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MULTICRITERIA OPTIMIZATION OF SIMULATION MODELS

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ABSTRACT

Most simulation models output multiple responses. Yet little research has been done in the area of multicriteria optimization of simulation models. This paper suggests a framework for the multicriteria optimization of simulation models by, first, discussing the unique difficulties of this problem area along with important problem characteristics, and, second, discussing the way that these problem characteristics would affect the choice of a particular technique.

Keywords: Multicriteria Optimization, Goal Programming, Decision Analysis, Multiattribute Utility Theory.

1 INTRODUCTION

Simulation is one of the most useful modeling tools for the design of manufacturing (and other) types of systems. For example, Harpell, Lane, and Mansour (1989), in a survey of large corporations, noted that simulation ranked second in utilization from among eight modeling tools. Simulation was ranked first in terms of utility from among twelve modeling approaches, in a survey of the OR Division members of the Institute of Industrial Engineers (Shannon, Long, and Buckles, 1980).

There are several reasons for the popularity of simulation modeling, including the large number and variety of specialized languages and software packages to aid in the simulation modeling effort, such as SLAM (Pritsker, 1986) and SIMAN (Pegden, Shannon and Sadowski, 1990), and the fact that most graduates from an IE curriculum have had at least one, and possibly two, courses in simulation modeling. Also, the concepts associated with the building of a simulation model are perhaps easier to understand than those associated with many other modeling techniques (e.g., queueing theory).

Perhaps the main reason for the popularity of simulation modeling is the fact that it provides the most accurate modeling methodology. As noted by Evans and Haddock (1991), "simulation models can incorporate a greater level of detail and capture the time dynamic behavior of the system." This differs from other modeling techniques such as queueing theory and mathematical programming, which model a system from a steady state, or a static viewpoint. The ability to model the time dynamic behavior of a system is especially important when such things as machine breakdowns, balking, starving, and other transient aspects of systems must be analyzed.

Although simulation can provide a very accurate model of a system, it is only an evaluative (descriptive) as opposed to generative (prescriptive or normative) modeling tool (Suri, 1984). As such, optimization of a simulation model is typically done in an ad hoc, trial-and-error fashion. That is , the designer may input a "good design" to the model and by examining the output (e.g., performance variable values such as utilizations, production rates, etc.) decide which variables to change in the design to improve this output. However, there is no formal guidance as to how these values should be changed between runs. This formal guidance, or procedure, is especially important when one realizes that there are usually many performance variables (criteria) which are output from a simulation model, and which must be considered.

The fact that many of these performance variables are conflicting makes the optimization process even more difficult when there is no formal guidance between iterations. For example, suppose that an analyst is using a simulation model of a job shop to analyze the routings of particular types of parts. He notices that because of a bottleneck at a particular work center the production rate for one part type is not what he would desire. (This is accomplished through analysis of the output from a simulation model). So, he decides to change the routing for that part type so that the operation(s) that were being performed at the bottleneck work center will now be performed at a different work center. Yet, upon running the model with the new routing, he discovers that the production rate for another part type, which also uses the new workcenter, is not what he would desire. That is, the production rates for the various part types are conflicting in nature since the various part types must share resources. When there are hundreds or even only 5 to 10 part types to consider, expecting the analyst to implicitly trade off the various production rates, without any formal guidance, is unrealistic.

The purpose of this paper is to suggest a framework, or set of guidelines, for the multicriteria optimization of simulation models. This is accomplished by examining the various characteristics associated with this problem, and suggesting particular techniques based upon these characteristics. The problem of manufacturing system optimization will be specifically addressed. However, the principles described will extend to optimization of any general system described by a simulation model.

Many authors have discussed the general areas of simulation optimization and response surface methodology. For examples, see Smith (1973 and 1976), Biles and Swain (1979), Meketon (1987), Wilson (1987), Jacobson and Schruben (1989), Myers, Khuri, and Carter (1989), and Safizadeh (1990). These reviews/discussions have generally not emphasized the multicriteria aspect of the problem, as is done in this paper. Related discussions can also be found in Bengu and Haddock (1986), Cochran and Chang (1990), Hopmans and Kleijnen (1980), Law and Kelton (1991), Porta Nova and Wilson (1989), and Shannon and Prakash (1990).

