

RETROSPECTIVE SIMULATION RESPONSE OPTIMIZATION

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ABSTRACT

An approach to simulation response optimization is presented where a simulation experiment is run in such a manner as to generate optimal solutions. Once the stochastic sample path of a simulation run has been generated, it is sometimes possible to retrospectively solve a deterministic optimization problem or a closely related problem. As demonstrated with some examples from communications and manufacturing, this approach can greatly simplify both the simulation experiment and the simulation model.

1 BASIC APPROACH AND NOTATION

1.1 Conventional Response Surface Experiments

The objective of a conventional simulation response surface experiment might be to find the values for a set of controllable factors such that some function of the (random) simulation response is optimized. Specifically, let $P(\theta)$ be a scalar sample performance measure of a system where $\theta \in \mathcal{D}$ is a vector of decision factors selected from the set of feasible designs, \mathcal{D} . $Z[P(\theta)]$ is some characteristic of the performance which is called the system response at θ . For example, $Z[P(\theta)]$ might be the expected long-run cost of operating the system with factors, θ . One experimental goal might be to find θ^* such that

$$\theta^* = \underset{\theta \in \mathcal{D}}{\operatorname{argopt}} Z[P(\theta)] \quad (1)$$

Conventional simulation response optimization experiments typically involve

1. selecting successive values of θ from the set \mathcal{D} according to some design or algorithm,
2. replicating the simulation at each selected value of θ to estimate the distribution of $P(\theta)$ and $Z[P(\theta)]$, and finally,

3. deciding to quit and select the best factor as an estimate of θ^*

In designing such an experiment, many decisions must be made such as

1. what values of θ to run,
2. how many replications to run at each of these values of θ ,
3. how to initialize each run,
4. how long to make each of the replications,
5. what data to collect, and
6. when to stop the experiment.

The design of simulation optimization experiments is an active area of research. However, there has yet to be developed a method that works well for a broad class of simulation optimization problems; techniques may be spectacular for some problems while failing in others (see Jacobson and Schruben 1989). The approach presented here, as presently developed, is no exception. We will give some examples where it works well and also illustrate where it can give poor results. Structuring the approach into an algorithm and discovering its appropriate domain of application is a focus of current research.

1.2 A Retrospective Approach

"Monday morning quarterbacks" always look smart. Correct decisions are easy after the fact, but hindsight is not possible in the real world. However, for some important engineering design decisions, 20/20 hindsight is possible - in a simulated environment. In management education, *retrospective* analysis of case studies and management decision games are often used to develop *qualitative* principles and guidelines for problem solving. The focus here is on *quantitative* engineering decisions such as setting production schedules, inventory levels, capacity planning, etc.

The notion is to look at the output from simulation runs retrospectively in order to generate only optimal

answers. That is, we try to solve a deterministic optimization problem with respect to an observed sample path as if the outcomes of all uncertainties were known in advance. Of course, the sample path we record is different than that obtained by merely recording system performance. Once a sample path is determined though, it may be possible to reconsider various decisions and design factors without changing the sample path.

Specifically, we define the random variable

$$\tilde{\theta} = \underset{\theta \in \mathcal{D}}{\operatorname{argopt}} P(\theta)$$

with the intent of estimating θ^* by some function of $\tilde{\theta}$. In fact, the problem we will be solving is going to be different from (1). The retrospective optimization experiments proposed here result in an estimate of

$$\hat{\theta} = \underset{\theta \in \mathcal{D}}{\operatorname{argopt}} Z[P(\theta)] \tag{2}$$

which is generally different from (1) but often trivial to compute. While (1) might be regarded as "the problem", (2) has some virtues beyond the fact that it is sometimes easier solve. In recurring situations, (1) might be more appropriate for evaluating long-term performance (although it might involve unrealistic risk taking - playing the lottery might make sense if you can afford to keep playing until you win).

