}} \;=\; \frac{(\alpha A)^j e^{-\alpha A}}{j!}. \tag{4}$$

The probability of searching A without executing a false target attack is

$$P_{\overline{FTA}}(A) \;=\; P_{FT_{0,A}} \;=\; e^{-\alpha A}. \tag{5}$$

We can now formulate and solve an expression for the probability of encountering a target within A_S.

$$P_E(A_S) \;=\; \int_0^{A_S} \frac{e^{-\alpha A}}{A_S} dA \;=\; \frac{1 - e^{-\alpha A_S}}{\alpha A_S}. \tag{6}$$

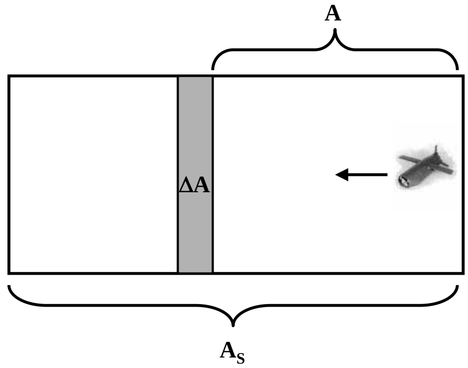

Figure 1: Single Target Search

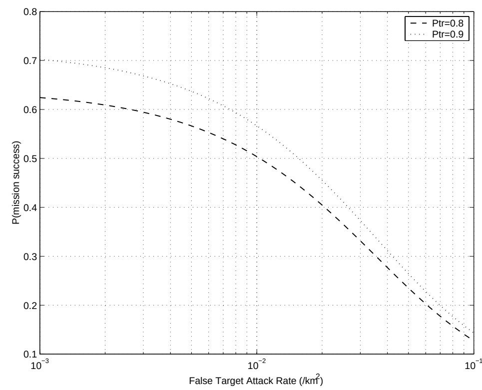

Figure 2: Single Target - Uniform Distribution

Note that the expression above assumes that the target is contained within A_S with probability one. For the case of uniform target/Poisson false target distribution, $P_E(A_S)$ can simply be multiplied by the probability that the target is contained within A_S, P_C. For cases of non-uniform target distributions a simple multiplication factor is no longer sufficient because the order of the search can affect the probability of encountering the target.

Figure 2 shows the sensitivity of mission success to the FTAR (α) and probability of correct target report (P_{TR}) for $P_k = 0.8$ and $A_S = 50\ km^2$. As shown, the probability of success begins to drop off rapidly for $\alpha > .01/km^2$. The problem is more sensitive to P_{TR} for low values of α than it is for higher values. While the probability of success may seem low, it can be improved by assigning multiple munitions to the same search area as will be discussed later.

2.2 The Single Munition/Multiple Target Case

There are several ways of looking at the multiple target case. If the objective is to find a specific target within a field of other targets, this could be treated in the same manner as the single target case; the other targets merely serve to increase the density of false targets. If any of the targets is considered valid then we need to be able to evaluate the probability of a successful encounter with any one of the targets. The single target case allowed us to determine the probability of finding and recognizing a target within a searchable area as

$$P_{RT}(A_S) \;=\; P_{TR}\ P_E(A_S). \tag{7}$$

For that case P_{TR} did not appear in the formulation for $P_E(A_S)$. For the multiple target case we will formulate it in a slightly different fashion. Referring back to Figure 1, the ability to find and recognize a target in the element of area ΔA is now conditioned on no false target attacks *and* no real target declarations/attacks prior to getting to ΔA. Assuming a Poisson distribution for both real and false targets (with $\lambda_{real} \neq \lambda_{false}$), our new formulation for the elemental probability of recognizing the target is

$$\Delta P_{RT}(A) = P_{TR}\ P_{\overline{FTA}}(A)\ P_{\overline{RT}}(A)\ \eta_T\ \Delta A \tag{8}$$

where η_T is the uniform target probability density. Implicit in this formulation is the assumption that $\eta_T \Delta A$, loosely interpreted as the probability of finding a target in the elemental area ΔA, is sufficiently less than one. This assumption is typically met for munitions with small instantaneous sensor footprints relative to the average target density in the area. $P_{\overline{RT}}(A)$, the probability of not having recognized a real target after searching A, is obtained in the same manner as $P_{\overline{FTA}}(A)$. Specifically, $P_{RT_{k,A}}$ represents the distribution of k, the number of target recognitions that would be reported by the seeker in a non-commit mode, as a function of the area searched, A. It is a Poisson distribution with parameter $\lambda_{real} = P_{TR}\ \eta_T\ A$:

$$P_{RT_{k,A}} = \frac{(P_{TR}\ \eta_T\ P_{LOS}\ A)^k e^{-P_{TR}\ \eta_T\ A}}{k!}. \tag{9}$$

