

## BATCHING ANNEALING OPERATIONS TO OPTIMIZE QUEUEING TIMES AND FURNACE EFFICIENCY: A SIMULATION MODEL

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### ABSTRACT

This paper describes a simulation model representing the annealing operations in a rolling-mill facility. The model was developed using the discrete event simulation language SIMAN. The model integrates a friendly user interface through which the user interacts with the simulation model without programming or recompiling. The simulation model also integrates external subroutines to optimize the batching of jobs and their sequencing so as to minimize flow-times and lateness and maximize the annealing furnaces efficiency. The optimization approach relies mainly on a data-driven adaptive estimation of the lot-sizes based on a queueing model formulation.

### 1 INTRODUCTION

In this paper, we present a discrete event simulation model constructed to simulate a group of heterogeneous annealing furnaces in a rolling-mill facility. The goal of the simulation model is to represent the annealing operations in detail so as to evaluate (i) an adaptive approach for estimating lot-sizes and (ii) a job sequencing heuristic. The adaptive lot-sizing approach relies on a multi-objective function that seeks to minimize flow-times and maximize the furnaces efficiency subject to specific capacity constraints. The job sequencing approach developed seeks to control day-to-day operations on the job floor; it incorporates rules to account for capacity constraints and due-dates.

The simulation model provides a friendly user interface that incorporates much of a Lotus-based decision-support system already developed by the user. The interface is used to generate different configurations of the annealing operation environment without programming or recompiling. All the decision support functions that relate to lot-sizing and job sequencing are integrated to the simulation model using external C-coded subroutines.

The content of the paper stands as follows. Section 2 first describes the manufacturing environment. Section 3 then presents the adaptive approach we have developed to optimize lot-sizes. Section 4 then provides a description of the simulation model; the user interface and the SIMAN model are particularly discussed. Section 5 finally presents concluding remarks.

### 2 THE MANUFACTURING ENVIRONMENT

The simulation model is based on the manufacturing environment of the rolling-mill facility of the Société d'Aluminium Reynolds du Canada, at Cap-de-la-Madeleine, Québec, Canada. This facility has been the subject of other studies; for example, see Lefrançois *et al.* (1991, 1989)). The rolling-mill transforms raw aluminum ingots into industrial and domestic foil products within a typical job-shop setting. As represented in figure 1, during the transformation process, the aluminum may visit several workstations: hot or cold rolling-mills, annealing furnaces, slitters, edge conditioners, etc. Sequencing problems are numerous as the product mix may account for dozens of products with different routings. Lefrançois *et al.* (1989) described a SIMAN-based visual simulation model used to optimize the sequencing of jobs arriving for processing at cold rolling-mills. Sequencing in this case raised the problem of minimizing the flow-times while minimizing the number of sequence-dependent set-ups.

Sequencing the operations at the annealing furnaces also presents numerous problems. The processed metal is effectively characterized by physical properties (rigidity, hardness) as well as by mechanical properties (ductility, elongation, modulus, resistance to thermal shocks, compression and fatigue). The rolling operations considerably decrease the elasticity of the aluminum and reduce its malleability. The annealing operations acts upon the aluminum physical and mechanical properties (i) to eliminate or reduce the

undesirable effects from a previous rolling (termed: intermediate annealing) and (ii), to give to the final product the physical and mechanical characteristics needed for a specific application (termed: final annealing).

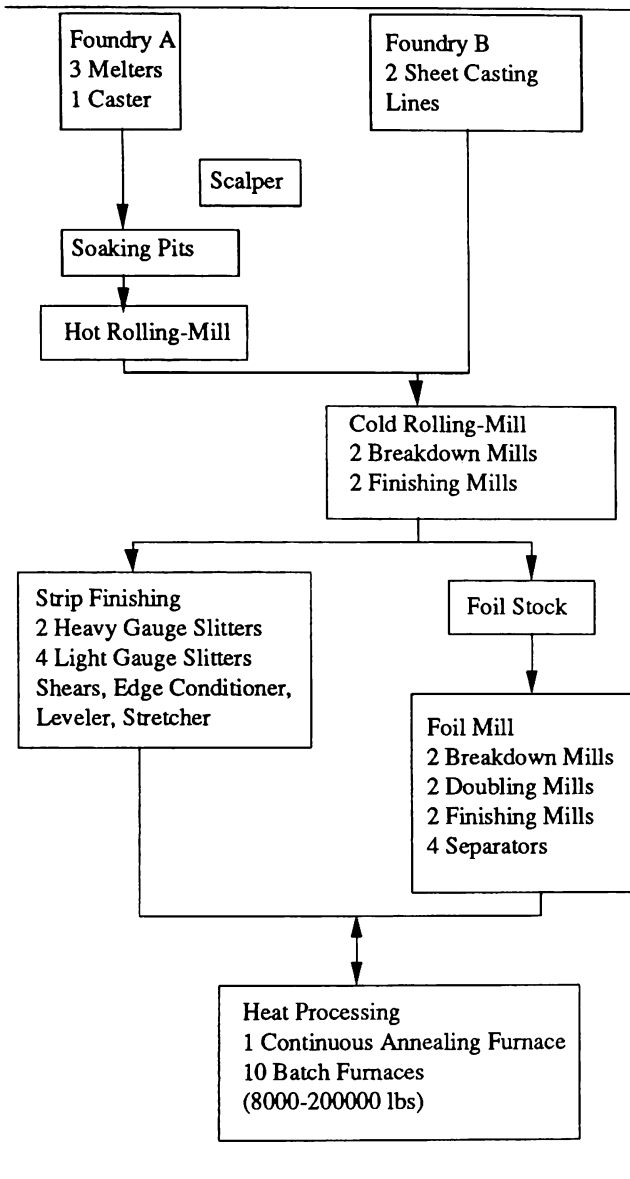


Figure 1: Typical sequence of operations in a rolling-mill facility.

