# ADVANCED INPUT MODELING FOR SIMULATION EXPERIMENTATION

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# ABSTRACT

We discuss ideas useful to simulation practitioners when specifying the probability models used to represent stochastic behavior. Emphasis is on situations in which the classical simple models are inadequate. After discussing some general modeling issues, we consider univariate distributions, nonnormal random vectors and time series, and nonhomogeneous Poisson processes.

# **1** INTRODUCTION

We assume that the reader is familiar with Nelson and Yamnitsky (1998), last year's excellent advanced tutorial on simulation input modeling. That tutorial is *advanced* in the sense that it focuses on nonclassical univariate distributions and on dependence, both statistical and through time. In addition to being advanced, their tutorial is practical in that it focuses on ideas supported by easily obtainable software. (For an introductory tutorial, see any of Leemis 1996, 1997, 1998, 1999). Simulation textbooks often discuss input modeling, but typically not the advanced topics considered in Nelson and Yamnitsky (1998).

Rather than repeating the content of their advanced tutorial, we expand upon it via comments and opinions. Some comments are historical, some point to additional references and ideas, some concern common errors. The opinions, we hope, are obviously opinions and that the reader, at least upon reflection, agrees with them. Such agreement is sometimes difficult to attain, however, as evidenced by Kelton et al. (1990). In the spirit of a tutorial, no ideas here are new. A fairly extensive list of references is provided, with the large number from recent Winter Simulation Conferences indicating that input modeling remains a topic of substantial interest.

We organize our thoughts beginning with some general issues, followed by sections on univariate distributions, random vectors and time series, and the nonhomogeneous Poisson process.

# 2 GENERAL ISSUES

Underlying our discussion of input modeling is our view of simulation experiments (Nelson and Schmeiser 1986, Schmeiser 1990). We assume that the purpose of an experiment is to estimate a performance measure  $\theta$ , a (not necessarily scalar) constant with unknown value. The simulation experiment can be represented by the sequence of transformations

$$G \to U \to X \to Y \to \hat{\theta}$$

where  $\hat{\theta}$  is a point estimator of  $\theta$ , which is a characteristic (e.g., mean or quantile) of the unknown probability model of the output data Y. The output data Y are a known function g of the input data X, whose probability model is assumed to be known. The input data X are generated as random variates (see, e.g., Devroye 1986) using the U(0, 1) random numbers U, which are generated from a (pseudo)random number generator G.

### 2.1 The Logical and Input Models

We state this formalism to emphasize that a simulation model has two application-dependent components: the deterministic *logical* model g, which transforms the input data X into output data Y, and the probabilistic *input* model, which represents that stochastic behavior. Given a logical model and an input model, the simulation experiment can be run to obtain the point estimator  $\hat{\theta}$ , which typically goes to  $\theta$  in the limit as the simulation run length goes to infinity. Whether conclusions about the value of the model's performance measure  $\theta$  apply to the real-world system of interest depends directly upon the degree to which the logical and input models are valid (i.e., match the real-world system).

Commercial simulation software provides extensive support in creating the logical model, with relatively little support for creating the input model. General-purpose commercial statistical software (such as SAS or SPSS) can be helpful for input modeling, but is not designed to focus on the issues that arise in simulation input modeling. Specialized commercial software designed for simulation input modeling includes the ARENA input modeler (Kelton et al. 1998), BESTFIT (Jankauskas and McLafferty 1996) and EXPERTFIT (Law and McComas 1998). They focus on the important, but relatively simple, task of fitting classical univariate distributions to data and incorporating the fitted distributions into simulation software. The last few years have produced some easily attainable noncommercial software and ideas for creating more-complex input models, including Avramidis and Wilson (1994), Cario (1996), Cario and Nelson (1997a, 1997b), Chen (1999), Kuhl, Damerdji and Wilson (1997, 1998), Kuhl, Wilson and Johnson (1997), Song and Hsiao (1993), Song, Hsiao and Chen (1995), Stanfield et al. (1996), and Wagner and Wilson (1995, 1996a, 1996b). These (and some other ideas) are discussed in Nelson and Yamnitsky (1998).