The probability of searching A without executing a real *or* false target attack is

$$P_{\overline{RT,FTA}}(A) = P_{RT_{0,A}} \cdot P_{FT_{0,A}} = e^{-(P_{TR}\ \eta_T\ +\ \alpha)\ A}. \tag{10}$$

We can now formulate and solve an expression for the probability of recognizing a target within A_S:

$$\begin{aligned} P_{RT_m}(A_S) &= \int_0^{A_S} P_{TR}\eta_T e^{-(P_{TR}\eta_T+\alpha)A}\ dA \\ &= \tfrac{P_{TR}\eta_T}{(P_{TR}\eta_T+\alpha)}(1-e^{-(P_{TR}\eta_T+\alpha)A_S}). \end{aligned} \tag{11}$$

Figure 3 shows P_{MS} vs. α for the Poisson distributed multi-target case, with $\eta_T = .1/km^2$, $P_{TR} = 0.8$ and $P_k = 0.8$. As one would anticipate, it is far less sensitive to α than the single target case. Of greater interest is that the sensitivity to P_{TR} is greater for low values of α than it is for higher values; the opposite of the trend for the single target case. The reason for this is that a missed target is no longer a failed mission because there are other targets to be found. Further, the probability that these other targets will be encountered is high if the FTAR is sufficiently low.

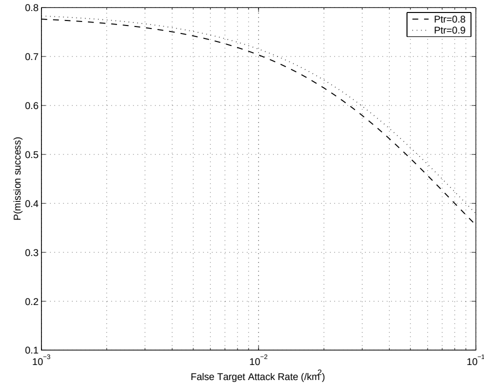

Figure 3: Multi Target - Uniform Distribution

One of the assumptions stated earlier was that of Poisson fields of false targets and, for the multi-target case, valid targets. Although a Poisson distribution will yield an expected number of false and/or valid targets within a specified area, the actual number in that area is a random variable. This is important from a simulation standpoint because too often Monte Carlo simulations are performed with the same number of targets and false targets for each repetition. Assuming a Poisson field of false targets, a uniform distribution of N targets will yield a higher probability of mission success than a Poisson field of targets with an expected number of targets equal to N. As N gets large, the two cases will converge. Similar differences can be shown for uniform distribution (fixed number) vs. a Poisson field of false targets. Although $P_{FTA|FT}$ and η appear in equation (6) as the product $\alpha = P_{FTA|FT} \cdot \eta$ only, it is incorrect to simulate the process by setting $\eta = \alpha$ and $P_{FTA|FT} = 1$. This effectively simulates a uniform distribution of false targets as opposed to a Poisson field. The more accurate way to simulate the Poisson distribution is to specify $P_{FTA|FT}$ using a Confusion Matrix (described in the next section) and distribute a sufficient (and variable) number of false targets to achieve the prescribed false target attack rate. Recall that η represents the expected spatial density of objects that could possibly be confused as real targets. Operationally it is determined by what target you are looking for and the environment in which you are looking. $P_{FTA|FT}$ is a measure of performance for the sensor and ATR algorithm. Because η is scenario dependent, the false target attack rate α will also be scenario dependent.

