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ACCURACY IN QUALITATIVE DESCRIPTIONS OF BEHAVIOUR

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ABSTRACT

Qualitative representations provide a method for obtaining coarse-grained descriptions of dynamic system behaviour. These can be valuable where a general result is required, or where the available information is not adequate for a numerical or analytic representation. If qualitative methods are to find wider use, it is clearly important to understand the limitations in accuracy which are inherent in the representation. This paper discusses the relationship between qualitative representations and results obtained from more conventional mathematical approaches. A simple second order system is used to illustrate the use of qualitative methods, including a qualitative calculus. An extension to cater for a non-linear case is also shown. A clear correspondence is demonstrated between qualitative and conventional results.

1. INTRODUCTION

The use of abstract descriptions of reality has been well established for a considerable time as a basis for predicting states of dynamic systems. Impressive results have been obtained in all fields in which dynamic situations figure, with simulation becoming recognised as a particularly important tool. The ability of computers to perform extensive calculations has made them natural partners for simulation since their earliest appearance. More recently, interest has grown in symbolic computation: the direct use of abstract representations within a computer.

This paper is concerned with a part of Artificial Intelligence sometimes termed Deep Knowledge Based Systems (DKBS). The main aim of DKBS is to enhance the capabilities of intelligent systems beyond the simpler rule-based approaches which are now commonplace. The limitations of such first-generation systems are becoming well known, although they are perfectly capable of providing valuable service in a wide range of applications.

Observation of humans dealing with dynamic situations shows that they often reason with approximate or qualitative values. Emulation of this capability by machine therefore involves the use of similar non-numerical values, and methods of calculation which use them. Although a relatively new field, a number of applications for qualitative reasoning have been proposed, of which the following are typical.

- *Simulation*: the generation of a description of system behaviour under a specified set of constraining conditions [Kuipers 1985].
- *Envisioning*: the identification of all possible behaviours of a system, possibly indicating conditions under which certain behaviours may arise [de Kleer and Brown 1984].
- *Understanding*: the production of soundly based explanations of the relationships and processes operating in the system [Forbus 1984].
- *Control*: the selection of actions based on qualitative values, possibly producing qualitative inputs to the controlled system [Clocksin and Morgan 1986].
- *Diagnosis*: the recognition of incorrect system behaviour and the identification of possible causes of that behaviour [Pan 1984].

The first two of these are of particular interest here, and are obviously related: for example, a simulation can be considered as a selection of behaviours from a larger envisionment set. The accuracy of simulation results is obviously a matter of concern. Deployment of qualitative reasoning systems for operational use will require some confidence that results produced are "correct". This paper explores some of the issues involved, and shows how qualitative techniques can produce accurate behavioural descriptions, using a typical dynamic physical system as an example.

2. QUALITATIVE MODELLING

It is clear that the actions of humans in many complex situations can be explained on the basis of a mental model, which constrains and guides the mind in reaching decisions or conclusions [Gentner and Stevens 1983]. One possible explanation for the limitations of some current approaches to intelligent systems is the lack of a corresponding model. Much of the work in qualitative reasoning is based implicitly or explicitly on the use of a model, and this has important implications when considering accuracy.

Models are a simplified version of reality. However, the simplification must preserve the aspects of reality which are relevant to the task at hand. The designer of the model has to make choices about the model scope, since there are always trade-offs involved; normally size or speed *versus* accuracy or resolution. The issues involved include:

- what facets should be represented in the model? (electrical, thermal, mechanical, *etc.*),
- the extent of the model; what should be included and what left out?,
- the relationship between the model and reality; typically, scaling of variables or time,
- the level of detail and closeness of approximation required,
- any standards or conventions which must be adhered to by the model.

A prime consideration is the *accuracy* of the model: the fidelity with which it describes reality. Accuracy must be distinguished from *resolution*, which relates to the ability to distinguish between similar behaviours. In a qualitative model, the resolution is clearly going to be lower than in a model based on (say) real number representations. Although a low resolution imposes limits on the degree of accuracy which can be expected, the model should be accurate within these limits. Two criteria are relevant. These are:

- *Completeness*: all real behaviours should be identified without omissions.
- *Consistency*: no spurious behaviours should be introduced by the reasoning process.

