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EVALUATION OF A (R,s,Q,c) MULTI-ITEM INVENTORY REPLENISHMENT POLICY THROUGH SIMULATION

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ABSTRACT

In this paper, the case of a cardboard box marketing firm is studied and a (R,s,Q,c) inventory replenishment policy is proposed and evaluated by means of discrete event simulation. The complexity of this multi-item inventory problem requires a fast and reliable method of determining the operating conditions that optimize the inventory control. Simulation techniques can be effectively used to determine an adequate ordering policy for this type of problems. Several ordering options were analyzed and compared to find the policy that best accomplishes the firm's organizational objectives. The simulation model built allows the dynamic change in the demand pattern for each item of the inventory. The results revealed that the implementation of the multi-item replenishment policy can reduce total investment and maximize customer service, while maintaining the business efficiency.

1 INTRODUCTION

Inventories constitute an important investment in all type of firms, from a merchandise distributor to a manufacturer of products. Huge quantities of materials are sometimes kept on stock to deal with productive constraints or to fulfill dynamic demand patterns. In this sense, it is vital to have effective information to aid the management for the decision making process to maximize the customer service, minimize total investment and maintain the operating efficiency. The situation turns complex because these objectives are in conflict with each other, and tradeoffs occur when trying to improve one of them. For example, to maximize customer service, a relatively high investment in inventories is required, and due to capital constraints, these funds could have the opportunity of better profit in some other investment. The conflict finds its solution when applying an efficient inventory control, leveling these tradeoffs between investment and costs to find an adequate policy for the operation of the business. This principle is well known and simple in concept, however the complexity of real situations makes it difficult to apply. Most of the real situations not only face a one item problem, but multiple items with several periods of replenishment. These inventories are frequently managed in aggregate due to the complexity of handling each individual item.

2 BACKGROUND AND PROBLEM STATEMENT

The case studied in this paper is that of a cardboard box marketing firm located in Toluca, Mexico. The firm operates two retail stores and a warehouse, supplied directly from a single source. Initially, the firm was oriented to the domestic market with a line of approximately 100 different products for household and office applications. The inventories were then managed observing the market conditions. Reordering cycles were two weeks to satisfy the expected demand for a two week time frame. This practice worked well at the time because the demand was relatively small and predictable due to gradual market penetration, and constraints in capital and space as well.

As demand and sales increased, the firm opened a second retail shop and the product line increased to 137 products, with the reordering cycle reduced to one week. Inventory control was then made in each store with the aid of a detailed record of the weekly operations. The results were accumulated at the end of each week, and reordering quantities were established for next Monday's shipment, which was made by an external transportation company. This reorganization and the market opportunities resulted in a growth in sales, and thus the necessity for improved customer service. At this moment, the inventory control played an important role for the firm. The stock of products grew, and consequently the associated costs. However, the customer service did not increased as a result of this. Shortages of some products frequently occurred, while others remained on the shelves in big quantities. Lost sales and carrying costs presented complex trade-offs for the profitable operation of the business. Due to the accelerated growth of the firm, the inventory control turned more complicated. Intuition and commonsense rules with the aid of historical data have been useful in the past, but certainly not efficient.

Given this situation, the firm's manager decided to find a fast, reliable and safe method of ordering the quantity of each of the products to minimize total inventory cost, (sum of ordering and carrying costs) and to maximize two customer service measures (defined by the firm as P1 and P2). Service level measure P1 refers to the percentage of items stocked out from the total quantity of items demanded. Service level measure P2 relates to the percentage of items stocked out from the total number of items carried. Initially, these measures of performance were estimated at an average of $409 for the total inventory cost, 80% for P1 and 85% for P2, on a weekly basis.

After conducting a research, the manager found that the proposed ordering policy could not be evaluated by means of theoretical inventory models due to the complexity of the real system and the variables involved. In this sense, simulation can provide a powerful tool for evaluating the performance of a proposed system and choosing the right alternative before actually implementing the solution. A simulation project was then undertaken by the firm for this purpose.