The next section of this paper contains a formal definition of the problem, along with a brief discussion of the unique aspects of this problem as compared to the related area of multiobjective mathematical programming. The third section contains a discussion of the various characteristics of a problem involving the multicriteria optimization of a simulation model that would affect the specific technique chosen for solving the problem. Following this, in the fourth section of the paper, various techniques along with their advantages and disadvantages are discussed. These techniques are categorized according to the timing of the articulation of the required preference (tradeoff) information with respect to the optimization. Finally, the fifth section of the paper contains some general conclusions.

2 THE PROBLEM

A simulation can be viewed as a black box, for which decision variable values and parameter values are input, and output variable values are obtained by running the model. Often, because of the stochastic nature of the model (e.g., resulting from the use of random variable values as input), the output variable values are actually random variables. This is depicted in Figure 1.

In Figure 1, the decision variables are denoted as $X_1, \ldots, X_n$, the parameters as $P_1, \ldots, P_o$ and the output variables as $Y_1, \ldots, Y_p$. The designer is assumed to have no control over the

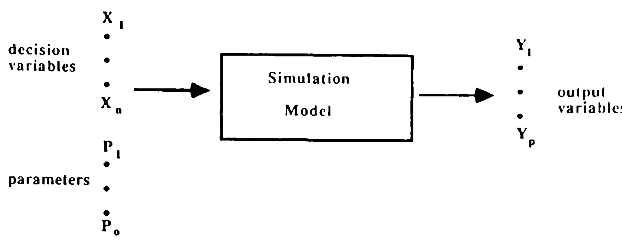

Figure 1. Input and Ouput for a Simulation Model.

parameter values.

The objective of the designer is to choose the values for $X \equiv (X_1, \ldots, X_n)$ so that he is most satisfied with the output variables values $Y_1, \ldots, Y_p$; that is, optimize $(Y_1(X), \ldots, Y_p(X))$ subject to $X \in X^c$ where the Y_i's are functions of X as expressed through the simulation model, and X^c implicitly represents any constraints on X. (We will assume, without loss of generality, that each of the Y's are to be maximized).

This problem is comparable to the type of problem addressed by multiobjective mathematical programming algorithms (see Evans (1984) or Rosenthal (1985)). It does differ however in three important aspects:

1) The relationships between the X's and the Y's are not of a closed form,
2) The Y's may be random variables (i.e., stochastic as opposed to deterministic in nature), and
3) The response surfaces may contain many local optima.

These characteristics result in unique difficulties, which may affect the particular technique chosen to solve the problem.

In the discussion that follows, we will use the following terminology.

1) Attribute: a measure associated with a performance variable (e.g., the mean hourly production rate for a part over a one day run in parts per hour),
2) Goal: an aspiration level associated with an attribute (e.g., achieve a mean hourly production rate of 15 parts),
3) Objective: a direction associated with an attribute (e.g., maximize the mean hourly

production rate),

4) Criterion: anything that can be called an attribute, goal, or objective,

5) Outcome: a point in the Y space, associated with some input X.

The definitions for the first four terms are basically the same as given in Zeleny (1982).

3 IMPORTANT PROBLEM CHARACTERISTICS

The multicriteria optimization technique chosen for a particular simulation model should depend upon several important problem characteristics, including:

1) the number of decision variables and criteria,
2) the nature of the response surfaces (e.g., convex or nonconvex),
3) the nature of the response variables (deterministic or stochastic),
4) the run time for the model, and
5) the ability/desire of the decision maker to articulate various types of preference information, concerning tradeoffs between the various criteria.

Obviously, the larger the numbers of decision variables and criteria, the more complex the problem will be. Of greater concern is the number of criteria which must be handled. When this number gets very large, many formalized techniques for the articulation of the required preference information "break down", and less formalized techniques (e.g., goal programming) must be employed.

The number of decision variables is of great concern only to the search technique; since the designer must only provide preference information in the outcome space.

