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BATCHING METHODS IN SIMULATION OUTPUT ANALYSIS: WHAT WE KNOW AND WHAT WE DON'T

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ABSTRACT

As an advanced tutorial, we discuss batching methods for determining point-estimator precision for steady-state simulation experiments. We emphasize batching methods in which each batch provides a point estimator analogous to that of the experiment, but we mention other methods that use batches, especially the more-general idea of standardized time series. Despite the preponderance of literature on confidence-interval estimation for the mean using adjacent nonoverlapping batches, we focus on estimating the point estimator's standard error and consider both general point estimators and general batching relationships. Literature on multivariate batching exists, but we focus on the univariate problem. We consider the initial-transient problem only in passing. Specific issues include form of the point estimator, definition of the batch statistics, form of the batch-statistics estimator, optimal batch size (including various definitions of *optimal*), and determining batch size. This paper is a short summary of the issues, with a fairly complete bibliography.

1 INTRODUCTION

Consider a simulation experiment that produces steady-state data $\{Y_i; i = 1, 2, \ldots\}$. The purpose of the experiment is to estimate θ, a property of the steady-state distribution function F_Y. The property θ is a performance measure of the model being simulated, often a mean or variance or quantile.

2 POINT ESTIMATION

Point estimation is typically straightforward. Early observations whose distributions might differ substantially from F_Y are weighted lightly, often discarded entirely. The point estimator, $\widehat{\theta}$, is chosen to mimic θ, typically the sample mean, sample variance, or a simple function of the relevant order statistics. Although sometimes point estimators are defined to be the average of batch statistics, the grand estimator (obtained by using all of the data at once) is often simpler to compute and has better statistical properties, in particular smaller bias. (The average of batch means is the grand mean, so in that special important case the point estimators are identical.) The purpose of batching is to estimate the precision of the point estimator, not to improve the point estimator.

3 BATCH STATISTICS

The i^{th} *batch* is composed of the observations $Y_i, \ldots, Y_{i+m-1}$ for $i = 1, 2, \ldots, n-m+1$, where m is the batch size and n is the run length. The i^{th} batch statistic $\widehat{\theta}_i$ is a miniature version of $\widehat{\theta}$, again typically a sample mean, sample variance, or simple function of the relevant order statistics. When appropriate, a grand estimator can be used in the batch statistic; for example, centering a batch variance on the grand mean is better than centering it on the batch mean.

4 STANDARD-ERROR ESTIMATION

Batch statistics are combined to estimate the variance of $\widehat{\theta}$, or its square root the standard error. The batch-statistics estimator for the variance is

$$\widehat{\text{Var}}_A[\widehat{\theta}] = \frac{\sum_A (\widehat{\theta}_i - \widehat{\theta})^2}{d|A|},$$

where d is a function of n and m, chosen, for example, to yield $\text{E}\widehat{\text{Var}}_A[\widehat{\theta}] = \text{Var}_A[\widehat{\theta}]$ for independent data. A is a subset of $\{1, 2, \ldots, n-m+1\}$ and $|A|$ is the cardinality of A. Not surprisingly, the choice of A has little effect on bias, and variance is minimized if $A = \{1, 2, \ldots, n-m+1\}$, the case of overlapping batch statistics. Maybe surprisingly, computation of batch means and batch variances is $O(n)$; batch quantiles require $O(n \ln n)$ computation.

Rather than thinking of $\widehat{\mathrm{Var}}_A[\widehat{\theta}]$ as an estimator of $\mathrm{Var}_A[\widehat{\theta}]$, one can think of estimating the asymptotic constant $n \lim \mathrm{Var}_A[\widehat{\theta}]$. The change in the batch-statistics estimator is simple: multiply by n. But this relationship is true only asymptotically, and the difference can be noticeable for surprisingly long runs.

5 OPTIMAL BATCH SIZE

Optimal batch size is usually discussed in one of two contexts: confidence-interval estimation or standard-error estimation. Criteria for confidence-interval procedures include probability of covering θ, expected interval length, variance of interval length, and (more generally) the probability of covering an arbitrary point θ_0. Because confidence-interval estimation is inherently multi-criterian, seemingly contradictory advice abounds in the literature. Choosing ten to thirty batches is fine for fixed n if only coverage probability and expected length are considered, but does not yield the consistency needed for the asymptotic results underlying sequential methods. And more batches are useful when using control variates or obtaining smaller variance for the confidence-interval length.

Standard-error estimation is simpler because it allows a single criterion. The usual criterion is the mean squared error (mse) of $\widehat{\mathrm{Var}}_A[\widehat{\theta}]$. If $\widehat{\theta}$ is the sample mean, then asymptotic arguments lead to an mse-optimal batch size

$$\vec{m}^* = \left(2n \frac{c_\mathrm{b}^2}{c_\mathrm{v}} \frac{\gamma_1}{\gamma_0}\right)^{1/3} + 1,$$

where c_b^2 and c_v are the known bias constant and variance constant of the batch-statistics estimator, $\gamma_j = \sum_{h=-\infty}^{\infty} |h|^j \operatorname{Corr}(Y_i, Y_{i+h})$, and the additive constant is chosen to yield batch size of one for independent data. Much less is known when $\widehat{\theta}$ is not a sample mean, but that $\vec{m}^*$ grows with the cube root of n seems quite general.

6 DETERMINING BATCH SIZE

Many papers over the last few decades have suggested methods for choosing the number of adjacent nonoverlapping batches to obtain reasonable confidence intervals for the mean, often based on tests of independence. In the last few years papers have appeared on estimating $\vec{m}^*$, the central problem being how to efficiently estimate γ_1/γ_0, which can be interpreted as the autocorrelation's center of gravity. Very recently some papers have considered mse-optimal batch sizes for specific point estimators that are not means. Given n saved observations, $O(n)$ batch-means algorithms exist that can be used without tuning; no such algorithms exist for general point estimators.

7 WHAT WE DON'T KNOW

Despite knowing quite a lot about batch means, no method is available for estimating the variance of $\bar{Y}$ when the data come one at a time and are not saved. Such a method should use finite memory and somehow increase batch size as the run length increases. It should not need to know the run length *a priori*.

Little is known about batch sizes when the point estimator is not $\bar{Y}$. Creating a literature for each possible point estimator seems daunting. General methods will probably be created from two general results: that $\mathrm{Var}_A[\widehat{\theta}]$ is $O(n^{-1})$ and that $\vec{m}^*$ is $O(n^{1/3})$.

How to transfer the technology is also an open question. In practice, there are often tens of performance measures θ, all needing a point estimator and associated measure of precision. The practitioner has not the time, and often not the knowledge, to use any method that is not completely automated. Batch means has an advantage compared to other methods of estimating point-estimator precision because it generalizes so well to non-mean point estimators, which might allow the same method to be used for all point estimators.

More fundamentally, the research community does not agree on a set of criteria for good algorithms, and the research community and the practitioner community too seldom discuss the criteria. For example, the authors think that a good beginning would be the creation of an automated method to allow statistically meaningless point-estimator digits to be deleted from simulation output reports. Even such a minor suggestion for indicating point-estimator precision in practice leads, however, to interesting, non-trivial issues.

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