The attitudes of a practitioner toward the level of detail in the logical model and in the input model are often quite different. The inclination to include unneeded detail in the logical model is pervasive, despite ubiquitous warnings in discussions of simulation modeling. The inclination to simplify the input model (via use of classical distributions, statistical independence, and time homogeneity) is equally pervasive. Warnings against such simplification are easily found: Bratley, Fox, and Schrage (1987), Kelton et al. (1990), Johnson (1987), and Wilson (1997); warnings are implicit in sensitivity analyses such as Gross and Juttijudata (1997), Gross and Masi (1998), and Reilly (1998).

Why the different attitudes? Certainly a reason is that the logical model is often visible and supported by commercial software while the input model is less visible in that it is hidden by its own essence: observations are random. In addition, there is the wide-spread sense that  $\theta$  depends only weakly on characteristics of the input model beyond the population mean E(X). Even in the simple case where the performance measure is a mean (i.e.,  $\theta = E(Y) = E(g(x))$ ), typically  $E(g(X)) \neq g(E(X))$ ; that is, model performance is not obtained by substituting mean behavior. For example, a serial production line with deterministic arrival and service times might need no buffers, but buffers are needed when stochastic behavior is present.

### 2.2 Stochastic and Subjective Uncertainty

Two types of uncertainty are represented in the input model. Helton (1996) refers to them as stochastic (or aleatory) and subjective (or epistemic). For example, suppose that for any particular simulated the time between arrivals is modeled as a Poisson process with a constant rate  $\lambda$ . In addition, suppose that for each day  $\lambda$  is chosen from a particular Weibull distribution. The arrival times are clearly stochastic uncertainty. The randomness of  $\lambda$ , however, might be either stochastic or subjective. It would be stochastic if the arrival rate depends upon the day's weather; it would be subjective if the arrival rate is an unknown constant and the Weibull distribution is reflecting the modeler's lack of knowledge about the true value. Notice that the simulation computation is identical in either case; only the interpretation differs. In the latter case, the simulation results correspond to no realworld system. Sometimes more appropriate is a sensitivity analysis that checks the change in  $\hat{\theta}$  for a change in  $\lambda$ .

Barton and Schruben (1993) discuss explicit methods for incorporating uncertainty about the model in the simulation experiment. Here again, the interpretation of the simulation results must be carefully stated because there is no real-world system that corresponds to the simulated model.

### 2.3 Goodness-of-Fit Tests

As a final general comment, we mention the inappropriate use of goodness-of-fit tests to determine whether an input model is adequate (and, more generally, whether a simulation model is valid). The problem with such tests is that they deal with statistical significance while the relevant issue is *practical* significance. Remember that, except for some trivial situations, we know before any data are collected and/or analyzed that the typical simple null hypothesis of a goodness-of-fit test is false. The question is not whether the input model is absolutely correct; it is whether the input model is adequate for the analysis at hand. A particular model might be quite adequate for a quick study over the weekend and quite inadequate for a six-month study, even if the real-world system is the same. Similarly, an input model might be adequate for a simulation experiment with a small number of replications but inadequate when run with many replications. The fallacy of the goodness-of-fit test is made obvious when a large real-world data set it fitted to many classical distributions and all are rejected; all are rejected because the large sample size yields large power and the error in the model is indeed statistically significant. The tyranny of the goodness-of-fit test is such that many practitioners fear using a distribution that has been rejected, even after confirming visually and conceptually that it provides an adequate fit.

The preceding paragraph is not meant to criticize the use of goodness-of-fit statistics for heuristic ranking of competing models (for example, Cheng et al. 1996). It should be remembered, however, that different statistics can provide quite different rankings. But if comparing alternative input models is the goal, then a Bayesian approach (e.g. Chick 1997) has substantial advantages.

# **3 UNIVARIATE DISTRIBUTIONS**

Sometimes the method of hypothesizing a classical distribution, estimating parameter values, and testing adequacy leads quickly to a good model. Certainly the three inputmodeling programs mentioned earlier automate this task well. But what to do when the classical distributions are not adequate? Schmeiser (1977), in an input-modeling tutorial before commercial input-modeling software, emphasized classical four-parameter families of distributions such as the Pearson and Johnson, both elegant in that they provide one and only one distribution for any first four moments and inelegant in that they use more than one functional form. Also discussed were two relatively new four-parameter family designed explicitly for use in simulation experiments (Ramberg and Schmeiser 1974, Schmeiser and Deutsch 1977), elegant in that they used only one functional form and are relatively easy to fit, but inelegant in that the one-to-one relationship with moments is lost. These distributions are all inadequate for data sets having anomalies as simple as being bimodal with tails.