Customer requirements dictate a large variety of product characteristics; thus the annealing operations have a significant variability both in their duration and in their technological requirements. A typical annealing cycle is shown in Figure 2. Annealing cycle parameters such as level temperature, time at level temperature, type of alloy, oxydizing or non-oxydizing environment, etc. vary considerably. It is difficult to process multiple jobs within a single annealing operation, as only those jobs with a similar annealing cycle can be batched. However,

optimizing the batching of jobs and their sequencing for the annealing operations is an important requirement to help the firm maintain and improve its competitive advantage. Optimization must account for the highest maintainable efficiency of the annealing furnaces while minimizing both the time spent by a job in the plant and the work-in-process inventories.

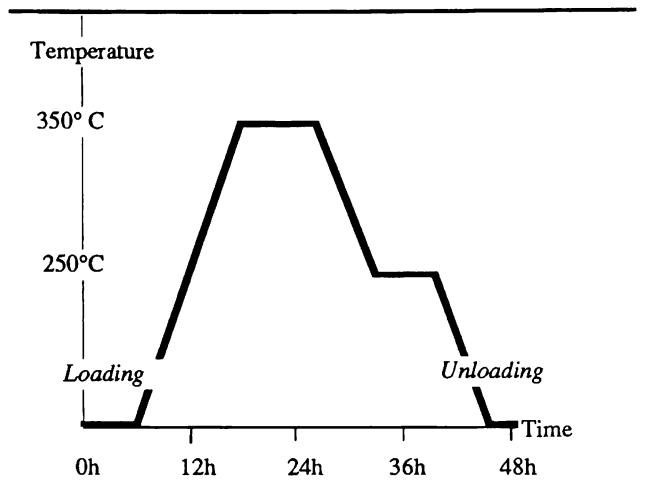


Figure 2: A typical annealing operation within the manufacturing process.

### 3 THE LOT-SIZING APPROACH

In the recent years, numerous research papers have been published on both lot-sizing and sequencing approaches. Among those, there has been a substantial amount of research work based on the application of queueing network models to represent the behavior of the manufacturing systems analyzed. Buzacott *et al.* (1986) and Suri *et al.* (1989) present comprehensive reviews of queueing models of manufacturing systems. There has also been a substantial amount of this work dealing with lead time estimation, work-in-process, lot-sizing and tactical production planning issues. Examples of these are the work of Lefrançois *et al.* (1990), Karmarkar (1989, 1987, 1985a, 1985b), Zipkin (1986) and Bertrand (1985).

The basic construct presented here for lot-sizing and tactical sequencing is a multi-objective function derived from a queueing network model formulation of the annealing operations. The multi-objective function accounts for the flow-time of the jobs, the efficiency of the furnace loading and a relaxation of the furnace capacity constraints. The model formulation is presented below.

#### 3.1. Queueing Network Model Formulation

To model the annealing operations, we consider an annealing station processing jobs belonging to multiple