Some compromise is usually necessary when mapping reality onto a model, since it is not practical to build fully comprehensive models. Models are built for a purpose, typically to solve a particular problem or range of problems. Representing facets of reality not relevant to the problem at hand would involve a great deal of unnecessary work. It is therefore normal for the designer of a model to look for simplifications which can reduce the modelling task, along the directions of choice outlined above. Although the resulting model will clearly be deficient in some respects when it comes to representing reality, the designer will have determined that these deficiencies are acceptable for the current task. Kuipers [1986] discusses the need for a "gold standard" by which results may be judged.

These issues must be taken into account when considering the accuracy of a reasoning system. The compromises selected in building the model will restrict the scope of the results which can be produced. The best that can be achieved is the production of results which are consistent with the model. In fact, nothing less than this should be acceptable. The position is summarised in Figure 1. Discrepancies between positions A and B in the figure are acceptable for the reasons outlined above. However, there should be no discrepancy between positions B and C in the figure, since this would indicate some shortcoming in the reasoning system and/or the model itself. Before looking at examples of possible sources of such problems we first need to look at methods for qualitative representation.

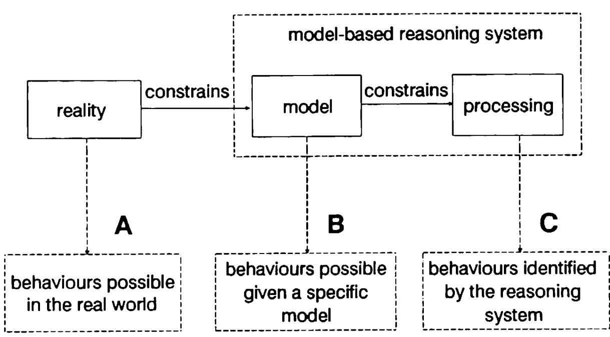


Figure 1. Relationship between Model and Reality

3. QUALITATIVE VALUES

In a qualitative reasoning system, variables can be represented as scalars in a coarse system of measurement, defined by partitioning the range of possible values. The simplest useful division splits the range into two partitions. In this case three symbols are required: one for each of the partitions, and one for the boundary between them. Any possible value can then be described with just one of the three symbols. The most common set of qualitative reasoning symbols uses exactly this representation. The symbols **+** and **-** are used to denote the two partitions, and the symbol **0** to denote the boundary between them. The set is extended to include the symbol **?**, representing a value which is presently unknown but which must ultimately be one of **+**, **0**, or **-**.

In a scalar qualitative system, values are represented by symbols drawn from the { **+ 0 - ?** } set. Both straightforward quantities, such as distance, and rates of change of quantities, such as velocity (which can be expressed as a change in distance per unit time), can be represented. These are sometimes referred to as "levels" and "rates" in the literature on systems dynamics; see [Coyle 1977] for example.

This scalar qualitative mathematics can be seen as a kind of interval arithmetic, with the symbol **+** representing the open interval (0, ∞), the symbol **-** representing the open interval (-∞, 0) and the symbol **0** representing the closed interval [0, 0]; see [Struss 1987] for an extended discussion. An important point to note is that the identification of the symbol **0** with a boundary between ranges implies that zero is a range of infinitesimal size. A corollary of this is that dynamically changing values will occupy the zero "range" for an infinitesimal time. Any qualitative values representing such states must therefore be transitory. In contrast, the **+** or **-** ranges have a significant extent, and values can therefore be persist in the range for a significant time. Engineers, as opposed to mathematicians, may prefer to accept a very small but finite zero band, to take account of the practicalities of measurement devices (the transition rules discussed in Section 6, below, were framed with this in mind).

Qualitative mathematical operations can be defined, corresponding to their quantitative counterparts. As an example, Table 1 shows the qualitative addition and multiplication of two values, [a] and [b].