3 REORDERING POLICY DEVELOPMENT

The inventory problem was initially reduced from the original 137 products to only 22 applying the ABC classification method. These 16% Class A items account for 79.2% of the total inventory value. The problem can then be described as follows:

- 2 selling points.
- 22 items to control.
- Stochastic demand.
- Joint replenishment of the 22 items.
- One supplying source.
- Fixed ordering lots for each item.
- Limited freight transportation capacity.
- Limited warehouse storage capacity.
- Fixed unit costs, no quantity discounts.
- Ordering and carrying costs relevant.
- Shortage costs not estimated.
- Backordering not considered.
- Weekly review of inventory levels (Saturdays).
- Inventory replenishment on Mondays.
- Availability of one truck per week maximum.
- Instantaneous delivery time.

3.1 A (R,s,Q,c) Model

The replenishment problem faced in this paper is stochastic in nature, with warehouse and transportation constraints present. Since several items are ordered at the same time, it is necessary to consider a (R,s,Q,c) model to find the solution. This model offers the following advantages that can eliminate some of the problems mentioned before:

- Warehouse and transport capacity rationalization.
- Unit ordering cost reduction.
- Easy to implement and use.
- Responsive to changes in demand.
- Low operating cost.
- Computationally efficient.
- Easy to maintain.

In general, the (R,s,Q,c) model can be stated as: "review the inventory level every R units of time, if the inventory is less than or equal to s you *must-order* Q, if the inventory is less than or equal to c you *can-order* (Q - c)". This is called *coordination* in inventory theory, which means that the ordering of one item is done "in coordination" with the inventory of other items. The level of coordination for each item depends on the parameters s and c.

3.2 Notation

R = units of time between inventory revisions.
s_j = must-order level for item j.
Q_j = lot size for item j.
c_j = can-order level for item j.
Oh_j = On hand inventory level for item j.
$d_{j(avg)}$ = average demand for item j.
W = truck capacity.
w_j = truck capacity of item j.
h = holding cost.
k_j = ordering cost of item j.
A = total ordering cost.
v_j = unit cost of item j.
n = number of items.
$|x_j|$ = closest amount (to x_j) feasible to be shipped taking into account the lot size and the capacity restrictions.

3.3 Replenishment Algorithm

The proposed algorithm for replenishing the inventory consists of the following steps:

STEP 1: Let CO={items to control sorted in descending order according to the ABC classification}. Review the inventory levels R units of time after last replenishment and let:

CA={items in CO such that $Oh_j \quad s_j$ }.
If CA= then STOP (do not order this period).
Else go to STEP 2.

STEP 2: Rank the k items in CA following the same sequence in CO, such that:

$$CA=\{[1], [2], [3], ... , [k]\}$$

STEP 3: Let CB={first t elements in CA (t k)} such that:

$$\text{for } t < k: \quad \sum_{j=1}^{t} Q_{[j]} \leq W \quad \text{and} \quad \sum_{j=1}^{t} (Q_{[i]} + Q_{[j]}) > W$$

for each i > t and i CA

Go to STEP 4

$$\text{for } t = k: \quad \sum_{i=1}^{t} Q_{[i]} \leq W$$

Go to STEP 5

STEP 4: Order Q_j for each element in CB. STOP.

STEP 5: Let CC ={items such that OHj cj for j= 1,2,...,n and j not in CA}. Let:

$$W' = W - \sum_{i \in CB} \left| Q_{[i]} \right|$$

STEP 6: Rank the m items in CC following the same sequence than CO, such that CC={[1], [2], ... , [m]}.

STEP 7: Let CD ={ first p elements in CC} such that:

$$\text{for } p < m: \quad \sum_{i=1}^{t} \left| Q_{[i]} - OH_j \right| \leq W'$$

$$\text{and} \quad \sum_{j=1}^{t} \left| Q_{[j]} - OH_j \right| + \left| Q_{[i]} - OH_i \right| > W'$$

for each i > t and i CA

$$\text{for } p = m: \quad \sum_{i=1}^{t} \left| Q_{[i]} - OH_j \right| \leq W'$$

$$\text{let} \quad Q_j' = \left| Q_j - OH_j \right|$$

Order Qj for each element in CB and Qj’ for each j in CD. STOP.