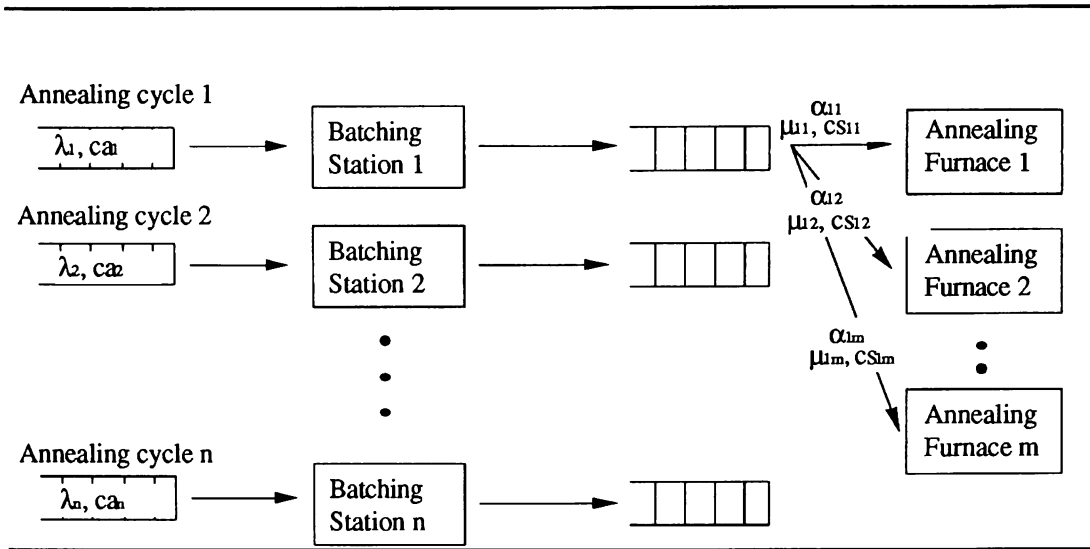


Figure 3: Annealing operations as a queueing network

product classes which are defined according to the annealing cycle needed (see Figure 3). The interarrivals of jobs within each class  $i$  are assumed to be general with rate  $\lambda_i$  and squared coefficient of variation (scv)  $c_{a_i}$ ,  $i=1,\dots,n$ . The annealing is in batches of size ( $r_i \geq 1$ ) for jobs of class  $i$ , with a first come, first served (FCFS) discipline. We assume that the annealing can start only after the batches have been formed. The annealing times are assumed to be batch-size independent with product class and furnace dependent rates  $\mu_{ij}$  and scv's  $cs_{ij}$ ,  $i=1,\dots,n, j=1,\dots,m$ . We also consider ( $0 \leq \alpha_{ij} \leq 1$ ) the proportion of batches of type  $i$  processed in furnaces of type  $j$ ,  $j=1,\dots,m$ . Our analysis of the annealing system with batch processing is similar to the case presented in Bitran and Tirupati (1989). To model the flow of jobs within the queueing system, we assume a fictitious station for each type of annealing cycle where the forming of batches takes place. Once a batch of size  $r_i$  have been formed, it is transferred immediately to a queue where it waits until a suitable furnace is available for annealing.

Furnace operation is approximated using a GI/G/1 queue with multiple product classes and where each customer from class  $i$  represents a batch of  $r_i$  jobs. Estimates developed in Lefrançois *et al.* (1990), Bitran and Tirupati (1989) and Kraemer and Langenbach-Belz (1976) are used to describe mean waiting time; the Appendix describes the estimates in detail. In the remainder of this paper, we denote by  $W_{qi}(r_i)$  the mean waiting time for annealing of jobs of cycle  $i$  with batch sizes  $r_i$ .

### 3.2 Nature of Multi-Objective Lot-Size Decision Function

The multi-objective lot-size decision function

incorporates both a minimal flow-time and a maximal furnace utilization criterion and also incorporates a relaxation of the furnace capacity constraints. The following parameters and estimates are assumed known:

- $\lambda_i$ : arrival rate of jobs of annealing cycle  $i$ ,  $\lambda = \sum_{i=1}^n \lambda_i$
- $h_i$ : inventory holding costs for jobs of annealing cycle  $i$  measured in \$ per unit/time unit
- $\alpha_{ij}$ : proportion of jobs of annealing cycle  $i$  processed in furnace  $j$
- $r_i$ : lot size decision for jobs of annealing cycle  $i$  measured in jobs/batch
- $Q_i$ : average size of a job of annealing cycle  $i$  measured in units
- $c_j$ : capacity of furnace  $j$  measured in units
- $W_k$ : weight for criterion  $k$  in multi-objective function

The problem is formulated as follows:

$$\text{Minimize } F = W_1 \left\{ \sum_{i=1}^n (\lambda_i/\lambda) h_i Q_i W_{qi}(r_i) \right\} + W_2 \left\{ \sum_{i=1}^n \sum_{j=1}^m (\lambda_i/\lambda) \alpha_{ij} [1 - Q_i r_i / c_j] \right\} + W_3 \left\{ \sum_{i=1}^n \sum_{j=1}^m (\lambda_i/\lambda) \alpha_{ij} [Q_i r_i - c_j] \right\} \quad (1)$$

The first component of the multi-objective function estimates the inventory holding costs for the overall annealing operations. The second component estimates the furnace-efficiencies; the overall efficiency of the annealing furnaces is obtained by weighting the efficiency of each furnace for each type of annealing cycle. The last component is a weighted relaxation of the furnace capacity constraints. Problem (1) is solved according to the following approach.

### 3.3 Optimal Searching Heuristic

The multi-objective function of problem (1) is a rather complex function and is not easily differentiable on the decision variables  $r_i$ . The complexity of the function implies many interactions among the decision variables which makes it nearly impossible to identify the impact of a given lot size on the expected waiting times of other jobs or on the overall furnace efficiency.

The optimization problem being nonlinear, an in-depth analysis of the behavior of the multi-objective function clearly is essential for the development of an effective solution technique. An analysis of the function, and in particular of its inventory costs estimation component shows similarities with the M/G/1 queueing time based lot-size decision function of Yang and Deane (1989). Based on their results and those from Zipkin (1986) which shows that queue length estimates for GI/G/1 queueing system with batch arrivals are convex in regards to the batch size, it is reasonable to expect at least general quasi-convexity of the function for selected sets of weights  $W_1$ ,  $W_2$  and  $W_3$ .