Table 1. Qualitative Arithmetic

(a) Addition

		[a] +	[a] 0	[a] -
[b]	+	+	+	?
[b]	0	+	0	-
[b]	-	?	-	-

(b) Multiplication

		[a] +	[a] 0	[a] -
[b]	+	+	0	-
[b]	0	0	0	0
[b]	-	-	0	+

Qualitative vectors are an extension to the use of scalar qualitative values. The description here is necessarily brief: a detailed treatment is given in [Morgan 1988]. The vector representation expresses not only the value of the variable but also its successive derivatives. Scalar values can be seen as the zeroth derivative and the initial element of the vector. Successive vector elements represent the first derivative, the second derivative, and so on. The notation used here is to enclose qualitative values in square brackets, and to denote qualitative vectors in the form [d^0 d^1 d^2 ...], where the d^n stand for qualitative values drawn from the set { **+ 0 - ?** }.

For a function of the general form y = f(x), a change in x will lead to some corresponding change in y. Taking increasing x as a convention, we can express a corresponding change to y in the form of a qualitative vector. In graphical terms, this corresponds to a description of the shape of the curve of y against x.

Various functions will have their own characteristic curve shapes, and some illustrative examples are given in Table 2 for simple x-y relationships. The qualitative vectors are produced by successively differentiating the function and noting the sign of the derivative. Note that the coarse resolution of the qualitative values does not always allow functions to be distinguished, because their curve shape may be similar, as can be seen by comparing $y = x^{-1/2}$ and $y = e^{-x}$ in the table.

Table 2. Typical Functions and Vectors (positive x, y)

y =	successive derivatives					qualitative vector
x^2	2x	2	0	0	...	[+ + + 0 0 ...]
e^{-x}	$-e^{-x}$	e^{-x}	$-e^{-x}$	e^{-x}	...	[+ - + - + ...]
$x^{-1/2}$	$-1/2x^{-3/2}$	$3/4x^{-5/2}$	$-15/8x^{-7/2}$	$105/16x^{-9/2}$	...	[+ - + - + ...]
$x^{5/2}$	$5/2x^{3/2}$	$15/4x^{1/2}$	$15/8x^{-1/2}$	$-15/16x^{-3/2}$	...	[+ + + + - ...]

Since each vector can be viewed as a segment of a curve, consecutive vectors can be seen as consecutive curve segments. In physical systems, quantities normally change value in an unbroken way, *i.e.* they are continuously differentiable. This implies that adjacent curve segments must meet in a way which preserves continuity. A series of rules can be defined to express the continuity conditions governing adjacent curve segments, as described in [Morgan 1988]. For example, any element cannot change sign in adjacent vectors (from **+** directly to **-** or *vice versa*), since the element must pass through zero in transition. Vectors containing zero derivatives are transitory; this follows from the definition of "zero" as a boundary of infinitesimal width between **+** and **-**.

The qualitative mathematics of the scalar case can be extended to support corresponding operations on vectors. As examples: vector addition is achieved by adding corresponding elements; multiplication of a vector by a scalar by multiplying each vector element by the scalar, and multiplication of two vectors by application of the general rule for differentiation of products.

An important additional property arising from the vector representation is the ability to integrate or differentiate vectors. Since the vector elements represent successively higher derivatives, each element is the derivative of its preceding element. Differentiation is therefore achieved by shifting the vector elements to the left; integration by shifting to the right. With each differentiation, an additional element has to be inserted at the right (left for integration) to maintain the vector length. It may be possible to infer the value of the new element from information available, otherwise the value ? must be inserted. The application of the rules for continuity and qualitative mathematics to typical first and second order systems is discussed in [Morgan 1987].

4. A CONVENTIONAL SYSTEM MODEL

In order to illustrate the issues involved, a simple second-order case will be taken as an example. The same mathematical description can be applied to several physical systems; the easiest to visualise is probably a mass connected to a fixed frame by a spring and a damper, so that the mass is free to move under the influence of any force that might be applied. The forces on the mass (fm), the damper (fd) and the spring (fs) can be found from a definition of their basic properties:

$$fm = M.a,\quad fs = K.x,\quad fd = B.v,$$

where acceleration, velocity, and displacement are indicated by *a*, *v*, and *x* respectively. The applied force (F) is equal to the sum of the forces on the mass, the damper and the spring.