3.4 Obtaining Model Parameters

Parameter R (time units between inventory revisions): depends on the specific problem addressed, considering the revision policy of the firm.

Parameter s (must-order level): obtained as the first integer that satisfies the desired service level based on the probability distribution of its demand. For example, with a 95% service level:

$$F_j(s_j) \geq 0.95$$

Parameter Q (lot size): obtained solving the following equations simultaneously:

$$Q_j = \sqrt{\frac{2k_j d_{j(avg)}}{hv_j}} \qquad (1)$$

$$\frac{k_j}{Q_j} = \frac{A}{W_j} \qquad (2)$$

Equation (1) is the well-known *economic lot size* (Plossl 1985), and (2) expresses the relationship between the total ordering cost and the cost to order item j only. Solving for k_j in (2) and substituting in (1), we finally find:

$$Q_j = \frac{2Ad_{j(avg)}}{hv_j W_j} \qquad (3)$$

Parameter c (can order level): the choice of c depends on the desired level of coordination for the items in the inventory. Typically ranges from zero (independent replenishment) to some value, e.g., the economic lot size Q_j (totally coordinated replenishment). See Love (1979) for suggested values of the parameter c.

For the specific inventory problem in this paper, the model parameters were established as R=1 week, s was given by each corresponding demand distribution with a 95% service level. Q was calculated using expression (3) for each item, and c taking four different values as shown in Table 1, to represent different replenishment coordination levels.

Table 1: Replenishment Policies to be Evaluated

Replenishment Policy	Value of c
A	0
B	s
C	s + Q_j / 2
D	Q_j

4 SIMULATION MODEL DEVELOPMENT

The simulation project undertaken had the following objectives in order to understand the performance of the proposed inventory ordering policy:

- Evaluate the (R,s,Q,c) ordering policy for the stated inventory problem.
- Compare alternative replenishment policies by changing the *can-order* parameter c.
- Evaluate the selected ordering policy performance before its actual implementation.

Considering that the inventory reordering in the firm studied is done weekly (R=1), the *fixed-increment* time advance approach (Law and Kelton 1991) was selected for advancing the simulation clock.

Initially, the inventory problem was modeled to represent the current situation and get a validated model of the real world system. Once validated, this model was modified to include the proposed ordering policy, making simulation runs with each of the four values selected for the parameter c. Finally, the results of these simulations were compared statistically.

4.1 Data Collection and Model Definition

Daily transaction reports carried by each retail store were the main source for the data required to build the simulation models. Sixty weeks of daily demand for each of the items in the product line were available in these records. However, the sum of the individual demands of the two stores was considered for this study, since transfer of items is made between the stores to balance inventories. No transaction cost was considered relevant for the model. Other important sources of information were price lists, accounting records, organization manuals, and useful conversations and interviews with employees.

Considering the demand as a discrete random variable, the Chi-Square goodness of fit test was applied to each of the demand data sets for the 22 class "A" items. One item was found to fit the binomial distribution, 2 the Poisson, 15 the negative binomial, and 2 the geometric, all with a 95% confidence level. Two items failed to fit a theoretical distribution, for which empirical distributions were obtained. The demand of items was the only source of randomness considered to represent the inventory problem.

4.2 Data Validation

Data collected for building the simulation models were validated against the current operating figures of the firm. A structured walkthrough of the conceptual model was also performed to make sure that the assumptions were correct, complete and consistent with the real world system.

4.3 Model Building

For the model construction, the general purpose language *Turbo Pascal* (ver. 5.5) was selected because of its flexibility and low cost to model this type of problems.

The inventory system model uses the types of events shown in Table 2. Of these five events, only four actually involve state changes (the end of simulation being the exception).

