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SIMULATION-BASED CONSTRAINT GENERATION WITH APPLICATIONS TO OPTIMIZATION OF LOGISTIC SYSTEM DESIGN

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ABSTRACT

We present in this paper a general framework for a combined optimization/simulation approach where constraints to be satisfied are identified from results of simulation evaluation of the proposed system alternative, and then these constraints are added to the optimization model for re-optimization. The proposed cutting-plane-like procedure is iterative and terminates when an “optimal" solution of a mathematical program is obtained which passes all conditions of performance criteria set for simulation evaluation. A case of the real large-scale logistic system design is taken as an example, and the proposed approach is shown to work efficiently for the case, and looks promising for other problems especially in the field of logistics and scheduling.

1 INTRODUCTION

There exist many complex systems of prime importance which have to be designed so that certain goals are optimized. Modeling and analyses are often critical for the successful design of a good system. Mathematical programming provides a powerful optimization tool, whereas simulation is well-suited for performance evaluation with greater flexibility.

If alternatives are limited in number, each of them could be evaluated via simulation. As the number of alternatives increases, however, repeating many simulation runs could be time-consuming and costly. Mathematical programming is the proper choice for such a situation. Yet, modeling all details of the target system by a mathematical program may be inappropriate or impossible computationally. A natural choice then would be to combine optimization via mathematical programming with evaluation via simulation.

Presented in this paper is a general framework for combined optimization/simulation where simulation results are used to identify constraints to be satisfied, and then these constraints are added to the optimization model for re-optimization. The procedure is iterative and can be regarded as a class of cutting plane approaches, which terminates when an “optimal" solution of the mathematical program is obtained which passes all performance criteria set for simulation evaluation. A case of the real large-scale logistic system design is taken as an example, and it is shown that the proposed framework works efficiently for the case where optimization via mixed integer programming is combined with evaluation via discrete-event simulation.

The paper is organized as follows: The next section presents the general cutting-plane-like framework for combined optimization/simulation, and identifies key elements needed for the approach. After describing the case of logistic system design of postal service in Section 3, an optimization model is presented in Section 4. After describing the outline of the detailed simulation model, Section 5 describes how constraints are generated, together with the performance of the proposed approach for the logistic system design case. Conclusions and future works are given at the end of the paper.

2 A GENERAL FRAMEWORK FOR COMBINED OPTIMIZATION/SIMULATION

Optimization and performance evaluation can be regarded as the two main functions of operations research techniques. Correspondingly, mathematical programming and discrete-event simulation are the two most powerful tools of OR/MS with many commercial software available.

Weaknesses and strengths of these tools are complementary. That is, mathematical programming is weak in such factors as unknown input/output relationship, system dynamism, nonlinearity, randomness, and system details, whereas simulation is weak in its optimization capability. It then is natural to think of their combination, provided that there exist appropriate problems suitable for the combined optimization/simulation.

In fact, there are many problems of importance which are optimization in nature, yet too complicated to include all their details. For these problems, we propose the following cutting-plane-like framework for combined optimization/simulation:

2.1 Basic Framework

Step 1 (Basic Model Construction) Construct a basic optimization model which takes essential factors of the problem into considerations and also a simulation model which may include details of the system.

Step 2 (Optimization) Solve the resultant optimization model and obtain an "optimal" solution.

Step 3 (Simulation) Simulate the "optimal" solution obtained in Step 2, and evaluate its performance to see if the solution is acceptable or not.

Step 4 (Constraint Generation) If the solution is acceptable, the "true" optimal solution is found and stop. Otherwise, identify constraints necessary to make a solution acceptable. Add the constraints to the current mathematical program, and return to Step 2 for re-optimization.

Some comments are in order concerning the above basic framework:

1. The general path from Steps 1 through 3 is natural and commonly used, but one often takes a heuristic approach to make a solution feasible when the proposed solution was found to be non-acceptable as a result of simulation.
2. The optimization model often includes economic considerations in the form of cost minimization or profit maximization.
3. After a solution candidate is evaluated via simulation in Step 3, one checks its feasibility. Feasibility or infeasibility will normally be determined by checking if specified conditions are satisfied or not. This check can often be made by a form which conforms with mathematical programming models, namely, as constraints expressed by (hopefully linear) inequalities.
4. Constraints identified in Step 4 are nothing but "cuts" in mathematical programming, which eliminate "the current solution" found to be non-acceptable in Step 3. The addition of the cuts makes the current solution infeasible just like cutting plane algorithms of integer programming which eliminates a part of (LP-)feasible region as in Fig.1a.
5. In our framework, the mathematical programming model generally is a relaxation of the "true" model in the sense that the "true" model is well-defined, but is not completely defined (but only partially defined) by the mathematical programming model. An acceptable "optimal" solution is a solution which is optimal with respect to the objective function defined in the mathematical program, and satisfy all well-defined conditions specified in the "true" model. Refer to Fig.1b. Acceptable solutions are those "points" in the funny-shaped region which is included in the mathematical programming (say, integer programming) "relaxation" of the "true" model. Note that feasible integer points of the mathematical programs generally changes as we add cuts.

Combining optimization with simulation has been explored in many past research in several different ways. In particular, when performance measures appearing in the objective function are evaluated via simulation, several approaches have been developed and are often called as simulation optimization. See, e.g., Glover, Kelly, and Laguna(1996), Morito and Lee(1994).

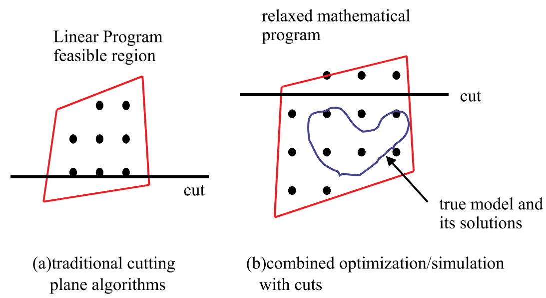

Figure 1: Cutting Plane Approaches

2.1.1 Conditions Required for Generated Constraints

Conditions required for generated constraints are essentially same as those for traditional cuts, and the constraints (cuts) give restrictions to the decision variables in such a way that:

1. constraints make the "current optimal" solution infeasible, and
2. constraints do not eliminate any solutions of the "true" model.

How this can be achieved depends on the scenario of interest, and a general theory of constraint generation is to be developed. In Section 5, we describe how this is done for the case of logistic system design.

3 PROBLEM DESCRIPTIONS: A CASE OF THE POSTAL LOGISTIC SYSTEM DESIGN

Postal service can be regarded as a gigantic logistic system. Efficiency improvement has been regarded as one of the important missions of the Japanese Postal Service in today's tighter economy and more competitive environment. The case described below reflects a quantitative study to analyze alternative plans for equipment allocation to post offices, and to find an "optimal" allocation plan. The analysis is performed on the basis of optimization of equipment allocation via a mixed integer programming problem, and also of simulation to analyze feasibility of the optimization-generated equipment allocation plan under a more realistic and dynamic environment.

Expanded 7-digit postal code together with bar code encoding have been introduced in Japanese postal service in February 1998. Efficient, yet cost-intensive, automatic sorting machines have been utilized at major regional as well as large-scale city post offices.

Under manual sorting, one can imagine that sorting operations at individual acceptance post offices used to be more preferable. With the introduction of machines for reading numerical postal codes and/or bar codes and for sorting, however, one naturally wants to process more mails with a smaller number of machines. One possible alternative to increase machine utilization is to centralize mail processing at post offices equipped with sorting machines.

Those post offices equipped with sorting machines, however, are only a small portion of post offices which perform sorting operations. In some offices, sorting operations are still performed manually.

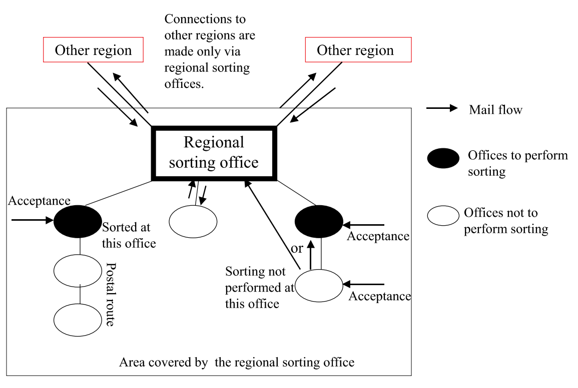

Figure 2: Postal Logistic System Design

With the introduction of more powerful sorting machines, one possibility is to centralize some sorting operations at a fewer offices equipped with powerful machines. German postal service, for example, uses such a "centralized" approach. Switch from expensive manual labor to machine sorting, together with very scarce space in major city post offices make the centralization appealing.

