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INFORMATIONAL MACRODYNAMICS: SYSTEM MODELLING AND SIMULATION METHODOLOGIES

Vladimir S. Lerner

Computer Science National University
9920 So. La Cienega Blvd., Inglewood, CA 90301, U.S.A.

ABSTRACT

Informational Macrodynamics (IMD) presents a unified informational systemic approach with *common information language* for modeling, analysis and optimization of a variety of *interactive* processes including a human being. IMD contains formal mathematical and computer methodology to describe transformation of random information processes (at microlevel) into system dynamic processes (at macrolevel) to model the observed processes on the basis of both discovery of their information dynamic regularities, and the identification of macrodynamic equations. Informational description of interactions includes not only physical connections between the processes, but also the information superimposition of processes of different nature with transferring information from one interacting process to others and creating the defect of information. IMD model follows from solution of the variation minimax problem for information macrofunctional that evaluates the average set of stochastic interactions as a Path Integral. The synthesized macrodynamics give rise to the self -organized structures, becoming ordered in the process of optimal motion. IMD defines the informational structured network of dynamic macronodes (IN) formed by the unification of collective chaotic attractors. Computer methodology of systems modelling, simulation and control with IMD software package were applied to concrete processes in Technology, Biology, Economics, and Artificial Intellect.

1 INTRODUCTION. BASIC IMD CONCEPTS

Simultaneously occurring and interacting processes in complex systems create indeterminacy and randomness which are the sources of dynamic properties not inherent to any of the processes in the absence of the interactions. The random nature of interactions and the dynamic laws determined by them lead to a bi-level structure of complex objects with stochastic processes at the microlevel and dynamic processes at the macro-level. Observed process is represented by a set of random interactions connected via a Marcovian chain with a path integral evaluation. This functional accumulates information contributions from the local functionals of the interacting microlevel processes as a collective information functional. Transformation from microlevel model to macro level is performed by using the informational form of the variation principle (VP).

The dynamic model follows from the solution of the VP problem for the information macro functional. The irreversible microlevel stochastics and the VP as a mathematical tool for the information transformations distinguish this approach from a number of known works with deterministic reversible micro- and/or macrolevel processes. The essence of this approach consists of: an accumulation and evaluation of Marcovian stochastics via macro functional of the stochastic process; use of informational form of that integral as an entropy macro functional; use of informational form of the VP (for this functional) as a mathematical mechanism for discovering and modeling the informational regularities; transfer of the Marcovian statistical properties of the dynamic level (and back) through created informational channels between micro- and macrolevels; filtration of the randomness of this transformation; and identification the dynamic macro model via micro level statistics.

The entropy functional as a measure of information *process* marks a major departure of this approach from traditional information approaches that use an entropy function. Traditional information theory uses maximal, minimal, or minimax principles for the entropy function which is able to select the optimal states and create a static information model. Applying the variation principle to the entropy functional allows us to obtain the irreversible *macro dynamic* model that microlevel stochastics transfers into the macro level. The IMD model contains an explicit form of the integral functional responsible for the particular variation problem as proper information functional (PF) of the identified process.

This macro functional characterizes the relative order (with respect to stochastic Wiener process); it is able to extract order from micro level randomness and transfer the order to the macro level. Macro movement occurs along segments of the initial n-dimensional extremal of an entropy functional. These segments are successively joined at discrete points (DP) effectively shortening the initial dimension and ultimately leading to renovation of dynamic process. Extremization of the PF is performed by applying controls that symbolize the input of negentropy into a closed system at the expense of high quality energy or unstable stochastic dynamics. The macro states are created by cooperative stochastic dynamics in the locality of DP. Stochastics impose a new form of differential constraints (DC) on the Hamiltonian dynamic equation at the localities of DP that reflects a "deterministic impact" the stochastics on Hamiltonian mechanics. This constraint is responsible for the state integrations and consolidation which forms the macro systemic connections and dynamic hierarchical structures.