The approach used to solve (1) is based on a multidimensional search without derivatives. The search method was adapted from the approach of Hooke and Jeeves (1961). The method performs two types of search on the lot sizes  $r_i$ : (i) an exploratory search from a vector of lot sizes  $(r_i)_k$  along the coordinate directions to find a new vector  $(r_i)_{k+1}$  and (ii), a pattern search along the direction  $d=(r_i)_{k+1} - (r_i)_k$  which leads to a new point  $y$ . The exploratory search procedure used is based on a modified integer-solution Fibonacci search. From point  $y$ , another exploratory search gives the new vector  $(r_i)_{k+2}$ . The next pattern search is along the direction  $d'=(r_i)_{k+2} - (r_i)_{k+1}$ , yielding  $y'$  and so on. The process is repeated until convergence. Typical optimal solutions are obtained within three to five iterations.

## 4 THE SIMULATION MODEL

The preceding section has presented the solution approach that addresses the problem of determining lot-sizes for the annealing operations. The research project also involved the development of a detailed simulation model of the annealing operations to ascertain the validity of the solution proposed and to evaluate the benefits resulting from dynamically adapting the lot sizes as the simulation goes on. The present section describes this simulation model.

The simulation model was developed with flexibility in mind. The model is thus structured so as to adapt with minor changes to variations in the production environment, product specifications, annealing cycles and lot-sizing/sequencing rules. The discrete-event simulation language SIMAN was used with some modifications to the conventional model and experiment frames to increase the flexibility of the simulation

model. A customized configuration file was first designed to serve as a user interface to the model. The simulation model interfaces with C-coded subroutines triggered by issuing event calls. Two such subroutines are used, one for adaptively determining the lot sizes for each type of annealing cycle and the second for the day-to-day sequencing of the annealing operations. The structure of the model is presented in Figure 4.

### 4.2. The User Interface

The standard approach for a SIMAN model requires that the structure of the manufacturing environment simulated be described in the model frame and that the specific variable values which the user wants to control through its experimentation be entered in the experimental frame. These two frames are linked together and generate the simulation model under study. Changing any of these two frames necessitates recompilation and relinking before running the revised model. This may represent a considerable amount of work and time if the volume of data to be entered is large.

An interface was thus designed to provide an easier means of entering or changing the parameters of the manufacturing environment and to avoid recompilation. The interface relies on an ASCII file referred to as the configuration file. The configuration file is generated using a simple data acquisition program based on Lotus 123 macros. The set of Lotus macros is an extension of a Lotus-based decision support system developed by the industrial engineering department of the Société d'Aluminium Reynolds du Canada to estimate the duration of the annealing operations. Base-inputs are the annealing cycle needed, the critical dimensions of the aluminum coils to be annealed and the weight of the aluminum. From these parameters, macros calculate the duration for the annealing operation using estimates derived from a multiple regression model. Other inputs are the product mix, the initial values for the interarrival, service times for each type of annealing cycle and initial estimates of the lot sizes.

### 4.2. The SIMAN Model

All the data from the configuration file is integrated within the simulation model thus customizing the manufacturing environment modeled. Given the distribution of the job arrivals, the SIMAN model generates jobs and parameterizes them using basic informations on the distribution of the weight of aluminum within a job, the due dates, the number of remaining operations and the annealing cycles needed. The arriving jobs are then routed to a station macro used for forming batches. The macro consists of a range of individual batch forming stations, one for each type of annealing cycle. All the jobs flow through these stations and are queued at their appropriate station until

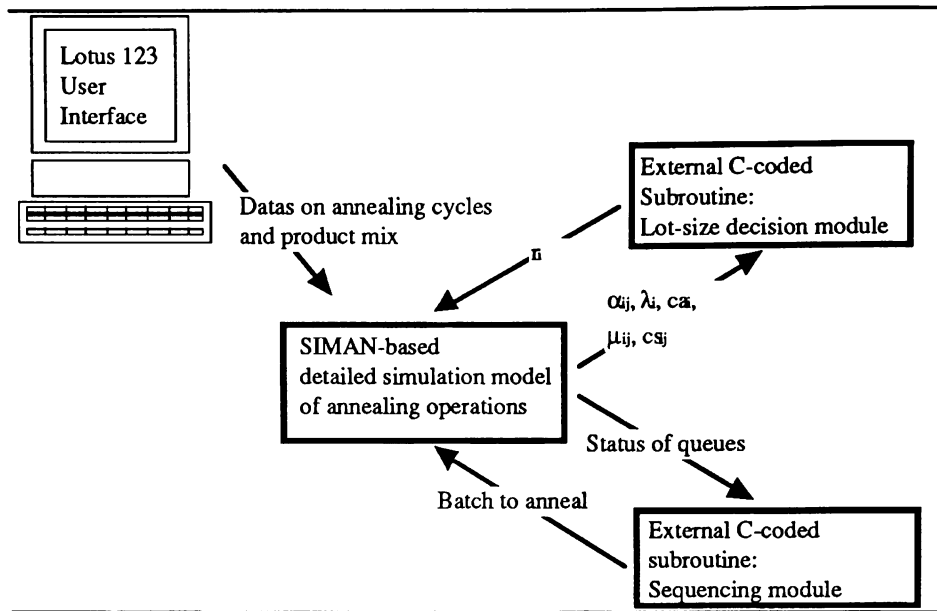


Figure 4: Model Structure.

the required number of jobs in a batch have entered the station. The number of jobs required to form a batch is obtained from an external subroutine which periodically updates the lot sizes when appropriate event calls are issued.