On the other hand, if $Z[\cdot]$ is the mode, then (2) can be interpreted as the solution with the *maximum likelihood* of being optimal for the next experiment or realization. The "next" experiment we are referring to is the real one, and it may only occur once. From this perspective, a solution to (2) perhaps makes more sense than a solution to (1). It also may be that the set of possible scenarios is very large. The approach proposed to solve (2) would let the simulation choose the scenarios naturally and find what would be best for these scenarios. Thus sampling would be concentrated in those scenarios that are most likely to occur.

While it may appear to be quite different, each run of the experiment we are proposing can be viewed as a so-called "trace driven" simulation. In a trace driven simulation, actual operating data is fed into the model. The best answer is found, retrospectively, for the particular sample path being studied. Most of the shortcomings and advantages of trace driven

simulations apply to this method with the exception that we are going to be able to run independently seeded replications. A conventional trace driven simulation would be like running only one replicate of the optimum generating simulations proposed here.

2 SOME EXAMPLES

2.1 Production Lot Sizing

The approach can be illustrated with a very simple example: a manufacturing process consisting of only one operation. There is a demand for D good finished parts in a production run and we need to determine how many parts to schedule. The answer would simply be D , except for fact that the operation has a variable yield of good product. We assume that this shrinkage is well documented but cannot be controlled exactly or predicted with certainty. It will be necessary to schedule more than ten parts in order to have a reasonable probability of producing at least ten acceptable finished parts. How many? The problem is pictured below,

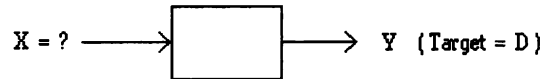


Figure 1: Yield Management Problem

Retrospectively, if the shrinkage, $(X - Y)$, were known the answer is easy; simply set X equal to the shrinkage plus D . What good is this answer? Clearly, this answer, by itself, is not particularly valuable. However, it would be trivial to simulate this system thousands of times and estimate the empirical probability distribution of the optimal production lot size, X . The question is then: what do we do with this sample of optimal values?

The problem is that we have made exactly hitting the production target our only concern; close is no better than "missing by a mile". Models of this sort typically assign costs or penalties to missing the target value of D good finished parts. If having ten parts is critical to some down-stream production process, there will be a large cost to a shortage; if the parts are expensive to produce, there will be a significant cost to any excess production. Let C_s denote the per unit cost of a shortage and C_e be the per unit cost of excess production. Then the cost of a production lot size of X is

$$C(X) = C_e \max(0, (X - D)) + C_s \max(0, (D - Y))$$

We will run the simulation and observe the function $C(X)$. The function will have its minimum at some value of $X \geq D$ since for $X < D$ the first term is zero and the second term is non-increasing. Also the function increases linearly once $Y = D$ so we know that we can stop the run once $Y = D$. The experiment is simply to replicate starting with $X = D$ and run until $Y = D$, recording the value of X where $C(X)$ is minimized. Pick the value of X that occurs most often.

Determining an appropriate number of replications to run might involve using some of the well-developed machinery of statistical selection and ranking procedures (see Goldsman 1988). The sample size problem is complicated by the fact that the upper limit on X is unknown and random. Otherwise, it would be easy to use a multinomial statistical selection and ranking procedure. If U were an upper bound on X , then the outcome of each independent replicate follows a multinomial distribution with $U - X$ possible classes. The selection objective might be to select the value of X with the greatest likelihood with a bound on the probability of making an incorrect choice.

For a specific example, assume that the target demand, D , is 10, the probability of a particular part being good is .7 (independent of the quality of the other parts), $C_s = 5$, and $C_e = 2$. The simulation described above was run giving us (after 2000 replicates) the marginal histograms for the optimal values of X and corresponding minimum costs shown in Figures 2, and 3.