Later models (Avramidis and Wilson 1994 and Wagner and Wilson 1995, 1996a, 1996b), based on polynomials in various forms, allowed distributions to be formed on the fly with an arbitrarily large number of parameters. The Bézier models of Wagner and Wilson are particularly elegant in that they combine direct tractable visual distribution fitting with an arbitrary number of parameters. The Bézier transformation of the polynomial to a function that can be dragged on a computer screen is fundamental to the idea; without the visualization, a direct polynomial fit is unwieldy because polynomials are not automatically monotonically increasing from zero to one. That arbitrarily difficult data sets can be modeled interactively and visually using only one functional form is a significant advance.

### 4 MODELING DEPENDENCY

Two fundamental forms of statistical dependency arise: Random vectors with dependent components and time series with autodependence. We comment briefly on each.

Other than the multivariate normal distribution, few random-vector models are tractable and general, though many multivariate distributions are well documented (Johnson 1987). The early approach to creating flexible multivariate models was to assume a particular marginal distribution, as done, for example, by Schmeiser and Lal (1980, 1982) and Lewis and colleagues (e.g., Lewis and Orav 1989). Wagner and Wilson (1995) discuss a generalization having Bézier marginal distributions and Stanfield et al. (1996) discuss a generalization having Johnson marginals.

A more-general idea is to transform the multivariate normal distribution to a multivariate uniform distribution. The components of the multivariate uniform distribution then are placed in the inverse transformations of the desired marginal distributions to obtain pseudorandom vectors. The resulting model has the desired marginal distributions and desired rank correlation, as discussed, e.g., in Clemen and Reilly (1999) in the context of decision-analysis modeling. For the harder problem of providing the desired Pearson correlation, see Cario and Nelson (1997b) and Chen (1999).

When the multivariate properties of the model are particularly difficult, a simple method is to "bootstrap" from a data set for real-world data. Customers might arrive according to a fitted input model but have characteristics assigned by choosing, with replacement, a real-world past customer. The advantage and disadvantage is that every customer in the simulation will be identical, except for arrival time, to a real-world customer.

Similar to random vectors, the most tractable timeseries models have normal marginal distributions. Lewis and colleagues (for example, Lawrance and Lewis 1981) developed a variety of ingenious simple time-series models having nonnormal marginal distributions, often gamma. More generally, and as with random vectors, transformation from the normal model works well. See Cario (1996), Cario and Nelson (1997a), Song and Hsiao (1993) and Song, Hsiao, and Chen (1995). In principal, transformation from processes with other marginal distributions could be used; for example, various simple queueing models provide time series with exponential marginal distributions, as discussed in Schmeiser and Song (1989).

### 5 POISSON PROCESSES

Point processes model events that occur at points in time (such as arrivals) or in space (such as defects). The Poisson process is the special case when arrivals, say, are independent (that is, the numbers of arrivals in nonoverlapping increments of time are independent of each other). Fortunately, a Poisson process model is often appropriate and can be verified by the physics of the situation (and without data collection) simply by asking whether there is any mechanism by which arrivals can "see" each other.

If a Poisson process model is appropriate, the only input-modeling issue is to determine the appropriate arrival rate, which can be any nonnegative function of time. If the rate is assumed to be constant, estimation is trivially easy. Only a bit more difficult is to assume that the rate is piecewise constant over intervals of time. Substantially more generally, Wilson and colleagues (e.g., Kuhl, Damerdji and Wilson 1997, 1998, Kuhl, Wilson and Johnson 1997) have investigated an assortment of increasingly complex functional forms to model trends and seasonalities.

Leemis (1991) and Arkin and Leemis (1999) fit piecewise-linear rates to real-world arrival times. This approach is nonparametric in that no model parameters need to be specified and generation of arrivals from a piecewise-linear rate function is straightforward (Klein and Roberts 1984).

# 6 FINAL COMMENT

Despite the recent advances in generality, automation via software, and visualization, simulation practitioners are often left to their own devices when the input model becomes complex (for example, Ware et al. 1998 and Pritsker et al. 1995). What to do, for example, when a nonnormal time series of random vectors is needed? What to do when non-Poisson defects in time and space are to be modeled? The ideas above are useful building blocks, but the state of the art is far from allowing novice practitioners to build complex input models in the way that they can build complex logical models with today's commercial software.

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