When a batch has been formed, all the jobs are grouped in a single entity; the new entity gets its attributes from the jobs it contains. These attributes include the total weight of the aluminum to be processed, the due dates of the batched jobs and the time when the batch was formed. This information will be needed for the sequencing at the annealing furnaces. The batched jobs are then sent to a second macro station where they are queued until their selection for processing at an annealing furnace. The selection of the batch to anneal is based on a sequencing rule described in the following subsection; this rule was embedded within a second external subroutine.

After the annealing cycle, statistical data is recorded and the batch is then released from the system. The statistical data includes the service rate, the service time scv's and the routing probability matrix (giving the  $\alpha_{ij}$ 's) which is used to update the lot sizes.

### 4.3 C Subroutines

The simulation model interfaces with two C-coded subroutines. The first of these is the annealing operations sequencing module and the second is the lot-

size decision module.

The sequencing module controls the day-to-day annealing operations; it is accessed through SIMAN *qpick* calls. The sequencing rule it contains is triggered whenever an annealing furnace is unloaded at the end of an annealing cycle. The module was needed because the information emanating from the lot-size decision module is not sufficient for a detailed control of the annealing operations.

Figure 5 presents a summary description of its structure. Batches that have been formed join a wait-for-annealing queue that depends on the annealing cycle to be performed. Whenever an annealing furnace is made available, the sequencing rule selects the first batch from each queue. From that group, all those batches that exceed the capacity of the furnace are rejected. The remaining batches are then sorted according to three criteria. The first criterion is the earliest arrival date for annealing; the arrival date of a batch is defined as the date when the batching operation ends. The second criterion is the lowest slack on the number of remaining operations. For a given job, the slack is obtained as the due-date minus the current time which is then divided by the number of remaining operations. For a batch of jobs, the lowest slack on the number of remaining operations of the jobs that form the batch is used. The last criterion used to sort the candidate batches is the furnace loading efficiency. The selected batch is obtained through a scoring model based on the ranking of a batch according to the three criteria.