The consolidated dynamics leads to an information structured network of macro nodes, formed by the initial information spectrum of the macromodel operator. The operator is renovated at each discrete point because of state consolidation and by the action of applied controls. The ordered macro states are stored and memorized at the DP. They are defined by the system of local invariants, which are a result of the VP solution. The controls, reduced to a state vector, are formed by duplication of the macro states at discrete moments, and are memorized at these same points. The macro structures accumulate defect information as an information mass which is spent for binding the integrated macro states at DP. Each new macro structure is the source of the controls for the next integration and ordering, forming a sequence of the nested states.

The spatial macro model trajectory is represented by the parametric equation of a spiral-shaped curve on a conic surface determined by the macro dynamics. Transition of the trajectory from one cone to another cone occurs at DP with the application of controls. The optimal cones stick at DP's in triplet form. Shortening of the macro model dimensions and formation of the singular points and surfaces at locality of DP are a consequence of the differential constraints. The IMD model has 4-levels of hierarchy: statistical micro level; quantum dynamic level (at the DP locality); dynamic (classical) macro level as a result of selection of the initial macro states and macro trajectories; and, hierarchical IN of macro structures. The quantum macro level with inherent uncertainty is a carrier of the cooperative constraints (DC), forming the hidden self-generated and ordering controls. IMD creates essentially new *systemic* results in with regard to Shannon's Information Theory [Kolmogorov,1987], contains constructive methodology with the applied software package for solution systemic problems.

Definition. *Information System is an interconnected set of interactions exchanging information capable of integrating them into a common information unit.*

2 THE IMD MATHEMATICAL MODELS

Microlevel process represents a result of observation of an ordered phenomena "wrapped" into stochastics which are modeled by the solution of the Ito stochastic equation

$$d\tilde{x}_t = \tilde{a}(t, \tilde{x}_t, u_t)\,dt + \sigma(t, \tilde{x}_t)\,d\xi_t, \quad (1)$$

with initial conditions $\tilde{x}_s = \eta$, drift function $\tilde{a}(t,x,u(t)) = a^u(t,\tilde{x}_t)$ as a contribution of the ordered phenomena at the observations. To extract order from the random *observation*, the process $\tilde{x}(t)$ is transformed into Wiener's process $\xi(t)$ with diffusion matrix $b(\tilde{x},t)=1/2\,\sigma\sigma^*$. As a relative information model , we consider the transformation of the Marcovian's diffusion process ($\tilde{x}(\bullet)=\tilde{x}_t$) into the Wiener process ($\tilde{x}'(\bullet)=\xi_t$)(as a model of a complete disorder) with the aid of the appropriate additive functional (ω_Δ). An informational difference between $\tilde{x}(t)$ and $\xi(t)$ measures a possible negentropy generation as an information source for such transformations. A numerical evaluation of transformation of $\tilde{x}(\bullet) \rightarrow \tilde{x}'(\bullet)$ is defined by functional

$$S(s,T,x(\bullet)) \overset{def}{=} M_{\tilde{x}_s,\tilde{B}}[\ln(q_{sT}^{-1}(\tilde{x}(\bullet)))] \geq 0,$$

$$M_{\tilde{x}_s,\tilde{B}}[\bullet] = \int_{\tilde{B}} [\bullet] P_{\tilde{x}_s,\tilde{B}}(d\omega), \quad (2)$$

where

$$q_{s,T}^{-1}(\tilde{x}(\bullet)) = \frac{dP_{s,T}}{dP'_{s,T}}(\tilde{x}(\bullet))$$

is the density measure of transformation of the probabilities P into P'. (Generally, the $q_{s,T}^{-1}(\tilde{x}(\bullet))$ can be defined via other measures, not only the probabilistic ones). For two Marcovian processes $\tilde{x}(\bullet): \tilde{x}'_t = \tilde{x}(\tilde{x}_s, t, u, \xi, T)$ and $\tilde{x}'(\bullet)$: $\tilde{x}'_t = \tilde{x}(\tilde{x}_s, t, \xi, T)$ with additive functional