The nature of the response surface (e.g., whether it is convex or nonconvex) is of concern because this will affect the search technique employed. For example, when there are many local optima, the use of an unmodified local search technique is clearly inappropriate.

Whether the response variables are deterministic or stochastic is another important characteristic. Obviously, the need to consider the stochastic nature of responses makes the problem much more difficult.

The run time for the model is especially important if one uses a technique which employs a progressive articulation of preferences. With this type of approach, the designer must provide preference information between runs of the model.

Finally, the ability/desire of the designer to articulate various types of preference information is perhaps of greatest concern. For example, the designer may only be able to express that one outcome is preferred to another, but he may not be able to give a marginal rate of substitution of one attribute for another.

4 TECHNIQUES FOR MULTICRITERIA OPTIMIZATION

Over the last 25 years, there has been a tremendous amount of research in the broad area of multicriteria optimization. Much of this research has addressed multiobjective mathematical programming in which objectives and constraints can be written as closed-form functions of a problem's decision variables.

Because of the existence of multiple (and conflicting) criteria, multicriteria optimization consists of two aspects. First, information must be gathered concerning the designer's preference structure over the multiobjective outcome space. This preference structure, which is subjective in nature, implicitly defines the tradeoffs that the designer is willing to make among the various criteria of the problem. Second, the optimization itself must be performed in order to identify the preferred design.

The many various techniques and methodologies which could be used for the multicriteria optimization of a simulation model might be categorized by the following characteristics:

1) the timing of the required preference information vs the optimization,
2) types of preference information required,
3) types of decision variables considered (all continuous, mixed, pure integer), and
4) types of objective and constraint functions handled (all linear or at least some nonlinear).

With respect to the first characteristic, the articulation of the designer's preference structure can occur:

1) prior to the optimization (prior articulation of preferences),
2) during the optimization (progressive articulation of preferences), or
3) after the optimization (a posterior articulation of preferences).

In the discussion that follows, we categorize various methods which could be employed in solving multicriteria optimization models, according to the timing of this preference information.

4.1 Prior Articulation of Preferences

Basically, a prior articulation of preferences indicates that all information concerning the tradeoffs that the designer is willing to make between the criteria is obtained prior to the optimization process. The three main types of formalized techniques/methodologies which could be included in this category involve the use of multiattribute value functions, the use of multiattribute utility functions, and goal programming.

4.1.1 Multiattribute Value Functions

A multiattribute value (MAV) function, v, is one which maps the outcome space into the space of real numbers

$v\colon Y_1,\ldots, Y_p \Rightarrow \mathbb{R}$

Usually v is scaled so that its range is [0, 1]. The MAV function has the characteristic that:

$Y' \succ Y'' \Leftrightarrow v(Y') > v(Y'')$

where $Y' \succ Y''$ indicates that the outcome Y' is preferred to the outcome Y". Hence once the MAV function is formed the problem is one of

$$\underset{X}{\text{Maximize}}\ v(Y_1, \ldots, Y_p),$$

subject to $X \in X^c$.

The MAV function is determined through detailed interview sessions between the designer and an analyst, in which the designer expresses the tradeoffs he is willing to make between the various criteria: $Y_1, \ldots, Y_p$. (Keeney and Raiffa, Chapter 3, 1976).

There are several difficulties associated with using this approach to optimize multiresponse simulation models. First, the relationship between the X's and Y's are not of a closed form nature. Therefore, a numerical search technique will typically be employed in the process.

Second, with a value function, the stochastic nature of the responses cannot be explicitly considered since the function can only rank deterministic outcomes. Hence, only the expected values of outcomes may be considered. One way to circumvent this difficulty is to consider the variance of an outcome variable as one of the attributes, or responses.

Third, when there are more than a few attributes (or responses) to consider, the assessment of a MAV function can be exceedingly difficult. One way to get around this problem would be to select 3 or 4 of the most important criteria, and assess a MAV function over these criteria. The other criteria could be considered implicitly through the use of constraints. For example, if the first three criteria were the most important, then the problem might be expressed as:

Maximize $v(Y_1, Y_2, Y_3)$ over x
subject to: $X \in X^c$, $Y_j \geq b_j$ for $j = 4, \ldots, p$.