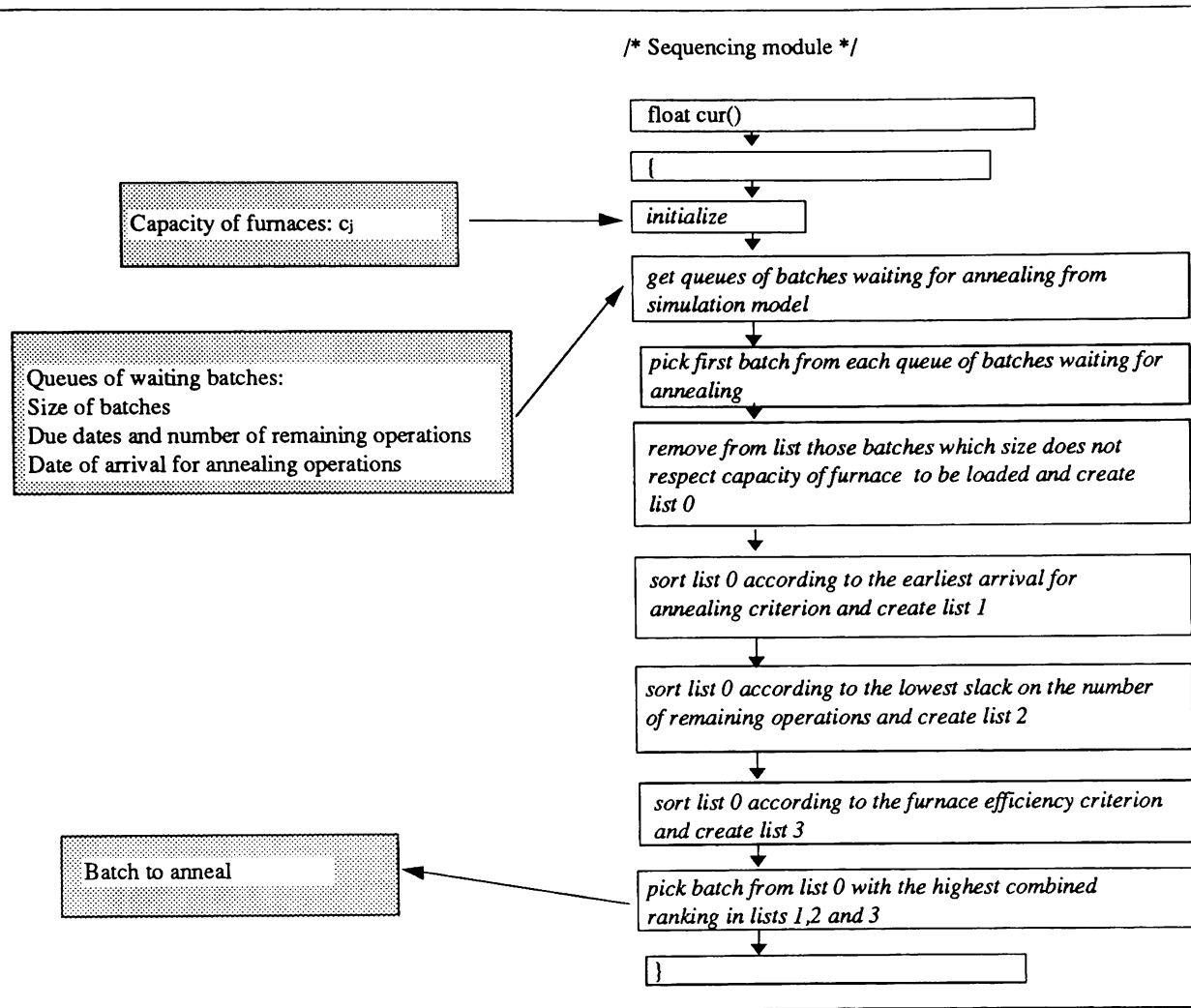


Figure 5: Structure of sequencing module.

The lot-size decision module then implements the queueing network model we presented in section 3.1 along with the optimal searching procedure described in section 3.3. Figure 6 presents the structure of the lot-size decision module.

Figure 7 illustrates when the sequencing and lot-size decision modules are used. As can be observed, the sequencing module is used each time a furnace has been unloaded. The lot-size decision module runs on a much less frequent basis thereby making the lot-size less reactive to transient behaviors of the manufacturing system. In its current configuration, the lot-size decision module is used after fixed periods of simulation time; other configurations will eventually be investigated. This investigation will include the particular case of rerunning the lot-size decision module whenever the state of the manufacturing system deviates significantly from what was observed in the past.

DOS versions of the lot-size decision module, the sequencing module, the user interface and the SIMAN-

based simulation model have been installed on an IBM PS/2-70 and on a SUN 386i Roadrunner<sup>®</sup> workstation at the Laboratoire d'Opérations et de Gestion Assistées par Ordinateur (LOGAO) at Université Laval. The model is currently used to validate the optimal lot-size searching heuristic and the tactical sequencing rule that controls the day-to-day annealing operations. The integrated model should eventually be implemented on IBM PS-2's by the industrial engineering department of the Cap-de-la-Madeleine plant of the Société d'Aluminium Reynolds du Canada. The latter implementation should lead to a broadening of the application of the model, particularly for capacity planning, bottleneck prevention and furnace-starvation avoidance.

## 6 CONCLUDING REMARKS

The detailed simulation of the annealing operations presented in this paper incorporates a custom-designed user interface along with optimization and rule-based

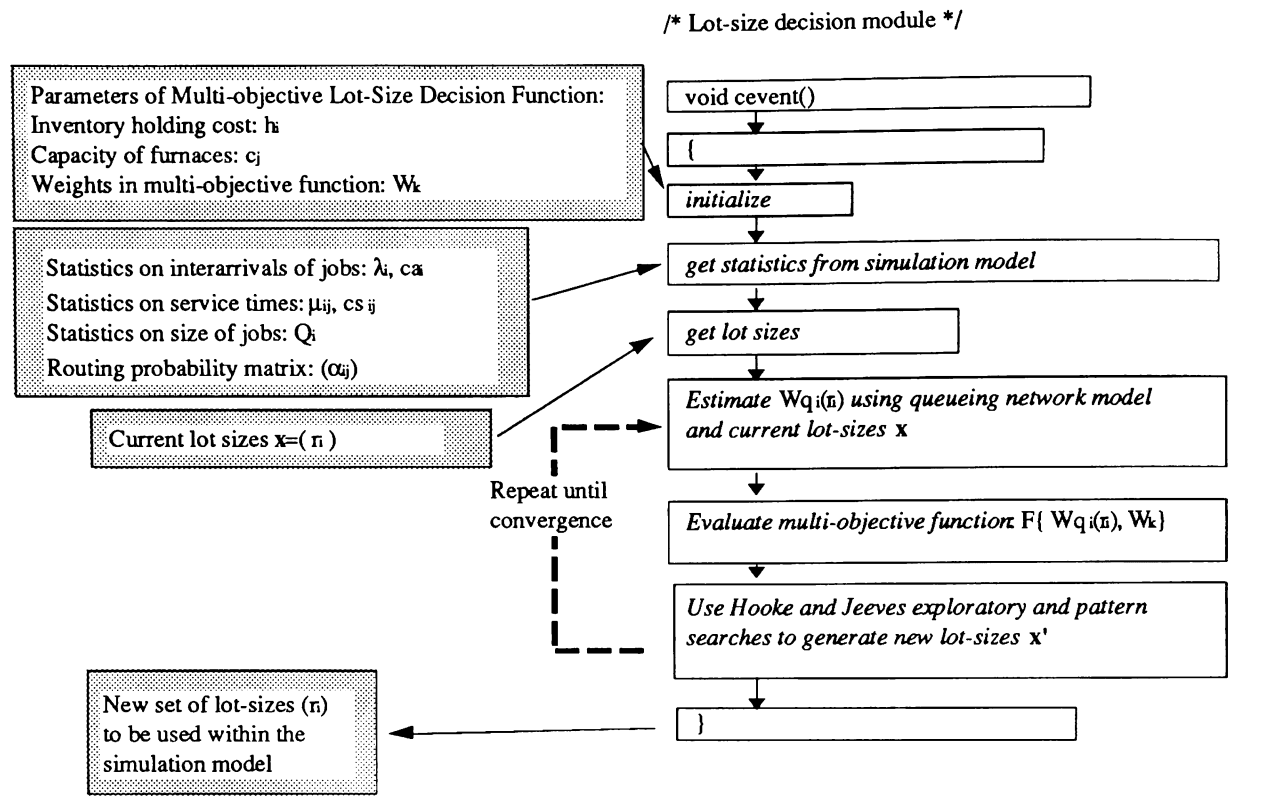


Figure 6: Structure of lot-size decision module.

external modules.