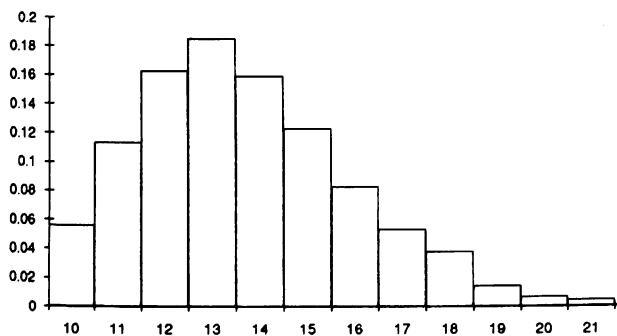


Figure 2: Optimal Lot Size

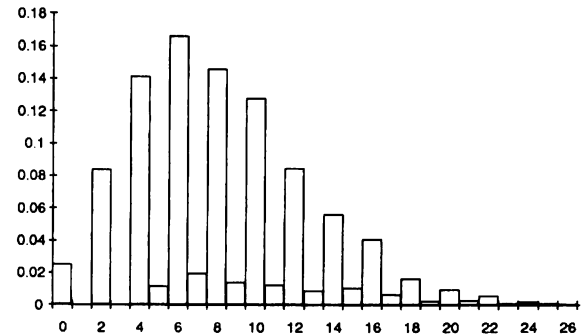


Figure 3: Min Cost

It is not at this time exactly clear what to do with this information. The most likely optimum value was $X = 13$ and the nearest integer to the average optimal solution is 14.

We are not ready to suggest that the solution to (2) generated by the above experiment be used without further study. There is the obvious danger mentioned above that we do not consider the "down side" to each possible solution; that is, how bad is the solution when it is not optimal. To illustrate: assume the cost structure was such that there was a very large penalty for a shortage and a very small cost of overproduction (not an unrealistic situation). The conventional experiment would report the largest value of X seen as the solution to (1) whereas the retrospective experiment would report a distribution of answers not unlike that in Figure 3. A more appropriate model for this highly asymmetric costs situation would be to try and minimize the lot size constrained by the probability of a shortage not being higher than a specified risk level.

Of course, for small problems with simple (independent) yields direct analysis is possible without resorting to simulation. However, any serious manufacturing problem is more likely to involve many operations in large complex branching flows, reworking of bad parts, and dependent yields. An analytical treatment of these events in all but the most elementary real problem is impossible or requires unreasonable assumptions. For real problems with realistic assumptions, direct analysis becomes intractable while the experiment described above remains essentially the same. There are several approaches to this problem, some of which are addressed by Schruben (1991).

2.2 Buffer Space Allocation

A problem frequently encountered in the design of manufacturing, communications, and computer systems

is how to allocate a fixed number of buffers spaces amongst the stations or nodes in the process to avoid frequent buffer overflows. Anantharam (1989) considers as a model an open Jackson network of exponential servers, Poisson arrivals and Bernoulli routing. The objective is to optimize some performance measure associated with the time to buffer overflow. He uses pathwise probabilistic arguments to reason that for *any* such performance criterion, the allocation should be made roughly in inverse proportion to the logarithms of the effective service rates.

The simple tandem arrangement of queues given below is an example of such a network where λ is the arrival rate and the μ 's are service rates. If there are 10 buffer spaces in all, this method allocates 6 spaces to the first station and 2 spaces each to the remaining 2 stations; we denote this allocation as (6,2,2).

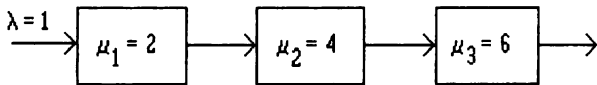


Figure 4: Tandem Queuing Network

A retrospective approach to such resource allocation problems might be as follows. The general idea is to run the simulation unconstrained by a resource until the total availability of the resource is exhausted. In this case, a simulation of the network is run with unlimited work-in-process buffer capacities while keeping track of the running maximum buffer contents at each operation. When the sum of these running maxima reaches the total buffer space available, the run is stopped. For a given run, the final values of the running maxima are necessarily the allocation that maximizes the time until blocking occurs since buffers were allocated whenever needed to keep constraints on total buffer availability from becoming binding for as long as possible. By repeating the experiment we generate samples from the probability distribution of the "optimal" allocation from which an empirical distribution can be estimated.