Table 2: Types of Events Modeled

Event Type	Event Description
1	Arrival of an order from the supplier
2	Demand of the product from a consumer
3	Update inventory level
4	Inventory evaluation (and possible ordering) at the end of a week
5	End of simulation after k years

Considering the *fixed-increment time advance* approach for the problem, the simulation program built has the following logic. The simulation clock is advanced in increments of exactly 1 week. Each time the clock is updated, a check is made to determine if any events should have occurred during the previous interval of length 1 week. If one or more events were scheduled to have occurred during this week, these events were considered to occur at the end of the week and the system state (and statistical counters) is updated accordingly. In addition to the usual language declarations, the model has the structure and flow outlined in Table 3.

In order to evaluate the performance of the different ordering policies (by changing the *can order* parameter c), the following performance measures were used:

- Average total inventory costs TIC (sum of ordering and carrying costs).
- Average service level measure P1 (% of total items demanded from inventory without stockout).
- Average service level measure P2 (% of the 22 items not presenting a stockout).

4.4 Model Verification

Well-known simulation techniques were used to verify the simulation model (Law and Kelton 1991): modular program development, structured walkthrough, output traces, running the model under simplifying assumptions and under a variety of settings of the input parameters. Animation was not considered relevant for the simulation due to the nature of the problem and the time advance mechanism used for the model.

Table 3: Model Structure and Flow

Subroutine	Purpose
INITCOND	Initialize system state and statistical counters.
CONSTDATA	Constants definition (costs, inventory levels, demands, lot sizes, etc.).
CONSTGEN	Definition of random number generator constants.
ROUND(Q)	Rounds orders to lot sizes.
LOTSIZE	Calculates lot size for each item.
RANDOM(Z)	Generates U(0,1) random numbers based on the linear congruential algorithm.
DEMANDGEN	Event routine to process demands.
CALCPARA	Calculates the (R,s,Q,c) model parameters.
UPDATEDEM	Event routine to update inventory levels.
ORDARRIVAL	Event routine to process arrivals and inventory evaluation.
STATCOUNT	Calculate weekly and yearly statistics.
NEXTWEEK	Updates inventory levels.
WARMUP	Performs warm-up runs.
REPORTGEN	Generates weekly and yearly reports.
MAIN	Advances the clock, checks next event type, invoke subroutines, process end of simulation.

4.5 Model Validation

The following steps were observed during the validation of the simulation model (Law and Kelton 1991): develop a model with high face validity, test the assumptions of the model empirically, and determine how representative the simulation output data are. This analysis showed that the model output is consistent and correct when compared to the real world system figures.

The *two sample Kolmogorov-Smirnov test* (Conover 1980) was applied to validate the demand generation subroutines. Initially, this analysis was conducted to each of the 22 items, comparing the historical data set with the demands generated by the computer program through the K-S test. After performing minor adjustments to the program code, the 22 items demand generators passed this statistical test and were considered valid.

4.6 Design of Experiments

Special care was taken when selecting the initial conditions, so that the performance measures do not depend directly on the system's state at time zero. Initially, the inventory level for each item is set to its maximum storage capacity, considering a full replenishment to start operations. A warm-up period of 15 weeks was selected to obtain steady state conditions. The inventory level reached its average values after this period, during which no statistics were gathered.

The run length was established at 50 weeks per year, to represent a typical business operating cycle. The number of replications was arbitrarily set to 50 years, and considering the small variance during pilot runs, this was considered acceptable and computationally efficient.

The variance reduction technique of *Common Random Numbers* was applied through the use of random numbers *synchronization* (Law and Kelton 1991). In this way, it is possible to compare the alternative configurations under similar experimental conditions. This assures that the observed differences in the performance are due to differences in the system configurations rather than to fluctuations in the experimental conditions. To achieve this, each inventory model configuration used the same seeds for the generated random variates.

5 ANALYSIS OF RESULTS

After performing simulation runs for each inventory replenishment policy, 95% confidence intervals for the means of the selected performance measures were obtained, as shown in Table 4.

