# SIMULATION OPTIMIZATION IN STRATEGIC LOCATION OF SEMI-FINISHED PRODUCTS IN A PULL-TYPE PRODUCTION SYSTEM

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#### ABSTRACT

In many manufacturing environments there are intermediate states of semi-finished products that are shared by several members of the product family. This is true, for instance, in machine tools manufacturing or manufacturing of several printed circuits from the same basic board. For these systems, processing parts ahead of time and locating them at strategic stages of the operation may improve the lead time with relatively low levels of work-in-process inventory. This is especially true when production stages do not have a dedicated resource and share a limited number of production equipment. In this paper a simulation model has been developed and interfaced with a simulation-optimization method to determine these levels. Furthermore, a method has been suggested to group optimization parameters such that fewer variables need to be optimized.

### **1 INTRODUCTION**

In many manufacturing environments several parts or products are made from the same raw material in several stages of manufacturing operations. After each process, the product assumes a particular state that is closer to its final shape. The closer the product is to its final state the shorter the lead time becomes to transform it to a finished product. However, as the product gets closer to its final state, its flexibility for being transformed into a larger variety of final products is decreased. Raturi et. al. (1990) provide a number of good examples in machine tools industry where the proposed problem exists and a solution will help the manufacturer to better satisfy customer demands. For uncertain demand patterns for final products, a good compromise among the overall inventory carrying costs, service levels, and lead times might be achieved if inventories of semi-finished products were kept in various intermediate stages. This consideration will have even a better effect when the system operates in a pull-type production environment.

The optimum levels of work in-process in general pulltype manufacturing systems have been studied by several researchers including Ohta and Miyazaki (1988), and Bitran and Chang (1987). So and Pinault (1988) presented some analytical results for determining the optimum number of inprocess inventories in a single product pull-type system. Azadivar and Lee (1985) showed the results for a multiproduct flexible manufacturing system. A closely related work to this study is that of Garg and Lee (1994) where the operating characteristics of multi-product production lines are investigated. Using a modified linear production rule by Tang (1990), they suggest target levels for inventory at each stage to be set to cope with uncertainties in demand.

This paper provides a heuristic method for determining these target values for a given multi-stage production system. However, the problem under study here has an additional complexity in that the resources are shared for various stages of production. This puts an additional constraint on allocating the in-process inventories, as stages that need more scarce resources need to be supplied with larger levels of semi-finished products.

Since each process does not have its dedicated resource and has to share it with other activities and because of the stochastic nature of demands and processing times, Tang's linear production rule or other analytical methods would not be easy to apply here. Thus for this study, the system is modelled through computer simulation and is formulated as a constrained stochastic optimization problem. A modeling and optimization procedure has been developed to determine the near optimum levels of in-process inventories to keep at each stage of operation. The objective of the optimization is to minimize the average lead time for satisfying a set of uncertain demands while keeping the inventory levels at a reasonable values and maintaining an acceptable service level. In addition, a software package has been developed that facilitates application of this methodology. Through a graphical interface the user supplies inputs on the parameters and the operating conditions of the system. The program uses these inputs and provides near optimum values of the work in-process at each stage.

# **2 PROBLEM DEFINITION**

To help in understanding the problem, consider the system shown in Figure 1. Four different products are produced from the same stock of bar through several operations utilizing four machines MC1, MC2, MC3, and MC4. In this figure, nodes represent the states of the product while arcs denote the processes of transforming a product from one state to another. Arcs are also marked by the manufacturing resource (e.g. machine) needed for the transformation. All processing times are random and are distributed according to given distributions. Orders for different products are also random and arrive at random intervals.



Figure 1: Production System for a Four Product Family

For the problem under study, it is assumed that the production period consists of just one cycle. The demand for products during this cycle is assumed to be uncertain with known probability distributions. It is also assumed that the lead time for obtaining the raw material is an order of magnitude larger than all due dates for orders. This means that if the plant runs out of raw material during a production cycle, there will not be enough time to order and receive the raw material in time and the corresponding orders will be lost. Thus all needed raw materials have to be ordered in advance. An example of this situation is that of a machine tool manufacturer ordering raw forgings for press brakes which would be customized for a variety of models. Further, it is assumed that the manufacturer orders as many sets of raw material as are sufficient to supply the expected value of the demand.