$$\omega_\Delta = \omega_s^T = \int_s^T \sigma^{-1}(t,\tilde{x}_t)\, a^u(t,\tilde{x}_t)\, d\xi_t +$$

$$\frac{1}{2}\int_s^T |\sigma^{-1}(t,\tilde{x}_t) a^u(t,\tilde{x}_t)|^2\, dt \geq 0, \quad (3)$$

the following relations are true

$$\frac{dP'}{dP}(\tilde{x}(\bullet)) = \exp(-\omega_{\Delta}) = q_{s,T}(x(\bullet)) \quad (4)$$

and the conditional mathematical expectation of the additive functional defines the entropy functional of the considered transformation:

$$S = \int_s^T M_{s,\tilde{x}_s} \tilde{L}(t, \tilde{x}_t, u(t, \bar{x}_t))dt \quad (5)$$

with the "Lagrangian"

$$\tilde{L}(t,x,u) = \frac{1}{2}\sum_{i,j=1}^{n}(2b_{ij})^{-1} a_i^u a_j^u \quad (6)$$

Extremal trajectories x_t associated with the solution of the VP for this functional, represent the dynamic model of the informational macro level process. The extremal values of this functional determine the PF on the macro trajectories. The main formal result is the Hamilton equations for the conjugate informational macro coordinates (x, X):

$$\frac{dx}{dt} = \frac{\partial H}{\partial X}, \ \frac{dX}{dt} = -\frac{\partial H}{\partial x}, \ X = \frac{\partial S}{\partial x}. \quad (7)$$

Those, together with the equation of the differential constraint (DC):

$$e(x,X,t') = \frac{\partial X}{\partial x} + 2X X^* => 0, \quad (8)$$

imposed on these equations, describe the model macrodynamic regularities. The macro mechanics of uncertainty represents an analog of physical mechanics. DC equation is based on joint solution of Kolmogorov and Hamilton equations [Lerner, 1996]. The DC changes the structure and value of dynamic macro model operator creating its dynamic dependency on observed data. The coupled effect of both the DC and Hamiltonian mechanics leads to an information structured network (IN) of macro nodes formed by informational spectrum of macro model operator. The IN nodes model an interaction of data and initial dynamics at particular points of distributed field. The macro dynamic process is characterized by the discrete extremal intervals , selected from Hamilton's solutions via DC with the time interval discretizations (DP) t', determined by VP invariants.

The Hamilton's equations determine, in general, the reversible dynamic solutions, and the DC equation imposes (at the t'- moments) the dynamic connection (through Hamiltonian H) with the Markovian stochastics, characterized by the diffusion matrix b=b(t,x):

$$H(t') = X^*(t')\frac{dx}{dt}(t') = -1/2\, b(t', x(t'))\frac{\partial X}{\partial x}(t'). \quad (9)$$

The Hamiltonian acquires the entropy production meaning, but only at the DP. Within the DP (when the DC is absent) time is reversible. Irreversibility arises at the moment of constraint imposture. Microlevel stochastics creates the macrolevel informational forces X, and macrolevel dynamics could originate the microlevel stochastics (through diffusion). Transformation of microlevel information into the macrolevel is the most effective when the H-function of information losses (9) is minimal (according to (8)). The DC is responsible for formation of optimal control v which includes a conjugate vector (with A as the macromodel matrix)

$$X = (2b)^{-1} A (x+v); \ Av=u, \ A=\{\lambda_i(t)\},$$
$$\lambda_i(t) = \alpha_i \pm j\beta_i, \ i=1,...,n, \quad (10)$$

with the control vector:

$$v(t') = -2x(t'); \quad t' = t_1, \ . \ t_j, \quad j=1,. \ . \ n-1, \quad (11)$$

which elements represent the inner controls, gets reduced to the state vector x; vector u represents the corresponding external controls. The DC connects differential operator A to statistical characteristics of the random process, in general, in the form of the nonlinear correlations which, in part, lead to the precise and simple identification equation:

$$A = 1/2b(t')[\int_{t'} b(t)dt]^{-1}; A = M\{x(t)x$$

$$\frac{dx}{dt}(t)^*\} \times [M\{x(t)x^*(t)\}]^{-1}; \ M = M_{t,'x}. \quad (12)$$