$$F = fm + fd + fs$$

Assuming an unforced response (F = 0) and using the basic properties of the components, the system can be expressed in differential equation form as:

$$M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx = 0,$$

Using the Laplace variable s, this can be written as:

$$(Ms^2 + Bs + K)x = 0.$$

In general $x \neq 0$, and so:

$$Ms^2 + Bs + K = 0.$$

Of course, this model is a simplification of reality. For example, it does not take into account the resistance of the air to the movement of the mass, the damping inherent in the spring, or the mass of the damper. The choice of this model implies that all of these minor effects can be ignored. All predictions of behaviour of the real system are therefore made on the basis of the "pure" version of reality represented in the second-order differential equation. This equation is known as the *characteristic equation* of the system, because it characterises the system's behaviour. The equation commonly appears in the form:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

where ζ is the damping ratio and ω_n is the undamped natural frequency. In terms of the component parameters here, these are

$$\zeta = B / 2(M.K)^{1/2},\ \text{and}\ \omega_n = (K/M)^{1/2}.$$

In conventional systems dynamics, a block diagram representation is commonly used. This captures the essential properties of the system; in this case in terms of the mass, spring, and damper parameters (M, K, and B respectively) and the integral relationships between acceleration and velocity, and between velocity and displacement. Figure 2 shows a typical diagram.

The fact that the system is second order can immediately be seen from the two integration blocks. It is important to note that the model implicitly assumes a non-zero value for the damping constant B. If the value of B were to be zero, the values fed around that branch of the network would be multiplied by the scalar constant zero, making all vector values zero. The branch containing B would then have no impact on the system behaviour, and would, in effect, be removed from the model. The resulting system has the characteristic behaviour of a simple harmonic oscillator: a continuous sinusoidal oscillation, with an amplitude determined by the initial conditions.

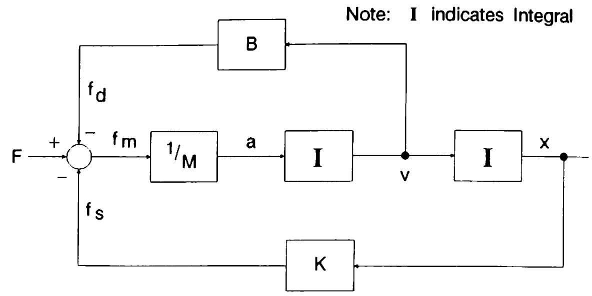


Figure 2. Block Diagram of Mass-Spring-Damper System

The dynamic behaviour of the system can be found from the characteristic equation and the initial values of displacement, x(0), and velocity, x′(0). A detailed derivation can be found in many standard references on systems dynamics, such as [Palm 1983] and [Shearer et al 1971].

The roots of the characteristic equation are given by:

$$s = -\zeta\omega_n \pm \omega_n(\zeta^2 - 1)^{1/2}$$

which may be real or complex. Determination of the roots requires a knowledge of the relevant parameters (M, K, and B in this case). For second order systems, three general behaviours can be distinguished, depending on the size of ζ:

underdamped: $0 < \zeta < 1$,
critically damped: $\zeta = 1$,
overdamped: $\zeta > 1$.

For a given set of parameters, the form of the system response will be similar for a wide range of input values. As an example, if the system is underdamped, the response to step inputs will be of much the same form regardless of the size or the sign of the step (although a negative step will clearly produce results in the opposite sense to a positive step). Also similar will be the free response, in which the system is allowed to "relax" from some initial starting position. Figure 3 shows a typical underdamped response.

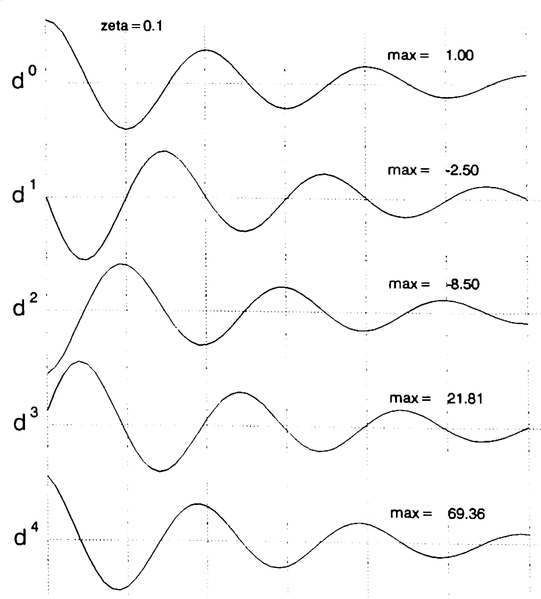


Figure 3. Free Response for zeta = 0.1

Figure 3 shows the free response from initial conditions:

$$x(0) = d^0 = 1, \quad x'(0) = d^1 = 0$$

with ζ – the damping ratio – equal to 0.1. The values of successive derivatives (d^1, d^2, *etc.*) have been scaled on the graph for convenience. A description of behaviour such as that represented by Figure 3 is valid only for particular parameter values. The aim of a qualitative representation is to find a more general description matching a range of parameter values.

5. A QUALITATIVE MODEL

Construction of a qualitative model is straightforward. The component parameters are positive constants. From Table 1, multiplication of a variable by a positive constant leaves the qualitative value unchanged. The blocks in Figure 2 representing 1/M, K. and B can therefore be ignored. The resulting qualitative equations are:

$$[v] = \text{int}[a], \quad [x] = \text{int}[v], \text{ and } [a] = [F] - [v] - [x],$$

where int[ ] represents qualitative integration. Each of the variables can be expressed as a qualitative vector, and the equations above constrain the combinations of vectors which are permissible. The resulting qualitative model is consistent with the conventional model, and makes the same assumptions about "pure" components. We should therefore expect to find a clear correspondence between results from the two models.

Conventional systems analysis assesses the performance of systems in relation to a number of standard test inputs; impulse, step, and ramp inputs are common. In a similar way, the qualitative equivalent can be assessed against standard inputs such as a step (*e.g.* [+ 0 0]) or a ramp (*e.g.* [+ + 0]). The impulse has no direct representation as a qualitative vector. In the qualitative case there is also a choice over the granularity of the response investigated; for example, whether to be concerned with the shape of the input as it rises from [0 0 0] to [+ 0 0].

A qualitative representation abstracts away from the particular values associated with an input but preserves the essential form of the result. Since the qualitative results are derived without knowledge of the system parameters, they will include all of the possible responses of the system; underdamped, critically damped, and overdamped. This paper concentrates on the underdamped response, which shows an oscillatory behaviour. Similar analyses can be performed for the overdamped and critically damped cases, which also show results corresponding to the analytic mathematical forms, although less interesting than the underdamped case.

Dynamic behaviours can be generated in a straightforward way by systematically testing all possible combinations of { + 0 -} vector element values against the constraining equations. This requires a choice of vector length appropriate to the task at hand. Bearing in mind that some functions require qualitative vectors of infinite length, this introduces another source of approximation. For example, choosing a vector length of 3, *i.e.* [d^0 d^1 d^2], implies that derivatives above second order are irrelevant.

To demonstrate the method of computation of the qualitative response of the system we can assume a zero forcing function, *i.e.* [F] = [0 0 0]. If we look at a possible value for [x] of [+ + +], the qualitative differentiation rule gives [v] = [+ + ?] and [a] = [+ ? ?]. Since [F] = [a] + [v] + [x] , this gives us [F] = [+ ? ?] , a contradiction to our earlier assumption. As another example, choosing [x] = [+ + -] gives [v] = [+ - ?] and [a] = [- ? ?]. Now [F] = [? ? ?] ; a solution which is consistent with our earlier assumption, since ? can represent any of the values **+**, **0**, or **-**.

An alternative method of computation relies on the propagation of selected scalar values from a given starting point around a component network (similar in concept to the conventional block diagram of Figure 2). Each component acts on values in a manner appropriate to its characteristics. A summation is normally required where network branches join together. Localisation of inference in this way creates difficulties in networks which have multiple branches, either feed-back or feed-forward. Since we are dealing with a continuous-time system, all values around a branch must be determined simultaneously. Failure to observe this condition is equivalent to a temporary opening of the branch, and requires a suspension of time within the model for resolution of propagated values. This difficulty is avoided with the qualitative vector method described above, since the consistency of vectors within the system is assured by the global test.