With these assumptions, the maximum service level, in terms of the fraction of the total demand that is satisfied can be achieved if all the stock is kept in the raw state at node 1. However, this may create long lead times because each order has to wait for the product to go through all stages of production. Alternatively, they can be processed ahead of time and kept at final states for each product. This will reduce the lead time to practically zero for orders that are satisfied, but some demands will go unsatisfied because raw materials needed are tied up in other products for which the demand did not turn out to be as expected. A solution between these extremes will result in a lead time in between while satisfying an acceptable portion of the demand.

#### **3 PROBLEM FORMULATION**

Before presenting the general formulation for the problem the following clarifications are in order:

1. Since the problem is considered in its general form with stochastic demand patterns and processing times, and the performance of the system is evaluated through computer simulation, the objective function and the constraints for this formulation are not available analytically. They can only be evaluated for a given set of decision variables and only through multiple runs of the simulation model.

2. The lead time has been defined as the average of the lead times for individual final products and is estimated for only those demands that are supplied. Thus, the demand for products that are not produced, due to lack of raw material, is assumed lost. This means neither the lead time nor the service level alone can be considered as the performance measure for the system. Otherwise, one could show a drastic reduction in the lead time by losing too many orders.

Based on these definitions and assumptions the problem in its general form can be stated as following:

Minimize
$$f(x_1, x_2, ..., x_n)$$

S.T.
 $g(x_1, x_2, ..., x_n) \ge b$ 

where,  $x_1$ ,  $x_2$ ,  $x_3$  ....,  $x_n$  are the decision variables representing the levels of work-in-process inventory at different nodes, f is the average lead time for customers as a function of work-in-process inventories, g denotes the service level in terms of the fraction of the total demand that is satisfied, also as a function of the in-process inventories, and b is the minimum tolerable service level.

The lead time obtained from one simulation run is only one stochastic observation on the response of the system. This observation is noise corrupted and consists of the theoretical mean (expected value) and some random error. The expected value of this function can be optimized by optimizing the theoretical regression function of the response defined by:

$$y(x_1, x_2, ..., x_n) = E[f(x_1, x_2, ..., x_n) | \forall x_i].$$

Since this function is not known analytically, its values have to be estimated based on several observations made on its simulation model.

Service level is also a stochastic variable. This makes it mathematically incorrect to require the service level obtained from the simulation model, which is a random variable, to be compared to a constant value. A more correct way to deal with this constraint is for the decision-maker to specify a probability indicating the acceptable risk for violating a given constraint. If we denote the risk for violating the minimum tolerable service level as  $\alpha$ , the modified problem can be rewritten as follows:

#### **4 OPTIMIZATION METHODOLOGY**

Azadivar (1992) provides an overview of the available simulations-optimization procedures. For this problem, SIMICOM algorithm developed by Azadivar and Lee (1986) has been employed. SIMICOM is developed for solving the optimization problems as defined above and is based on Box's (1965) complex search method. It can be interfaced with any simulation model automatically to seek the optimum.

#### 4.1 Example 1

To apply and test the procedure, a simple system for production of three products, belonging to the same family, was considered. The network presentation of this system is given in Figure 2. The system is assumed to operate in a pull-type production environment. All products are manufactured in two stages and all the state changes are performed by two machines denoted by MC1 and MC2.



Figure 2: Three Product Production System for Example 1

Orders for the products can come in batches for any of the final nodes. If there are finished products waiting at these nodes, orders are met right away. However, if there is no product waiting, a set of orders is sent to one or more of the supplying nodes based on the workloads on the machines. A tally is kept on the work already assigned and not yet completed for each machine. For instance, if an order comes to node 5 (Figure 2), it can be either satisfied by semi-finished products at node 2 using machine 2 through process 2-5 (arc 2 5) or by those at node 3 using machine 1 through process 3-5 (arc 3 5). Depending on the workloads on these two machines (time needed to process the products that are

waiting in the machine queue), the load will be assigned to the machine with the lighter load. Ties are broken arbitrarily.