3 THE CONFUSION MATRIX

The typical means for describing the ability of an ATR based system to make correct decisions, whether based on classification or identification, is the confusion matrix. To start, consider the simplest confusion matrix where the only discrimination is between target and non-target. The confusion matrix for this simple case is shown in Table 3. In a simulation, the numbers in the confusion matrix are used to determine the outcome of a random draw each time an object, target or otherwise, is encountered. The probability numbers in the matrix can vary between 0 and 1, with a perfect algorithm having a value of 1 for P_{TR} and a value of 0 for $P_{FTA|FT}$. Note that since an encountered target must be declared either a target or a non-target, the sum of the probabilities in any column must sum to one. Also note that the ATR terms required for evaluation of equation (3) appear in the confusion matrix.

Table 1: Binary Confusion Matrix

	Encountered Object	
Declared Object	Target	Non-Target
Target	P_{TR}	$P_{FTA\|FT}$
Non-Target	$(1 - P_{TR})$	$(1 - P_{FTA\|FT})$

A more complex confusion matrix for several target and non-target types is depicted in Table 3. The matrix can be expanded to accommodate any number of target types (TP1, TP2, etc.), any number of non-target types (NT1, NT2, etc.) assumed to be of similar characteristics to the mission targets, and a category of clutter targets that includes everything else. The zeroes in the rows of the declared objects for NT1, NT2, etc. indicates that there are no templates or models that would enable the ATR to make an NT declaration. Any encountered object is either declared one of the mission target types or clutter. If a template or model is included to aid recognition of certain non-targets, non-zero numbers would appear in the row associated with that non-target type. As in the simple case, any encountered object must be declared something, so all columns must sum to one. This requires

$$P_{TRC|TPi} = 1 \; - \; \sum_{j=1}^{N_{TP}} P_{TRj|TPi} \; , \tag{12}$$

$$P_{TRC|NTi} = 1 \; - \; \sum_{j=1}^{N_{TP}} P_{FTAj|NTi} \; , \tag{13}$$

$$P_{TRC|CT} = 1 \; - \; \sum_{j=1}^{N_{TP}} P_{FTAj|CT} \tag{14}$$

where N_{TP} is the number of mission target types. At this point there are some subtleties which need to be clarified for the equations above. For purposes of this paper and the modeling approach described herein, an attack on a target of type TP_i is not considered a false target attack just because it was identified as type TP_j , $i \neq j$. They are both valid mission targets. Having said this, the equivalent probability of target report for a given target type can be expressed as

$$P_{TR|TPi} = \sum_{j=1}^{N_{TP}} P_{TRj|TPi}. \tag{15}$$

The FTAR for this more complex scenario can also be evaluated, but we now require spatial densities for clutter and all non-target types not included in the clutter description. The expression for FTAR is as follows:

$$\alpha \; = \; \sum_{i=1}^{N_{NT}}((\sum_{j=1}^{N_{TP}} P_{FTAj|NTi}) \cdot \eta_i) \; + \; (\sum_{j=1}^{N_{TP}} P_{FTAj|CT}) \cdot \eta_c \; . \tag{16}$$

Including multiple non-target types allows evaluation of the expectation for collateral damage on civilian vehicles or objects expected to be in the target area. Further, multiple non-target types allows us to include correlated behavior at false target encounters, as will be discussed in a later section. For higher fidelity modeling, any number of non-target types may be included. However, the mission targets in the simulation should be restricted in both type and number according to what the munition is capable of. For example, if a given munition is only capable of processing a single model or template for a given mission, there must only be a single target type declaration in the confusion matrix.

One way of increasing the fidelity of the simulation is to consider target orientation in the definition of the confusion matrix. Any ATR based seeker will have some aspect angles for which it performs better than others. If the targets in the simulation have orientation, it makes sense to specify a confusion matrix that varies with aspect angle, assuming of course the availability of data and or analysis to support how it varies. This could be done with a separate matrix for each quantized aspect angle, or a single matrix specified as a function of aspect angle. This will also produce correlated behavior for the multi-munition simulation. Once again, the correlated behavior aspects will be discussed in a later section.