The incorporation of external modules within a SIMAN-based simulation model is well documented and has proved in the past to be extremely useful. Hood *et al.* (1989) is one known example of such an integration. In that case, the goals of the simulation design was to represent with great detail and flexibility a complex manufacturing system. External FORTRAN-coded subroutines were used to depict detailed aspects of the system such as the resource capacity changes, the expediting policies or the job selection rules. The case for integrating external subroutines within our simulation model is similar, even though the level of detail that could be attained with the SIMAN model was sufficient. The most important reason we found for integrating these external subroutines was the ability this setting offered for testing the optimal dynamic lot-size searching heuristic presented in section 3.

The development of real-time based tools for manufacturing planning and control is part of the new research orientations which should capture the focus of operation researchers and industrial engineers through the 90's (Nof *et al.* 1989). Static planning and control tools through which plans are made and then executed on a long term basis prove to be simply too slow, inflexible and unresponsive in today's dynamic environments. This raises the need for the dynamic lot-size searching heuristic of the type presented in this paper. The

complex dynamic resulting from adaptively optimizing the lot sizes using real-time data from the manufacturing environment, makes the development, the tuning and an in-depth testing of such heuristics very difficult, if not impossible. For the particular case under study, two factors clearly affect the performance of the proposed approach: (i) the optimizing heuristic used and (ii), the rules used to trigger an update of the lot sizes. Using a simulation model of the annealing operations with an external lot size optimization subroutine proves to be very helpful. It simplifies the development and the tuning of the optimization approach as this configuration ensures that changes in the heuristic can be integrated into the simulation model with minimal rework.

This integration also has major impact on the ability to refine the concepts behind the queueing network model and the optimal searching heuristic that was developed. Such an implementation using external modules helps separate within the simulation model those rules that trigger an updating of the lot sizes and those rules used to optimize their level. The latter reflects recent research trends in the modeling of reasoning within intelligent manufacturing systems where low level decision rules are used to trigger high level reasoning activities taking place within external knowledge sources (see for example Lefrançois and Montreuil 1990, Burns and Morgeson 1988 and O'Grady and Lee 1988).

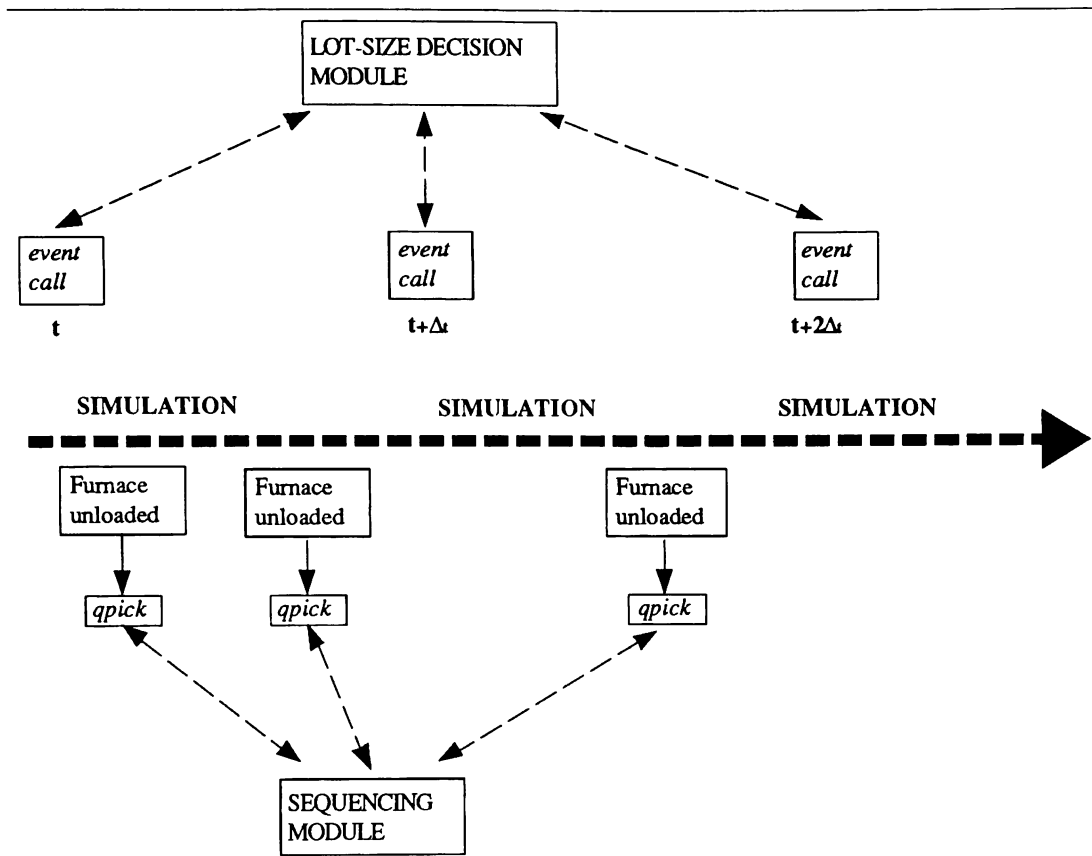


Figure 7: Reruns of lot-size decision and sequencing modules.