This problem was solved for a case where orders arrive with interarrival times that are distributed normally with a mean of 30 and a standard deviation of 5 time periods. Each order consists of a batch of 10 that could be for products 1, 2, or 3 with probabilities of 0.5, 0.35 and 0.15 respectively. The production cycle consists of 600 time units with an expected total demand of 200 units for the planning period. This will be the number of sets of raw material that will be ordered. The processing times were assumed to be all normal with parameters (4,1), (3,1), (5,2), (4,1), (4,1) and (3,1) time periods for processes 1-2, 1-3, 2-4, 2-5, 3-5, and 3-6 respectively. The minimum desirable service level was specified as 90%. The resulting work-in-process inventories are given below:

Nodes	2	3	4	5	6
WIP	4	4	9	16	16

This solution results in an average lead time of 38.07 time periods with an average service level of 93%. It took 44 evaluations of the simulation model to reach this solution.

Here, since there were only 6 variables involved, the solution could be obtained rather easily. However, in many situations, decisions should be made on a large number of work-in-process values where many stages are involved. In those situations, the simulation-optimization requires a large number of simulation evaluations which in some cases will demand an infeasible amount of computer time. Besides, this will put a heavier burden on the management in controlling a large number of decision variables.

One way to alleviate this problem is to group the nodes in a few groups according to a suitable set of criteria. Then, rather than solving the problem for the work-in-process for each individual node, it can be solved for the same amount assigned to each node of a given group. This could reduce the number of variables into a manageable size. The quality of the solution obtained in this way may, of course, be inferior to the one for individual nodes, but the optimization efforts will be reduced significantly.

The following section covers a discussion and analysis of some grouping criteria with a conclusion on the most suitable one for a given situation.

## **5 CRITERIA FOR GROUPING OF NODES**

In searching for common characteristics for nodes for grouping purposes, the following factors showed reasonable relevance.

#### 5.1 Stages of Production (A)

The nodes can be grouped according to stages of production. Stages are defined according to the relative position of the nodes in the network. The initial raw material node is considered to be at the lowest stage (stage 1) and the final product nodes are considered to be at the highest stage. The nodes that follow stage 1 are at stage 2 and those following stage 2 nodes are denoted stage 3 and so on. To incorporate this factor in grouping the nodes each node is given a weight according to the stage it is at.

### 5.2 Flexibility of Nodes (B)

According to this criterion, the flexibility is defined as the fraction of final products that can be reached from a node. Consider the network shown in Figure 3. Let us assume that when an order arrives the probabilities that it will be for products 1,2,3 and 4 are 0. 50, 0.25, 0.10, and 0.15 respectively. The fraction of the total expected demand for each of these products will be proportional to these probabilities. From node 9, only the demand for product 1 can be satisfied. The probability of an order being for product 1 is 50%. So, from node 9, 50% of the total demand can be satisfied. This gives node 9 a flexibility factor of 0.50. From node 6, we can satisfy any demand for products 1,2 or 3. The probability of an order being for products 1,2 or 3 is 0.85 (0.50+0.25+0.10). So, node 6 has a flexibility factor of .85. In this way all the nodes are assigned a flexibility factor. These factors for some of the nodes are shown in Figure 3.

#### 5.3 Combined Grouping Criteria (AB)

If nodes are grouped according to their flexibility, the optimization program will automatically allocate higher levels of inventory to the most flexible nodes. By keeping higher levels of inventory in the most flexible nodes, acceptable service levels will be maintained but satisfactory lead times may not be achieved. If nodes are grouped according to stages, the optimization program will automatically maintain higher levels of inventory in the final stages. By keeping high levels of inventory in the final stages, the optimization program can reduce lead time but that might not allow for meeting service level requirements. Since none of the two rules satisfy both lead time and service level criteria, a combination of these rules was also examined.