See Mollaghasemi, Evans, and Biles (1991) for an example of an approach which employed MAV function for the optimization of a simulation model.

4.1.2 Multiattribute Utility Functions

A multiattribute utility (MAU) function, u, is one which allows the ranking of probability distributions over the outcome space. The function itself is a mapping from the outcome space into the space of real numbers:

$u\colon Y_1, \ldots, Y_p \Rightarrow \mathbb{R}$

This function is also usually scaled so that its range is [0,1].

A MAU function has the characteristic that:

$Y' \succ Y'' \Leftrightarrow EU(Y') > EU(Y'')$

where $Y' \succ Y''$ denotes that the decision maker prefers probability distribution Y' to probability distribution Y", and EU(Y') denotes the expected utility of Y'.

Once the MAU function has been assessed, the problem can be stated as

$$\underset{X}{\text{Maximize}}\ EU(Y) \text{ subject to } X \in X^c;$$

i.e., choose the values of the X's which maximize the expected utility of the outcome.

The advantage of the use of a MAU function is that the stochastic nature of the responses can be explicitly considered.

The main disadvantage of the use of a MAU function is that the assessment procedure is even more difficult than the assessment for a MAV function. The reason for this is that the designer must express his tradeoffs over <u>lotteries</u> in the outcome space.

4.1.3 Goal Programming Approaches

Goal programming involves the determination of aspiration levels or goals for the various criteria, and then using as an objective function a weighted sum of the goal deviations.

The major difficulty with a goal programming approach is that the objective function may not represent the preference structure of the designer in an accurate fashion. For example, as noted by Goicoechea (pp. 116-118, 1982), a goal program will not necessarily lead to a nondominated solution; and, its effectiveness as a design tool relies on the ability of the analyst to capture the essential elements of the problem as goals and constraints.

A major advantage of a goal programming

approach is that the preference information required of the designer (e.g., aspiration levels) is easier to provide than the preference information required to assess a MAV or a MAU function. Clayton, Weber, and Taylor (1982) and Rees, Clayton, and Taylor (1985) give examples of the use of goal programming for the optimization of multiple response simulation models.

4.2 Progressive Articulation of Preferences

A progressive articulation of preferences typically involves sequential interactions between the designer and a specialized optimization algorithm. The designer specifies some "local information" (i.e., relative to a particular point in the outcome space: $Y_1, \ldots, Y_p$) about his preferences over the multidimensional outcome space. This specification of local information allows the algorithm to formulate a single criterion subproblem, which is then solved. The new solution point and outcome is then presented to the designer as a new reference point, from which he can provide some new local information.

A key requirement in any algorithm that would require a progressive articulation of preferences is that the run time for the simulation must be relatively short; otherwise, the designer may not be able to participate in the entire process. For example, a progressive articulation technique which required 12 iterations (which is not unreasonable) and one hour per run, could require approximately 11 to 12 hours of the designer's time.

There are many different types of information which could be required in a progressive articulation of preferences. These might include:

1) A ranking of points in the outcome space,
2) A readjustment of aspiration levels from one iteration to the next, or
3) Marginal rates of substitution between the various criteria.

In most cases however, a technique involving a progressive articulation of preferences will not require preference information as difficult to provide as that required to form a MAV or MAU function.

Montgomery and Bettencourt (1977) employed the Geoffrion, Dyer, Feinberg (1972) method (GDF method) to optimize a simulation model with four criteria. The approach suggested involved, first, the determination of response surfaces for each of the four criteria by running the simulation model several times; then, the GDF algorithm was interfaced with the response surfaces to find the preferred solution. Hence, there was no "direct" interaction with the simulation model.

In addition to the GDF algorithm, other progressive techniques which have been applied in a mathematical programming framework include those of Rosinger (1981), Zionts and Wallenius (1983), White (1980), and Masud and Zheng (1989). Of particular interest from the standpoint of the optimization of multicriteria simulation models is the algorithm developed by Sadagopan and Ravindran (1986) which allows nonlinear objective and constraint functions and varying types of preference information from the designer.