These results suggest that the A-matrix of the differential IMD equation (in simple form):

$$\frac{dx}{dt} = A(x+v), \ A = A(t',x) \quad (13)$$

which in turn is a differential operator of the classic mechanical analogy, acquires the irreversible qualities only at the DP, due to the second law , and is generated by the stochastics. Elsewhere (at the DP), A(t',x) is preserved within each DP, and changes by step due to constraint imposition. Introduction of controls allows to model the inner nonlinearities which leads to bifurcation's and Chaotic activities at points of interactions. Thus, by using these controls, one separates the linear and nonlinear parts of the model with reversible and irreversible processes. In this approach, we start with the stochastic model which through the Chaotic Dynamics with inner controls of discrete action at macro level, is supposed to initiate a new, second level of stochastics. Points of development of the

Chaotic Dynamics are predictable by the informational law of VP.

Two scales of the macro process are to be considered: large time intervals (t,t'-0) (outside of the DP), when the macro movement is analogous to the classic one, and the small intervals (in the vicinity of DP) | t' -0,t'+0 |, when the macro movement is defined by the DC equation. For the spatial distributive system the solution of VP problem leads to controlled diffusion form of macro equation, which is identified by the micro level:

$$\frac{dx}{dt} = r \quad (2b)^{-1} \quad go\frac{d^2x}{dl^2} + \frac{dr}{dt} r^{-1} \quad (x+v)^*, \quad M=M_{t,'x}$$
$$go=M\{\frac{dx}{dt}(x+v)^*\}\times[M\{\frac{dx}{dl}(x+v)^*\}]^{-1}, \quad (14)$$

In addition to the control functions (11) , the macro model possesses the "needle" controls acting as δ-function at the locality of discrete moments, and selecting the macro trajectory pieces where the VP and the Erdman-Weirstrass condition could violate.

3 THE IMD SIMULATION MODEL

The transfer on the quantum macro level and the DP vicinity is accompanied by equalization of the relative phase velocities of the macro variables:

$$\lambda_{it} = \frac{dx_i}{x_i dt}(t'-0) = \frac{dx_k}{x_k dt}(t'-0) = \lambda_{kt} \quad \text{or } \lambda_{it}\lambda^*_{it} = \lambda_{kt}\lambda^*_{kt} \quad (15)$$

and the selection of the extremals takes place, for which the condition (15) is satisfied. At the moments t' , the differential operator A is diagonalized successively. This procedure is accomplished upon equalization of all $\lambda_{it}, \lambda_{jt}$(at j=n-1). At the moments t', the controls (11) are applied, which results in transfer of the movement from the pair of the extremals to the joint extremal, with a new differential operator corresponding to the equal eigenvalues (15). The renovating operator is memorized at the moment (t'+0), and remains the same during along the next DP. At the same time, the macro state carrying the information maximum is memorized by its coping and doubling through the controls (11), on the basis of selection of the subsystem competing for the maximum information acceptation. This procedure corresponds to the successive macro model cooperation during the process of optimal movement, concurrently forming the hierarchical structure, which is memorized at the DP. At the moment (t'-0), a local minima of information production (defined by VP) :

$$\max H' = \min\frac{\partial S}{\partial t} = \min |-\frac{dx}{dt}(t')^* X(t')| > 0, \quad (16)$$