Part of an extensive research program on the development and the validation of adaptive production planning and control tools in collaboration with industries, the SIMAN-based simulation model presented in this paper illustrates how simulation can benefit from the integration of operations research techniques as aggregate and tactical decision tools. Such models are likely to offer a more adequate representation of the forthcoming generation of intelligent manufacturing systems.

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## APPENDIX

Derivation of the mean waiting time in the system is based on the following notation and estimations:

n: number of annealing cycles; cycles are indexed by i  
 m: number of annealing furnaces; furnaces are indexed by j

$\lambda_i$ : arrival rate of jobs of cycle i at batching stations

$ca_i$ : scv of interarrival times of jobs of cycle i at batching stations

$r_i$ : batch size

$\lambda_i^a$ : arrival rate of batches of cycle i:  $\lambda_i^a = \lambda_i / r_i$

$\lambda^a$ : arrival rate of batches:  $\lambda^a = \sum_{i=1}^n \lambda_i^a$

$ca_i^a$ : scv of interarrival times of batches of cycle i:

$$ca_i^a = ca_i / r_i$$

$ca^a$ : asymptotic estimate of scv of interarrival times of

$$\text{batches: } ca^a = \sum_{i=1}^n (\lambda_i^a / \lambda^a) ca_i^a$$

$\mu_{ij}$ : annealing rate for batches of cycle i on furnace j

$\sigma_{ij}^2$ : variance of annealing times for batches of cycle i on furnace j

$cs_{ij}$ : scv of annealing times for batches of cycle i on furnace j:  $cs_{ij} = \sigma_{ij}^2 / \mu_{ij}^2$

$\alpha_{ij}$ : proportion of batches of cycle i annealed on furnace j

$\beta_{ji}$ : proportion of batches annealed on furnace j of cycle i:

$$\beta_{ji} = (\lambda_i^a \alpha_{ij} / \lambda^a) / \sum_{i=1}^n (\lambda_i^a / \lambda^a) \alpha_{ij}$$

$\mu_j$ : annealing rate of furnace j:  $\mu_j = \sum_{i=1}^n \beta_{ji} \mu_{ij}$

$\mu$ : annealing rate:  $\mu = \sum_{i=1}^n \mu_j$

$\sigma_j^2$ : variance of annealing times on furnace j:

$$\sigma_j^2 = \sum_{i=1}^n \beta_{ji} \sigma_{ij}^2$$

$cs_j$ : scv of annealing times on furnace j:  $cs_j = \sigma_j^2 / \mu_j^2$

$cs$ : asymptotic estimate of scv of annealing times:

$$cs = \sum_{j=1}^m (\mu_j / \mu) cs_j$$

$\rho$ : annealing furnaces utilization:  $\lambda^a / \mu$

The batching process taking the form of a fictitious

station with a zero-duration service time, we obtain in a straightforward manner the following results for the mean waiting time of a job of cycle  $i$  at the batching station:

$$W_{qi}^b(r_i) = (r_i - 1) / 2\lambda_i, \quad i = 1, \dots, n \tag{A.1}$$

We model the annealing station as an ordinary GI/G/1 queue where each customer of cycle  $i$  represents a batch of  $r_i$  jobs. The estimate for the mean waiting time is derived from the Kraemer-Lagenbach-Belz (1976) approximations and from the Little(1961) formula (Lefrançois *et al.* 1991). It stands as follows:

$$W_q^a = \rho^2 (ca^a + cs) \psi(ca^a, cs, \rho) / [2\lambda^a (1 - \rho)] \tag{A.2}$$

where:

$$\psi(ca^a, cs, \rho) = \exp \left\{ -2(1 - \rho)(1 - ca^a)^2 / [3\rho(ca^a + cs)] \right\}$$

$$\text{if } ca^a \leq 1$$

$$\psi(ca^a, cs, \rho) = \exp \left\{ -(1 - \rho)(ca^a - 1) / (ca^a + 4cs) \right\}$$

$$\text{if } ca^a \geq 1$$

Combining equations (A.1) and (A.2), the mean waiting time in the system for a job of cycle  $i$  is obtained as:

$$W_{qi}(r_i) = (r_i - 1) / 2\lambda_i + W_q^a, \quad i = 1, \dots, n \tag{A.3}$$

The latter estimate is a somewhat crude approximation which should lead to exact results only for the cases that correspond to independent Poisson arrivals of the jobs and exponential service times. Similar approximations have however been used extensively in the decomposition approaches and have been shown to be fairly robust (see for example Suri(1983)).

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