Also of great interest would be the concept of interactive goal programming (Masud and Hwang, 1981), since the preference information required of the decision maker is not extensive.

4.3 Posterior Articulation of Preferences

A posterior articulation of preferences would involve the generation of all (or at least many) nondominated solutions, and then having the decision maker choose one of these nondominated solutions as being most preferred. Several researchers have addressed this problem for the case where all of the objective functions are linear (e.g., see Yu and Zeleny, 1975).

However, the problem of generating all of the nondominated solutions for a simulation model could very well be intractable, especially for the case where the uncertainty in the output variables is to be explicitly considered. One may be able to generate several nondominated solutions by solving p single objective problems of the type:

$$\begin{aligned} &\text{Maximize } Y_i(X) \\ &\text{subject to: } \quad Y_j(X) \geq b_j \text{ for } j = 1, \ldots, i\text{-}1, i+1, \ldots, p \end{aligned}$$

for $i = 1, \ldots, p$. The b_j's would act as aspiration levels, or required levels, for each of the performance variables. Assuming then that each of the p problems were feasible, one could attain p nondominated solutions to the problem. These solutions would then be presented to the designer who would choose his most preferred solution.

Of course, one would still have the difficulty associated with solving the individual, single objective problems.

5 CONCLUSIONS

Undoubtedly, the best general method for the multicriteria optimization of a simulation model would involve some aspects of all three types of approaches: prior, progressive, and posterior articulation of preferences. In fact, the method discussed above under posterior articulation actually would involve a combination of prior and posterior articulation (i.e., the designer would be specifying some information about his preference structure just by his determination of the aspiration levels, b_j).

Probably the most important point of this paper is that there is no single overall best approach for multicriteria optimization of simulation models. Characteristics such as the numbers of decision variables and responses, the nature of the response surfaces, the ability/desire of the designer to provide different types of preference information, and the run time of the model are all important in determining which technique to use for a particular situation.

REFERENCES

Bengu, G. and J. Haddock. 1986. A Generative Simulation-Optimization System. *Computers and Industrial Engineering* 10:4, 301-313.

Biles, W.E. and J.J. Swain. 1979. Mathematical Programming and the Optimization of Computer Simulations. *Mathematical Programming Study* 11, 189-207.

Clayton, E.R., W.E. Weber, and B.W. Taylor III. 1982. A Goal Programming Approach to the Optimization of Multiresponse Simulation Models. *IIE Transactions* 14:4, 282-287.

Cochran, J.K. and J. Chang. 1990. Optimization of Multivariate Simulation Output Models Using A Group Screening Method. *Computers and Industrial Engineering* 18:1, 95-103.

Evans, G.W. 1984. An Overview of Techniques for Solving Multiobjective Mathematical Programs. *Management Science* 30, 1268-1282.

Evans, G.W. and J. Haddock. 1991. Modeling Tools for Flexible Manufacturing Systems. To appear in *Production Planning and Control.*

Geoffrion, A.M., J.S. Dyer, and A. Feinberg. 1972. An Interactive Approach for Multicriterion Optimization with an Application to the Operation of an Academic Department. *Management Science* 19, 357-368.

Goicoechea, A., D.R. Hausen, and L. Duckstein. 1982. *Multiobjective Decision Analysis with Engineering and Business Applications.* New York: John Wiley and Sons

Harpell, J.L., M.S. Lane, and A.H. Mansour. 1989. Operations Research in Practice: A Longitudinal Study. *Interfaces* 19:3, 65-74.

Hopmans, A.C.M. and J.P.C. Kleijnen. 1980. Regression Estimation in Simulation. *Journal of the Operational Research Society* 31:11, 1033-1038.

Jacobson, S.H. and L.W. Schruben. 1989. Techniques for Simulation Response Optimization. *Operations Research Letters* 8:1, 1-9.

Keeney, R.L. and H. Raiffa. 1976. *Decision with Multiple Objectives: Preferences and Value Tradeoffs.* New York: John Wiley and Sons.

Law, A.M. and W.D. Kelton. 1991. *Simulation Modeling and Analysis.* New York: McGraw-Hill, Inc.