binds the dynamic macro structures at the moments (t' +0). Each discrete point is the result of joint solution of the 3 equations that could be nonlinear; such points are singular with the possibility of all kinds of Chaotic Dynamic Phenomena. The dynamic model operator is renovated at the points of discretization , where the extremals are "freely"' selected and stuck together by rules related to Quantum Mechanics. The renovated operator and macro states are sources of the new information, the new properties, and the structures at the points of consolidation. The procedure of changing the operator is directed on minimization uncertainty (with increasing order), and is predictable by the DP invariants. There exist two sub regions (RS) of the complex initial eigenvalues (λ_{io}*) of the differential operator. One of them corresponds to a positive direction of time (RS+), another one to the negative direction (RS-). At the phase trajectories approaching the RS+, the imaginary part of the eigenfunction turns into zero (or the pure imaginary eigenvalue is transferred to the real one). At the phase trajectories approaching the RS-, the real eigenvalues turn into zeros. RS represents the geometrical subspace where the curvature and symmetry change by step jumping. Within RS+ the real eigenvalues are cooperated; moreover, the optimal procedure is to join by threes with the following addition of a new pair to these, which were cooperated before a three. Such elementary information dynamic structure (triplet) can be also formed by the real eigenvalues (from RS+) cooperation with the two conjugate imaginary eigenvalues (from RS-), when these regions adjoin one to another. Zone of uncertainty (UR) is spreading between RS+ and RS- that is characterized by constant parameter of indeterminacy h. The quantum macro model is the carrier of the trigger effect which is evidenced in the step-wise control action (11), and trough existence of the "needle-controls", responsible for changing the operator sign. Along the trajectories, ending in RS+, the invariant:

$$\mathbf{a}=|\alpha_{io}|t_i, \alpha_{io}=\mathrm{Re}\lambda_{io}, \beta_{io}=\mathrm{Im}\lambda_{io}, \mathbf{ao}=|\alpha_{it}|t_i \quad (17)$$

is preserved; along the trajectories, transferring into RS-, the invariant

$$\mathbf{b}'_o=|\beta_{io}|\,t_i^-, \ \mathbf{b}'_o=g\,|\alpha_{io}|t_i^-, \ g=|\left(\frac{b_{io}}{a_{io}}\right)| \quad (18)$$

is preserved; moreover, **a** has a meaning of the quantity of the real information, formed at the moment t_i and $\mathbf{b}_o'$ represents the quantity of the imaginary information at moment t_i^- of the hitting in RS-. Considering here t'- as an imaginary time, and t' as a real time, we get the UR uncertainty :

$$h = \frac{jt}{t'} = \frac{t_i^-}{t_i}; h = \frac{\mathbf{b_o}'}{g\mathbf{a}} = \frac{\mathbf{b_o}}{\mathbf{a}}, \ \mathbf{b_o}(g) = |\alpha_{io}| t_i^- \quad (19)$$

At **a**=$\mathbf{b_o}$, h=1, and for the pure real eigenvalues uncertainty is h=0, taking into account the sphere of the admitted values of the invariants : **a**(g=0-1)= 0.765- 0.3, $\mathbf{b_o}$ (g=0-1)=0. 698 -0.3. The maximum value of the invariant **a**=ln2=ho characterizes the elementary information quantity within the single discretization interval. As long as a quantum macro level contains the imaginary information ($h \neq 0$), its probabilistic description is preserved. While a quantum macro model description can be applied, the information coordinates and their derivatives, the flows (the impulses) can not be determined simultaneously within the region RS. Algorithmization of the network node's information determines the minimal discrete spectrum of the local quantities of information {hio, i=1,...,n }for which is defined the systemic representation, and the hierarchical network of the structured information is reconstructed. The formed macro dynamic structure is evaluated by information complexity measure (MC), that depends on diffusion matrix of microlevel and is connected with the degree of the microprocess disordering . The MC is the most complete characteristic of macro system, defined by the basic parameters of the dynamics (**n , g**) and the cone geometry (**k**):

$$\text{MC}(\mathbf{n,g,k}) = \frac{SpA(t_o)}{dV / dt(t')}. \quad (20)$$

The MC defines the minimal code of the optimal algorithm which can be restored by the dynamic model , and MC may be written through the successive times of the macro state's memorization, carrying the maximum of information. At the fixed invariant, for the above equations we obtain

$$\text{H}' = \mathbf{a} \sum_{i=1}^{m} (t_i)^{-1}; \quad \mathbf{m}=\text{n}/2+1, \{\text{ti}\}=\text{t}', \quad (21)$$