#### **6 SUITABLE GROUPING CRITERIA**

As discussed before, grouping represents a tradeoff between the computational efficiency and the quality of the solution. By carefully selecting the suitable criterion for grouping, efficient designs can be achieved without sacrificing the quality of the solution significantly. To determine the suitable criteria for grouping, several experiments were performed.

The three criteria A, B, and AB for grouping were tested

on a set of 3-stage network problems possessing various specific characteristics. The characteristics that were considered are as follows.

### 6.1 Overall Flexibility of the Network

Here, the overall flexibility of a network is defined as the average flexibility of each node in the network. Figures 4 and 5 demonstrate two systems with high and low overall flexibilities, respectively. Two networks with average flexibility of approximately 40% and 70% were examined.

#### 6.2 Demand Patterns

Orders arrive at intervals distributed according to a distribution specified by the user. The probabilities for distributions of the orders among the products are also assigned by the user. If orders arrive with equal probabilities for all products we refer to the system as one with a uniform demand pattern. If probabilities for products are significantly different, the system is referred to as having nonuniform demand pattern. Two demand patterns; one with equal probabilities varying between 0.1 to 0.4 were considered.

#### 6.3 Batch Sizes

Orders for parts can come individually or in batches. For the experiments in this study two batch sizes of 6 and 9 were tested.

#### 6.4 Required Service Level

Two levels of 90% and 95% tolerable service levels were considered in conducting the experiments.

For each case, the inventory levels were determined using all the rules. The resulting lead time for each case was used as the measure of performance for comparison purposes. For each case, an analysis of variance was performed and using Fisher's Least Significant Difference (LSD) the significantly better (at 10% level) rule for each case was determined. The summary of conclusions from these experiments is as follows:

1) Rule B, grouping nodes according to their flexibility, performed well in all cases where the tolerable service level was at the lower level.

2) Rule B performed equally well for relatively high service level requirements for uniform demand patterns.

3) Rule B was also quite consistent when the network was highly flexible and demand patterns were uniform.

4) For high service level requirements and non-uniform demand patterns, Rule AB, grouping according to a combination of stage weight and flexibility, performed more consistently.



Figure 3: Assigning Flexibility Factors to Nodes



Figure 4: An Example of a High Flexibility Network

Figure 5: An Example of a Low Flexibility Network

5) Rule A, grouping the nodes according to their stages, never dominated the other two rules. Thus an argument could be made against using this rule by itself in any of the cases.

### 7 IMPLEMENTATION

A generic software has been developed that implements the procedure described in this paper automatically. Users provide values for the parameters of the system through an interactive interface. These include the number of final product types; the number and characteristics of each operation; possible routing for each product; resources required for processing each product; distribution of operation times; and the demand patterns for final products. A graphic interface has been developed through which these input parameters can be supplied by constructing the network and assigning resources and processing times to the arcs. The program automatically builds the simulation model, interfaces it with the optimization routine and determines the results based on the number of groupings specified by the user. Using this program the user can try various grouping criteria to choose the best for a given system.

## 8 CONCLUSIONS

This paper presented an application of simulationoptimization for strategically locating semi-finished products in a manufacturing system where several products are manufactured using the same raw material through several sets of manufacturing operations. The results showed that it is possible to service demand more efficiently and maintain reasonable levels of in-process inventory if the semi-finished products are manufactured in advance and are located in appropriate stages of operation and in appropriate quantities. The experience with this model showed that computation efforts for conducting such optimization process for relatively large problems is sometimes prohibitive. However, it was discovered that it may not be necessary to determine a separate optimum level for each variable individually. Depending on the structure of the system, many nodes represent characteristics by which they can be classified into various categories and the reasonable inventory levels could be specified only for each class. This drastically reduces the optimization efforts and the management requirements for the implementation of the results for a solution that still may be reasonably good to implement.

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