Table 2: Multi-Target Type Confusion Matrix

	Encountered Object				
Declared Object	TP1	TP2	NT1	NT2	CT
TP1	$P_{TR1\|TP1}$	$P_{TR1\|TP2}$	$P_{FTA1\|NT1}$	$P_{FTA1\|NT2}$	$P_{FTA1\|CT}$
TP2	$P_{TR2\|TP1}$	$P_{TR2\|TP2}$	$P_{FTA2\|NT1}$	$P_{FTA2\|NT2}$	$P_{FTA2\|CT}$
NT1	0	0	0	0	0
NT2	0	0	0	0	0
CT	$P_{TRC\|TP1}$	$P_{TRC\|TP2}$	$P_{TRC\|NT1}$	$P_{TRC\|NT2}$	$P_{TRC\|CT}$

4 ANALYTIC MULTI-MUNITION EXTENSIONS

The single target scenario can be extended to the multi-munition case in several ways. The easiest way is to divide the total search area by the number of munitions, and determine the P_K for the munition searching the subarea that the target appears in. All other munitions find nothing for the single target case. P_{MS} will increase because A_S will decrease for all munitions, including the munition searching the subarea where the target happens to be. However, because this method assumes zero overlap in the subareas being searched, the probability of mission success is ultimately limited by the P_{TR} and P_K for the single munition. If the product of these is not sufficient to provide the desired probability of mission success from a single munition-target engagement, then overlapping search areas and multi-munition engagements must be considered.

Extending expressions (1) and (6) above for multiple munitions becomes quickly complicated by path considerations and the degree of correlation assumed for the behavior of the munitions as they encounter either real or false targets. Considering only the terminal engagement for the time being, we can assume independent events for each warhead shot yielding an expression for $P_K^{[N]}$, the probability of kill for the case of N munitions finding the target:

$$P_K^{[N]} = 1 - (1 - P_K)^N \ . \tag{17}$$

Of course the assumption of independent warhead events means it does not account for cumulative damage resulting from multiple munition attacks. One could (incorrectly) assume a similar roll-up of P_{MS} for the case of N munitions all searching the same area for a single target:

$$P_{MS}^{[N]} = P_C(1 - (1 - \frac{P_{MS}}{P_C})^N) \ . \tag{18}$$

The reason this formulation is incorrect is because it assumes the placement of clutter, non-targets and the real target is re-randomized for each munition. This can never be true for the case of several munitions looking for the same target in the same location. To address the problem correctly requires some consideration of the search path followed by the individual munitions. For the distributions discussed in this paper, analytic expressions have been derived for the case of 100% overlap with identical paths, and 100% overlap with opposing paths (see Jacques, 2002). For the same total number of munitions, the opposing path case will produce the highest mission success value, the same path case will produce the lowest mission success value, and the simple multi-munition roll-up will produce a value in between the other two. The graph shown in Figure 4 is for two munitions, but it should be noted that the differences between the curves increases with the number of munitions used in the analysis.

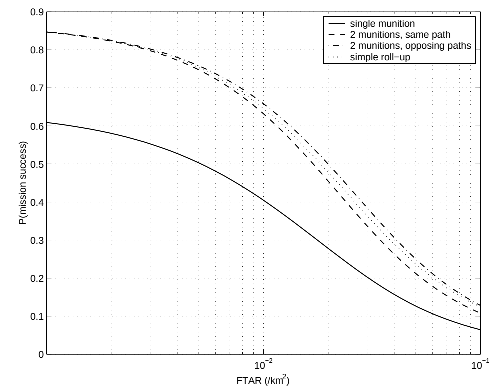

Figure 4: Path Considerations for Multi-Munition Case

It is worth repeating that the assumption of uncorrelated behavior (at either a real or false target) is not strictly valid, and we should expect a high degree of correlation for the case where the munitions are traversing the same path in the same direction. For scenarios where the potential false targets greatly outnumber the real targets, correlated behavior will degrade the overall mission success rate. For this reason, search patterns should be planned which decrease the degree of correlated behavior at false targets. This can be done through the use of lateral offsets between munitions and/or different approach vectors. While this does not make the assumption of uncorrelated behavior valid, it can reduce the degree of correlation at both targets and false targets.

Analytically it becomes intractable to define an expression for arbitrary numbers of munitions executing arbitrarily specified search patterns and degrees of correlation. However, for any realistic effectiveness analysis these are the cases we are most interested in. A numerical simulation with Monte-Carlo runs is the only practical way of performing this more general analysis, and the remainder of this paper will concentrate on the required elements of a practical yet realistic simulation.

5 MULTI-MUNITION/MULTI-TARGET MODELING

The preceding discussion provides all the elements needed for a reasonable simulation of the ATR processes in a multi-munition/multi-target scenario. As each munition moves around the search area, either independently or cooperatively, it encounters targets, non-target vehicles and/or clutter objects. Upon encountering any object, a random draw is accomplished and the confusion matrix is used to determine the outcome of the encounter and any subsequent engagement. What remains is to show that this approach is valid and will capture some of the known effects such as correlated behavior.