Table 4: 95% Confidence Intervals for Each Policy

Policy	TIC ($)	P1%	P2%
A	[211.1,216.3]	[89.6,90.4]	[87.1,87.6]
B	[215.5,220.0]	[94.0,94.8]	[95.5,95.9]
C	[238.5,243.2]	[96.7,97.5]	[98.1,98.4]
D	[241.7,246.5]	[96.7,97.5]	[98.1,98.4]

In order to compare these four policies adequately, *k(k-1)/2* confidence intervals for the difference of the means for each measure of performance were constructed, were *k* is the number of systems. In this way, with a confidence of $1-\alpha/[k(k-1)/2]$ for each interval, we can obtain a confidence interval of at least $1-\alpha$ for all the intervals together (Law and Kelton 1991). Then we compare the systems pairwise, and select the system with the best value for the measure of performance.

Considering k=4 systems and a significance level of α=.05 for the inventory problem, six 99.17% confidence intervals for each of the three performance measures were constructed, and the ordering policies were compared pairwise. For the total inventory costs (TIC) Policy A was selected for having a minimum average weekly cost of $214.11. For the service level measure P1, Policies C and D were selected for having a maximum average weekly service of 97.14%. Finally, for the service level measure P2, Policies C and D were selected for having a maximum average weekly service of 98.28%. These measures of performance are compared in Figure 1.

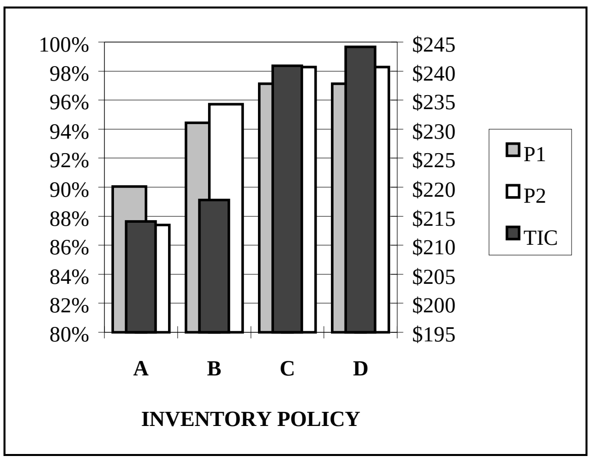

Figure 1: Comparison of Results

However, since we face a problem with multiple objectives, the three measures of performance must be considered simultaneously to select the best overall policy. If we consider a minimum acceptable service level of 95%, combined with a minimum inventory cost, we find out that only Policies C and D meet these targets. If we note that the service levels P1 and P2 for Policies C and D remain constant, but the total inventory cost increases $3.24 for Policy D, we finally decide that Policy C (with *can-order* parameter $c=s+Q_j/2$) is the best alternative for the proposed inventory model. The expected benefits if the (R,s,Q,c) inventory model is implemented by the firm using Policy C are shown in Figure 2.

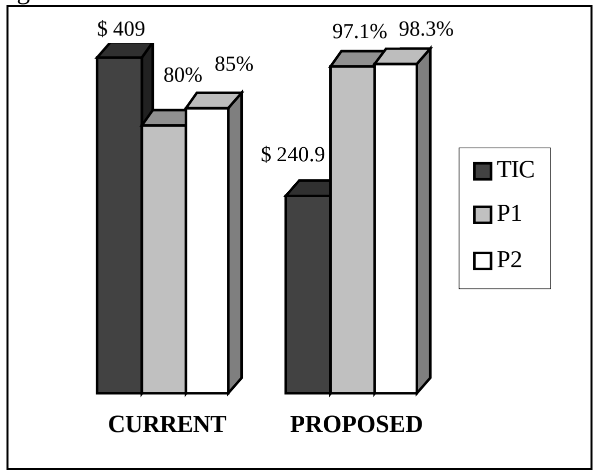

Figure 2: Expected Benefits of Policy C

6 CONCLUSION

The simulation study conducted revealed important information regarding the proposed inventory model performance, as it allowed the firm to gain valuable experience in implementing the system ahead of time.

The (R,s,Q,c) ordering policy proved to be an effective way of leveling the tradeoffs involved in this complex real world situation. The model structure offers also a reliable and efficient method to deal with the dynamic environment faced by the firm.

The simulation model developed assisted in the decision making process to reengineer the operation of the company in terms of customer service and operating costs. These improvements will eventually lead the company to fulfill its organizational and business objectives.

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