For the 10 buffer, tandem queue problem described above, 1000 (retrospectively) optimal allocations were generated using the simulation just outlined. Histograms of the marginal allocations for the three positions are shown in Figures 5, 6, and 7.

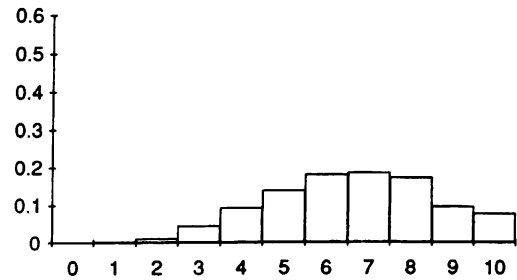


Figure 5: Buffer 1

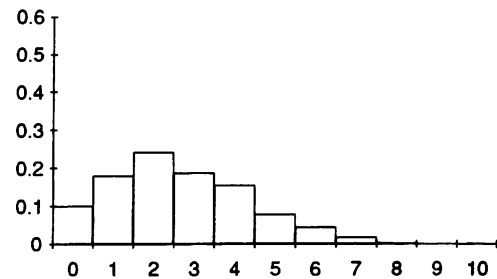


Figure 6: Buffer 2

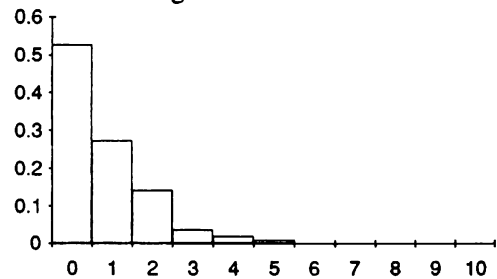


Figure 7: Buffer 3

The most frequent (run-wise) optimal allocation generated was (8,2,0), the closest feasible allocation to the optimal mean marginal allocations was (6,3,1). The expected time to buffer overflow for both these allocations as well as the allocation prescribed by the heuristic was estimated from 1000 simulated realizations. Common random numbers were employed in each set of runs. The sample means, T , and standard deviations, S , of the times until blocking are given in the table below.

Allocation	T	S
(6,2,2)	8.23	3.40
(8,2,0)	7.55	2.99
(6,3,1)	8.84	3.34

Like the previous example, the assumptions on which the analytically-based solution depend are very restrictive whereas the method of generating optimal

solutions presented here should be the same for almost any capacitated queueing network.

2.3 Inventory Management

An extensive theory has been developed to help establish policies for business decisions involving inventory and it continues to be an area of active investigation. An inventory policy is essentially a prescription for "when" and "how much" to order based on the fundamental tradeoff between the cost of managing inventory versus customer service requirements.

Here we consider a relatively simple infinite horizon, discrete time, periodic review model. That is, all activity (orders, demands, decisions, costs, etc.) occur at an indefinite number of fixed, equally spaced time points. The components of cost are c , the per unit cost of ordering, h , the per unit cost of storage per unit time, and π , the per unit cost of unmet demand per unit time. There is no fixed purchase cost or lead time associated with an order and full backlogging of shortages is assumed. The period demands are assumed to be continuous and identically distributed random variables with known distribution F . The goal is to minimize the long run expected cost per period.

It can be shown that the optimal policy can be characterized by a single "order-up-to" quantity, S^* , satisfying

$$F(S^*) = \frac{\pi}{\pi+h}$$

For given F , we can compare this solution with the retrospective approach to estimating S^* in which we repeatedly solve a fixed n -period problem for given realizations of the demand. We restrict the form of the solutions to single order-up-to quantities even though for known demand we can clearly do better by simply ordering an amount in each period equal to the demand for the coming period. If D_i is the (random) demand in period i , then the n -period sample path cost function can (after some algebraic manipulation) be written as follows:

$$C(S) = (c - n\pi)S + (h+\pi) \sum_{i=1}^n \max(S, D_i)$$

The cost function is, with probability 1, a continuous, piecewise linear, convex function of S . This becomes evident by examining the behavior of C for values of S in the intervals between successive values of the ordered demands, $D_{[1]} \leq D_{[2]} \leq \dots \leq D_{[n]}$.