As in any modeling approach, the end result will be no better than the numbers we put in the simulation. However, we are not trying to simulate the inner workings of the munitions, merely those statistical processes that drive the resulting effectiveness. For this reason we can get by with limited data requirements and still be able to capture the important results. Certainly we will need basic information regarding vehicle speed, sensor field of regard, and search patterns in order to support the dynamic elements of the simulation. Additional data such as munition reliability, dispense accuracy and delivery timelines may also improve the fidelity of the simulation. Beyond these basic data requirements, we need the values for the terms in equation (3) and the confusion matrix. From flight test, hardware in the loop (HIL), and/or 6-DOF simulations we can obtain estimates of guidance accuracy. Further, warhead tests and lethality analysis can provide estimates of P_K for given targets and given warhead/guidance accuracy. Captive sensor/ATR flight tests can provide data to fill out the confusion matrix, including sensitivity to target aspect angle, if any. The captive test results can also evaluate FTAR, although it is worth repeating that FTAR is both a function of what you are looking for *and* the terrain where you are searching for it. For example, the FTAR associated with looking for a SCUD missile launcher in the desert will be significantly lower than the FTAR associated with looking for a command van around a highway intersection. The $P_{FTA|FT}$ numbers can typically be estimated from sensor/ATR captive tests, but the density values, η, must be varied according to the different types of objects and terrain expected in the search area. Previous analyses have shown FTAR to be the single most critical factor determining the success rate of wide area search munitions, and any serious effort should include a sensitivity analysis for FTAR by varying $P_{FTA|FT}$ or η. Such a sensitivity analysis should consider the affects of P_{TR}, P_K and other critical factors as well.

5.1 Correlated Weapon Behavior

There are two types of correlation that we are concerned with. Correlation at a target and correlation at a clutter or false target object. At a real target, a situation which causes a first munition to miss a target is more likely, but not certain, to cause a second munition to miss if the situation still exists when the second munition arrives at the same target. Similarly, whatever might cause a first munition to falsely attack a non-target may cause a second munition to do the same. We will certainly want to account for the case where the conditional probability of false target attack given one or more previous false attacks is higher than the *a priori* probability $P_{FTA|FT}$. Conversely, the conditional probability given that previous encounters did not result in a false target attack should be lower than the *a priori* probability.

The discussion of the multi-target confusion matrix presented an expression for the overall FTAR, equation (16). For the present discussion we will ignore the separate category of clutter targets; they are considered to be represented as one or more of the NTi's. Given a non-target encounter of a specific non-target type, we can express the probability of false target attack as

$$P_{FTA|NTi} = \sum_{j=1}^{N_{TP}} P_{FTAj|NTi} \ . \tag{19}$$

We will also need to define the probability that an encountered non-target is of a given type, $P_{NTi|FT}$. This is merely the proportion of the total expected number of non-targets in the area that will be of type i. We can express the *a priori* probability of false target attack given an unknown non-target type is encountered as

$$\begin{aligned} P_{FTA|FT} &= \sum_{i=1}^{N_{NT}} P_{FTA|NTi} P_{NTi|FT} \\ &= \sum_{i=1}^{N_{NT}} ((\sum_{j=1}^{N_{TP}} P_{FTAj|NTi}) \cdot P_{NTi|FT}) \ . \end{aligned} \tag{20}$$

This is the *a priori* probability that results from using a random draw and confusion matrix at each non-target encounter of unknown type. The correlated behavior resulting from this approach is due to the fact that a given non-target will be of a specific type where, in general, either $P_{FTA|NTi} < P_{FTA|FT}$ or $P_{FTA|NTi} > P_{FTA|FT}$. Therefore, all munitions encountering this non-target will have a probability of false target attack either less than or greater than, respectively, the *a priori* value. Viewed another way, if a first munition has falsely reported an unknown non-target as a real target, there is a greater probability that it is a non-target type that has a higher probability of false target attack associated with it, and a second munition encountering this same non-target will therefore have a greater probability of falsely reporting it as a real target. To show this, we can derive the probability of false target attack for a second munition given two scenarios. The first scenario will assume a first munition encountering the same non-target falsely reported it as being a target, and the second scenario will assume the first munition did not falsely report the non-target as a target.