Centralization tends to increase machine utilization in general, but extra mail transportation and handling are expected to move mails from an acceptance office to an office where sorting operations are performed.

The basic elements of the problem are then as follows:

3.1 Decision Variables

1. how many sorting machines of each type as well as manpower to allocate for mail sorting to each station, and
2. which post office is to sort mails accepted at each post office.

3.2 Major Conditions

1. (**Time Restriction**) Depending on the time a mail arrives at a sorting office, sorting must be completed by prespecified time.
2. (**Space Limitation**) There is space limitation for sorting machines.
3. (**Available Machine/Labor Hours**) Time required to sort mails should match sorting capacity of the office. Machines or workers may not be used during certain hours of a day.
4. (**Dynamic Fluctuation**) Hourly total of accepted mails is known for each hour and for each office.
5. (**Truck Schedule**) Spoke-like truck routes go out from the regional office, and the associated truck schedules are assumed to be known.

3.3 Performance Measures

1. The goal is to minimize the total cost which consists of equipment cost, labor cost, and the cost of "unnecessary" transportation.
2. The cost of unnecessary transportation is assumed to be a linear function of cumulative distance traveled by such mails.

3.4 Basic Strategy of Analysis

The problem basically is that of optimization. Yet, considerations of dynamic and detailed factors make the mathematical program intractably large. Therefore, we opt to consider an optimization model which includes only 1,2,3 of the aforementioned conditions in a static fashion, and remaining conditions such as 4 and 5 are checked via simulation.

4 OPTIMIZATION VIA MIXED INTEGER PROGRAMMING

4.1 Notation

variables:

m_{ij}	:	the number of sorting machines at office i
n_i	:	labor hours at office i
x_{ij}	:	1 if mails received at office i are processed at office j, 0 otherwise

constants (input data):

a_i	:	equipment cost of sorting machine i per day per machine
b	:	labor cost per hour per head
c_{ij}	:	distance between office i to office j
q_{ij}	:	mail OD quantity from office i to office j
d_i	:	processing speed of a sorting machine i per hour
e	:	manual processing speed
v_i	:	space required for a sorting machine i
u_i	:	space allowance of office i

4.2 Performance Measure

The objective is to minimize costs. We consider costs for equipment, labor, and unnecessary transportation. To express these factors in the same monetary unit, we introduce two parameters, i.e., unit transportation cost coefficient s, and labor cost coefficient t. The cost of unnecessary transportation is assumed to be a linear function of cumulative distance traveled by such mails.

4.3 Assumptions and Constraints

1. Mails accepted at office i are to be sorted at a single office j, which may or may not be same as i.
2. A post office i which does not sort its own accepted mails may not sort mails accepted at other offices $j \neq i$.
3. The total mail quantity to be sorted at an office should be within the processing capacity of the office.
4. Space occupied by sorting machines allocated at a specific office should be within the designated space allowance.

4.3.1 Formulation

$$Min \quad \sum_i \sum_j (a_j m_{ij} + t * bn_i)$$
$$+ s * \sum_i \sum_j \sum_k x_{ij}(c_{ij} \sum_l q_{il} + c_{jk} q_{ik}) \tag{1}$$

$$\sum_j x_{ij} = 1 \qquad \forall i \tag{2}$$

(In(2), summation goes over offices j which are on the same route as i and are closer to the regional office.)

$$x_{ji} - x_{ii} \leq 0 \qquad \forall i, j \tag{3}$$

$$\sum_i (\sum_j q_{ij}) x_{ij} - \sum_l d_l m_{il} - e n_i \leq 0 \qquad \forall j \tag{4}$$

$$\sum_j v_j m_{ij} - u_i \leq 0 \qquad \forall i \tag{5}$$

$$x_{ij} \in \{0, 1\}\ ,\ \ 0 \leq m_{ij} \in R \quad \forall h, i, j \tag{6}$$

4.4 Results of Optimization

The mixed integer program as formulated in 4.3.1 can be solved by a standard optimization software. Figure 3 reflects results for the case of a regional post office in Tokyo with 14 truck routes and 30 post offices. Specifically, it shows which offices to sort mails assuming 3 different values of unit transportation cost. As penalty for unnecessary transportation increases (say, $s = 10$), one can see that mails tend to be processed at more offices. Many other