the minimal number of discretes **m** (corresponding to the **m**-triplet structures), as the necessary condition of the upper level of the hierarchical tree achievement within given optimal organization. The MC characterizes the information unchangeability of the optimal code, and at this meaning, is the quality measure of the accumulate information, as it's valuable. The *dynamics* and *geometry* of the macro model *are connected*. Irreversible macro dynamics of the IMD create evolution of their Informational Geometry with memorization the renovated *geometrical* structures. By the periodical rotation of the matrix A, each three eigenvectors form the rotating basis of the model geometrical coordinate system. The ranged sequence of the triplet eigenvalues preserves the pair-wise constant multiplicators:

$$\frac{\lambda_1}{\lambda_2} = \frac{\lambda_3}{\lambda_4} = G_1(g), \frac{\lambda_2}{\lambda_3} = \frac{\lambda_4}{\lambda_5} = G_2(g) \quad (22)$$

defined by the ratio of local entropies and invariants.

4 METHODOLOGIES AND APPLICATIONS

IMD methodology describes the creation of the dynamic properties formed by the micro level stochastics. The foundation of methodology one of the hypothesis's makes: an existence of PF, that creates some process as its extremal, or an existence of some process created by PF. Each observed process concretizes the PF form and intrinsic regularities by its identification. Similar to known differential equations, the IMD equations (1-22), act as a common tool for systemic modelling of complex systems. The VP as an information macromodel of a law, is defined by the identified microlevel stochastics.They are simulated by corresponding software and hardware, connected via information network (IN) as an "operating system".

The methodology of the system simulation includes the following procedures: identification of the macro model equations; selection of the information from the micro level as a procedure of the optimal filtering of the maximum information necessary for macro level; computer simulation of the macro level equations; consolidation of the macro states and of the macro processes; formation of the macro structures and selection of the macro structures with a different level of complexity; memorization of the renovated macrostates, carrying new information; formation feedback and minimizing the control indeterminacy; formation of the optimal predicted macro dynamics. Primary statistical information from the microlevel, as model's initial conditions, gives start to a formal procedure of identification of the macrolevel equations, and their solution, the determination of the discrete intervals, forming the optimal controls, and the procedure of macro state's consolidation. Procedure of identification, optimization and the state's consolidation is combined in time: identification occurs within the discrete intervals of the optimal controls action between the points of accepting the information from the microlevel. The dynamic system is able to predict: hierarchically structured organization of the different levels of complexity; development of the probabilities of the processes in time and space; discovering of some indeterminate events in the prognosis processes; decreasing uncertainty of information results; and construction of the family of system dynamic models of a given purpose and complexity. IMD model and its inherent procedures are sufficiently general and are applicable for different concrete objects as a common tool. The specialization of the model consists of defining the initial statistical information formed by the corresponding data based system; the flows of the first macrolevel description. The controls created in the self-organized process, serve as a discrete filter selecting the maximum quantity of information from microlevel at DP's. The selection can change the macrosystem peculiarities through adaptation at the DP's as the points of "mutation". Execution of the VP invariants (at DP's) works as the system "entry gate" with maximal information flows (channel capacity). The exchange of information is accompanied by increasing system's diversity. UR has the algorithmic complexity equal to number of bits to perform complete operations with an h-accuracy. Transformations within UR are non repeatable and can not be represented by a regular algorithms. The smallest algorithm of these operations is uncompressive. Geometrical transformations within UR are not symmetrical, and the formed geometrical structures are fractals.

The IMD numerical methods, *discrete* simulating procedures, and computer *software* have been elaborated and used in practice for constructive solutions in the concrete problems of modelling, prediction, optimization, structural organization, control, and design areas (Lerner, 1993, 1997).

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AUTHOR BIOGRAPHY

VLADIMIR S. LERNER is a professor in Computer Science Department at National University. He received his Ph.D. degree in Automatic Control from Moscow Power State University, and his Doctor of Science degree in System Analysis from Leningrad State University. He is an honorable member of *Encyclopedia of Life Support Systems*.