Suppose there are a total of NNT non-target objects in the scenario. Define the following quantities:

$$\begin{aligned}
N_{NT_j} &\equiv \text{expected \# false targets of type } j \\
&= P_{NTj|FT} \cdot NNT \\
N_{FTA_j} &\equiv \text{expected \# false target attacks of type } j \\
&= P_{FTA|NTj} \cdot N_{NT_j} \\
&= P_{FTA|NTj} \cdot P_{NTj|FT} \cdot NNT \\
N_{FTA} &\equiv \text{expected total \# false target attacks} \\
&= \textstyle\sum_{j=1}^{N_{NT}} N_{FTA_j} \\
&= NNT \cdot \textstyle\sum_{j=1}^{N_{NT}} (P_{FTA|NTj} \cdot P_{NTj|FT}) \\
&= NNT \cdot P_{FTA|FT} \ .
\end{aligned}$$

With these, we can state the probability that a given false target attack is of non-target type j as

$$\begin{aligned}
P_{NTj|FTA} &= \frac{N_{FTA_j}}{N_{FTA}} \qquad (21) \\
&= \frac{P_{FTA|NTj} \cdot P_{NTj|FT}}{P_{FTA|FT}}
\end{aligned}$$

and the probability that a given non-target is of type j given that a first munition did not falsely report it as a target is

$$P_{NTj|\overline{FTA}} = \frac{(1 - P_{FTA|NTj}) \cdot P_{NTj|FTA}}{1 - P_{FTA|FT}} \qquad (22)$$

where $\overline{FTA}$ indicates a false target attack did not occur. With these expressions, we can now state the two conditional probabilities we are interested in. First, the conditional probability of falsely reporting a target at a non-target given that a first munition also falsely reported it as a target.

$$\begin{aligned}
P_{FTA|FTA1} &= \sum_{j=1}^{N_{NT}} (P_{FTA|NTj} \cdot P_{NTj|FTA}) \qquad (23) \\
&= \frac{1}{P_{FTA|FT}} \sum_{j=1}^{N_{NT}} ((P_{FTA|NTj})^2 P_{NTj|FT}) \ .
\end{aligned}$$

The conditional probability of falsely reporting a target at a non-target given that a first munition did not falsely report it as a target is given as:

$$\begin{aligned}
P_{FTA|FTA0} &= \sum_{j=1}^{N_{NT}} (P_{FTA|NTj} \cdot P_{NTj|\overline{FTA}}) \qquad (24) \\
&= \frac{1 - P_{FTA|FTA1}}{1 - P_{FTA|FT}} \cdot P_{FTA|FT} \ .
\end{aligned}$$

Note that for both of these equations, it does not matter which of the munitions arrives first. We have symmetry with respect to the individual munitions (as we should). Also note that the correlated behavior is automatic as long as we have different values in the confusion matrix for different target and non-target types. We can use the values in the confusion matrix to create almost any degree of correlation that we wish. This holds for different target types as well as different angles of approach for the same target type.

A simple example will serve to illustrate the point. Consider a scenario where there are three types of non-targets, with $P_{FTA|NT1} = 0.05$, $P_{FTA|NT2} = 0.1$, and $P_{FTA|NT3} = 0.2$. Further suppose that 60% of the non-targets are of type $NT1$, 10% are of type $NT2$, and the remaining 30% are of type $NT3$. For a non-target encounter of unknown type, $P_{FTA|FT} = 0.1$. If a first munition has a false target report on the unknown non-target, it is twice as likely that the non-target is of type $NT3$ than it is of type $NT1$, even though there are expected to be twice as many targets of type $NT1$ as compared to $NT3$. Because of this increased likelihood of the non-target being of type $NT3$, the probability of a second munition falsely reporting the non-target (given that a first munition did) is 0.145, which is significantly higher than the *a priori* probability of 0.1. The probability of a second munition falsely reporting the non-target given that a first munition did not falsely report it is 0.095.