Masud, A.S. and C.L. Hwang. 1981. Interactive Sequential Goal Programming. *Journal of the Operational Research Society* 32, 391-400.

Masud, A.S. and X. Zheng. 1989. An Algorithm for Multiple-objective Non-linear Programming. *Journal of the Operational Research Society* 40, 895-906.

Meketon, M.S. 1987. Optimization in Simulation: A Survey of Recent Results. *Proceedings of the 1987 Winter Simulation Conference* (A Thesen et al, eds), 58-67.

Mollaghasemi, M., G.W. Evans, and W.E. Biles. 1991. An Approach for Optimizing Multiresponse Simulation Models. To appear in the *Proceedings of the 13th Computers and Industrial Engineering Conference*, Orlando.

Montgomery, D.C. and V.M. Bettencourt Jr. 1977. Multiple Response Surface Methods in Computer Simulation. *Simulation* 29, 113-121.

Myers, R.H., A.I. Khuri, and W.H. Carter. 1989. Response Surface Methodology: 1966-1988. *Technometrics* 31:2, 137-157.

Pegden, C.D., R.E. Shannon, and R.P. Sadowski. 1990. *Introduction to Simulation Using SIMAN.* New York: McGraw-Hill, Inc.

Porta Nova, A.M. and J.R. Wilson. 1989. Estimation of Multiresponse Simulation Metamodels Using Control Variates *Management Science* 35:11, 1316-1333.

Pritsker, A.A.B. 1986. *Introduction to Simulation and SLAM II*, Third Edition. New York: John Wiley and Sons.

Rees, L.P., E.R. Clayton, and B.W. Taylor III. 1985. Solving Multiple Response Simulation Models Using Modified Response Surface Methodology Within a Lexicographic Goal Programming Framework. *IIE Transactions* 17:1, 47-57.

Rosenthal, R.E. 1985. Principles of Multiobjective Optimization. *Decision Sciences* 16.

Rosinger, E.E. 1981. Interactive Algorithm for Multiobjective Optimization. *Journal of Optimization Theory and Applications* 35, 339-365.

Sadagopan, S. and A. Ravindran. 1986. Interactive Algorithms for Multiple Criteria Nonlinear Programming Problems. *European Journal of*

Operational Research 25, 247-257.
Safizadeh, M.H. 1990. Optimization in Simulation: Current Issues and the Future Outlook. *Naval Research Logistics* 37, 807-825.
Shannon, R.E., S.S. Long, and H.P. Buckles. 1980. Operations Research Methodologies in Industrial Engineering. *AIIE Transactions* 12, 364-367.
Shannon, R.E. and S. Prakash. 1990. Goal Directed Simulation Systems. *1990 International Industrial Engineering Conference Proceedings*, 345-350.
Smith, D.E. 1973. Requirements of an 'Optimizer' for Computer Simulation. *Naval Research Logistics* 20, 161-179.
Smith D.E. 1976. Automatic Optimum-Seeking Program for Digital Simulation. *Simulation*, July, 27-31.
Suri, R. 1984. An Overview of Evaluative Models for Flexible Manufacturing Systems. *Proceedings of the First ORSA/TIMS Conference on Flexible Manufacturing Systems* (K.E. Stecke and R. Suri, eds), 8-15.
White, D.J. 1980. Multi-Objective Interactive Programming. *Journal of the Operational Research Society* 31, 517-523.
Wilson, J.R. 1987. Future Directions in Response Surface Methodology for Simulation. *Proceedings of the 1987 Winter Simulation Conference* (A. Thesen et al, eds.), 378-381.
Yu, P.L. and M. Zeleny. 1975. The Set of All Nondominated Solutions in Linear Cases and the Multicriteria Simplex Method. *Journal of Mathematical Analysis and Applications* 49, 430-468.
Zeleny, M. 1982. *Multiple Criteria Decision Making*. New York: McGraw-Hill.
Zionts, S. and J. Wallenius. 1983. An Interactive Multiple Objective Linear Programming Method for a Class of Underlying Nonlinear Utility Functions. *Management Science* 29:5, 519-529.

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