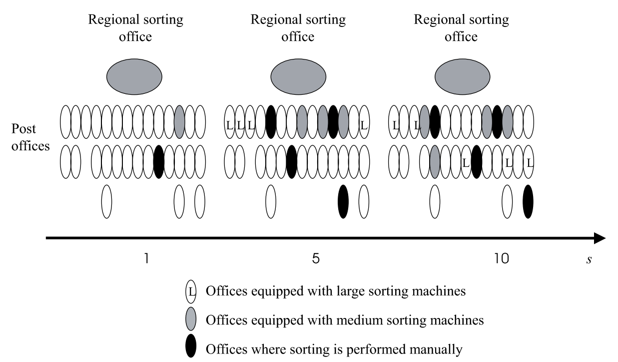

Figure 3: Optimization Results

analyses could be performed with the model and its variants, and will be described in Koida, et al.(1999).

5 SIMULATION IN REALISTIC AND DYNAMIC ENVIRONMENT, CONSTRAINT GENERATION, AND RE-OPTIMIZATION

5.1 Simulation

An “optimal" solution obtained from optimization will now be evaluated by simulation in a more realistic and dynamic environment. Specifically, hourly mail arrival patterns and detailed track schedules are included in simulation analysis to check if sorting finishes by the deadline. Key assumptions for the simulation model are as follows:

1. The simulation period is a 24-hour day.
2. Currently used truck routes and schedules are used. Normally, a truck travels each route 3 times a day. The smallest time period considered is 5 minutes, and no traffic delay is considered.
3. Mail receipts at post offices are assumed to occur every hour on the hour.
4. Preprocessing before sorting of received mails is assumed to take 30 minutes. without looking at the details.
5. Sorting machines and workers may not be used during certain hours of a day.

5.1.1 Simulation Results

Simulation traces dynamically the progress of mail processing as in Figure 4, and we can judge if sorting finishes before deadline or not.

5.2 Constraint Generation and Re-Optimization

We now consider a method of constraint generation when simulation results are not acceptable. In our case, infeasibility occurs at some post office(s) where mail processing is not completed by the predetermined time. This could occur

Figure 4: Simulation Results

when mails received at office i are sorted at other office j, and processing capability at office j is not sufficient.

It then is possible to calculate the necessary processing capacity, say c, at office j to meet the deadline, and the constraint which says that

> “the processing capacity at office j should be at least c if mails received at office i are to be sorted at office j "

is added. Note that this logical condition can be expressed as the following form of a linear inequality:

$$c * x_{ij} - \sum_{l} e_l m_{jl} \leq 0 \qquad (7)$$

Note also that when the capacity of an office which sorts only mails received at the office is not sufficient, use the inequality with $i = j$.

5.2.1 Finiteness of the added constraints

Obviously, there exist only finitely many patterns of processing those mails received at a certain office in other office. If a specific pattern of processing is given, the capacity of each office which performs sorting can be calculated, and thus parameter c above can be determined. Therefore, there exist only finitely many constraints of the above form, even though the value of parameter c depends on the particular combination and must be evaluated from simulation results. Therefore, the process of adding the constraints could not continue forever, and the approach terminates after a finite number of constraint additions.

5.2.2 Experimental results

Table 1 shows the results of computational experiment for the logistic system design case. The first optimization result was not acceptable as simulation revealed that 3 offices did not complete sorting by deadline. After cuts are introduced, the mathematical program is re-optimized to obtain a new solution with 0.6 % cost degradation. This solution still was not acceptable as one station still missed the deadline. A new cut was added, and re-optimization produced further 0.3

Table 1: Experimental Results

Iteration no.	No. of offices not completing sorting by deadline	Relative value of objective function
1	3	1
2	1	1.006
3	0	1.009

% cost increase, but the resultant solution was acceptable, and thus optimal.

Note that the approach is extremely efficient for the case considering the fact that the number of possible processing patterns is 2^{30}, which is astronomical (even though finite).

6 CONCLUSIONS AND FUTURE WORK

A natural yet novel framework for combined optimization/simulation has been proposed, and its successful application to the case of postal logistic system design is discussed. Specifically, key ingredients of the framework have been identified, among which a mechanism for constraint generation is critical.

Future research will be needed in the following areas to make the approach more attractive and efficient:

1. to demonstrate that the proposed framework in fact works for many other important and interesting scenarios, and
2. to make the approach computationally more efficient by, i.e., exploiting, in the optimization algorithm, the results of previous iteration of optimization.

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