6 APPLICATION

While the focus of this paper is clearly on the modeling considerations themselves and not the simulation results, it is useful to compare analytical and simulation results for the simple scenarios discussed here. A simple simulation was developed that allowed one or more munitions to search for a single target along either the same or opposing paths. A binary confusion matrix was implemented as the sensor model, and varying levels of warhead lethality and false target density were considered. The results of the simulation were compared to the analytical formulations discussed earlier. Table 3 shows the comparison results. There is good agreement between analytical prediction and the simulation results. Some differences between the two approaches are expected, as the simulation approximates a Poisson distribution of false targets with a uniform spatial distribution of a normally distributed number of false targets.

This basic simulation can also be used to evaluate the expected benefit of cooperative classification. A simple scenario consists of two munitions searching along the same path for a single target. For the non-cooperative case, any munition that declares an object to be a target will immediately initiate an attack. For the cooperative case, an initial target declaration by one munition must be confirmed by the second munition. If not confirmed, both munitions continue to search. If the target declaration is confirmed (correctly or incorrectly) by the second munition, both munitions initiate an attack with a combined lethality calculated as $P_{K_2} = 1-(1-P_K)^2$. Assuming independent events, the effective probability of target report and false target attack rate of the two munition system is

$$P_{TR_2} = (P_{TR})^2 \tag{25}$$
$$\alpha_2 = (P_{FTA|FT})^2\eta \tag{26}$$

respectively. Table 4 shows a comparison of results for the cooperative and non-cooperative case. While the cooperative case typically outperforms the non-cooperative case, there are scenarios characterized by high warhead lethality and low false target attack rate where cooperative classification is detrimental to mission effectiveness. The reason for this is that, for these scenarios, the reduction in FTAR is not sufficient to offset the detrimental effects of a reduced probability of target report for the two munition system.

7 CONCLUSIONS

This paper has presented modeling considerations for wide area search munition effectiveness analysis. Governing equations have been presented to provide a means for validating simulation results. Critical parameters were identified, and their role in the simulation were defined. Path dependence and the effect of correlated behavior on simulation results were discussed. Finally, it was demonstrated that the simulation approach described herein will provide a degree of correlated behavior at either false or real targets, and simulations resulting from this approach can be validated against analytical models.

REFERENCES

Gillen, D. and D. Jacques 2002. *Cooperative Behavior Schemes for Improving the Effectiveness of Autonomous Wide Area Search Munitions*. Cooperative Control and Optimization, Kluwer Publishing.

Jacques, D. 2002. *Search, Classification and Attack Algorithms for Cooperative Wide Area Search Munitions*. To appear in: Workshop on Cooperative Control and Optimization, Kluwer Publishing.

Koopman, B.O., 1980. *Search and Screening*. Pergamon Press, New York.

AUTHOR BIOGRAPHY

DAVID R. JACQUES is a Lieutenant Colonel in the US Air Force and an Assistant Professor of Aerospace Engineering at the Air Force Institute of Technology. His current research interests are in the area of cooperative behavior and control of autonomous and semi-autonomous air vehicles. Previous experience includes program manager of a technology demonstration program for an autonomous wide area search munition. His e-mail address is `<david.jacques@afit.edu>`.

Table 3: Simulation Result vs. Analytical Calculation

Search Pattern	P_{TR}	FTAR	P_K	Expectation of Mission Success: Sim Result	Calc Result	Δ (%)
Same Path	0.8	0.005	0.5	.516	.528	2.3
Same Path	0.8	0.005	0.8	.723	.748	3.4
Same Path	0.8	0.05	0.5	.143	.143	-0.1
Same Path	0.8	0.05	0.8	.204	.213	4.4
Opposing Path	0.8	0.005	0.5	.541	.533	-1.6
Opposing Path	0.8	0.005	0.8	.760	.759	-0.2
Opposing Path	0.8	0.05	0.5	.171	.162	-5.5
Opposing Path	0.8	0.05	0.8	.261	.252	-3.8

Table 4: Cooperative vs. Non-cooperative Classification

Search Pattern	P_{TR}	FTAR	P_K	Expectation of Mission Success: P_{MS} Non-coop	P_{MS} Coop	Δ (%)
Same Path	0.8	0.005	0.5	.516	.547	6.0
Same Path	0.8	0.005	0.8	.723	.703	-2.8
Same Path	0.8	0.01	0.5	.419	.523	24.8
Same Path	0.8	0.01	0.8	.613	.674	9.9
Same Path	0.8	0.05	0.5	.143	.375	162.2
Same Path	0.8	0.05	0.8	.204	.485	137.5