\underline{S}	$\underline{C(S)}$
$0 \leq S < D_{[1]}$	$(c - n\pi)S + (h+\pi) \sum_{i=1}^n D_{[i]}$
$D_{[1]} \leq S < D_{[2]}$	$(c - (n-1)\pi + h)S + (h+\pi) \sum_{i=2}^n D_{[i]}$
\vdots	\vdots
$D_{[n-1]} \leq S < D_{[n]}$	$(c - \pi + (n-1)h)S + (h+\pi)D_{[n]}$
$D_{[n]} \leq S < \infty$	$(c + nh)S$

The sample path optimum, \tilde{S} , occurs at the point at which the slope of the line segments change sign. This point is just $D_{[k]}$, the k^{th} largest of the n demands

where k is the smallest integer contained in $\frac{n\pi - c}{\pi + h}$. The

retrospective simulation experiment simply requires that we generate the k^{th} largest of n period demands. As an example, consider the case of uniform demand on

$[0, D]$ in which case $S^* = \frac{\pi}{\pi + h}D$. $\tilde{S} = D_{[k]}$ has a beta

distribution with $E[\tilde{S}] = \frac{k}{n+1}D$. As n , the number of

periods in the simulated finite horizon problem, increases the effect of the order cost on the solution diminishes and

$$\lim_{n \rightarrow \infty} E[\tilde{S}] = \frac{\pi}{\pi + h}D$$

Current work involves the extension of this estimation technique to classes of inventory problems for which only limited analytical results are available; in particular, those for which (s, S) policies are optimal.

3 CLOSING REMARKS

Of course, while retrospective experiments might give us suggested answers, it is important to evaluate and

refine these solutions. The solutions to (2) generated by the simulations might be thought of as starting points for an optimization algorithm. The spread in the histograms of the generated optimal solutions (like Figures 2 through 6) also give us a pretty good idea of how stable the solution is likely to be. If these histograms were more peaked, (measured, perhaps, by the sample kurtosis), then we might not feel that it is necessary to conduct a conventional optimization study using these solutions to initiate our search for a better answer.

Almost all engineering design studies involve simulation, if only to verify assumptions and predict performance. In these examples, the simulations are used to generate solutions, not only for "cut-and-try" and "what-if?" design and policy evaluation or merely for function evaluations in an optimization algorithm. When the solution to (2) is appropriate (which might be more often than the solution to (1)), the approach suggested here can be straight forward and robust changing the assumptions supporting the model.

Research questions include:

1. What is a correct interpretation of the results of these retrospective optimizations? How different can (1) and (2) be? When might they be close? When might (1) or (2) be the more meaningful?
2. What are some good probability models for analyzing these experiments. Clearly, the discrete (exclusive and exhaustive) outcomes of the experiments described above can be modeled as multinomial sampling. Guidelines on when to stop such experiments might be found along the lines of statistical selection and ranking procedures. There is a literature on the value of "prophets" in probability models that also might prove pertinent.
3. Combining retrospective optimization (to get promising parameter values) with conventional prospective experiments (to evaluate performance and sensitivity) seems like a potentially powerful manufacturing engineering problem solving approach with simulation models. How should an algorithm be constructed to do this? Should there be, say, 10 prospective runs for every retrospective run? When should the search terminate? How can the algorithm fail? How might the algorithm perform on real world problems?
4. What characteristics of a simulation model are necessary and/or sufficient for the retrospective

optimization approach suggested here to make sense? Perhaps a simulation that is not amenable to this approach can be made so by expanding the state space - this might make the technique theoretically feasible but computationally worthless.

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