A standard result from the state-space representation used in conventional analysis is that a linear system of order *n* requires a vector length of *n* for an adequate description. A corresponding conjecture is that a qualitative representation requires a vector length of at least $n+1$ elements. A vector length which is too short does not provide the discrimination necessary to distinguish between significantly different behaviours. This is illustrated in the next section.

6. QUALITATIVE BEHAVIOURS

In this section a consistent set of results is shown for the free response of the system, from an initial state where the system is held at a positive static value. The length of system vector generated can be varied to show the effect of the system constraints on the number of states identified. A system vector of length 1 gives information only on the level of displacement. Since the system can oscillate, the displacement can take any of the three qualitative values **+**, **0**, or **-**, with transitions between the **+** and **-** states taking place via the **0** state.

A more interesting result is shown in Table 3 for a vector length of 2.

Table 3. 2-element state vectors.

state	[x]	successors
1	+ +	(2)
2	+ 0	(3)
3	+ -	(6)
4	0 +	(1 5)
5	0 0	(4 6)
6	0 -	(5 9)
7	- +	(4)
8	- 0	(7)
9	- -	(8)

Nine states are possible, with transitions as shown in Figure 4. The states for the single-element vector are a simplification of the more complex states, as shown by the dotted boxes in Figure 4. The 2-element states preserve the same transitions as the simpler case, but add detail on state structure and transitions.

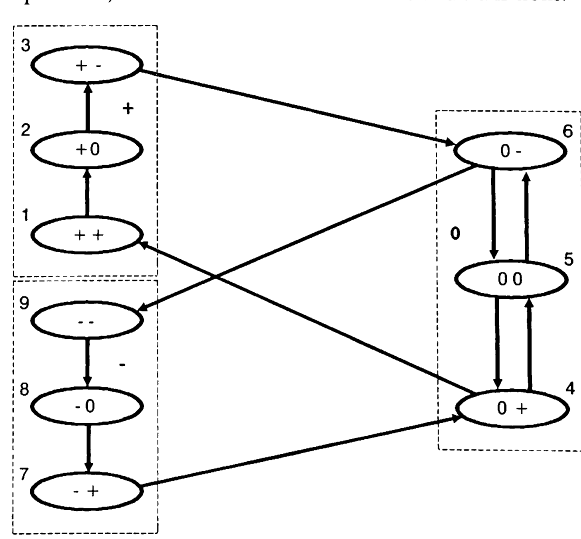


Figure 4. Transition Diagram for 2-element Vectors

A vector length of 3 produces 13 possible states, as shown in Table 4.

Table 4. 3-element state vectors.

state	[x]	successors
1	+ + -	(2)
2	+ 0 -	(5)
3	+ - +	(8)
4	+ - 0	(3 8)
5	+ - -	(4)
6	0 + -	(1)
7	0 0 0	( )
8	0 - +	(13)
9	- + +	(10)
10	- + 0	(6 11)
11	- + -	(6)
12	- 0 +	(9)
13	- - +	(12)

Each vector represents a feasible system state. A transition relationship between states can be determined from simple continuity rules [Morgan 1988]. For example, the only possible transition between the first two states is from State 1 to State 2. A consistent negative value for the third element of both vectors implies a decreasing value for the second element. This would be contradicted by a transition from State 2 to State 1, since the value of the second element would be increasing (*i.e.* transition from 0 to +). A systematic transition analysis of this kind reveals the set of feasible transitions shown in final columns of Tables 3 through 5.

State 7 in Table 4 represents the steady state, in which the system is completely static. It therefore has no transition path. Recalling that the oscillations shown in Figure 3 decay with time, it may seem strange that the steady state can not be reached from any of the oscillatory states. However, this is in exact accord with the conventional mathematical expression. The decay in the oscillations is governed by a negative exponential term which does not reach zero in finite time. The isolation of the steady state is therefore a consequence of the "pure" model selected initially; the qualitative transitions simply record this fact accurately.

The transitions are shown in Figure 5. The emerging pattern shows a symmetry between the positive and negative states, as might be expected from the sinusoidal nature of the under-damped response. Again, the simpler states of the 2-element vector behaviour are seen to be included within the 3-element behaviour. The exception is the resolution of the [0 0] case of Figure 4, which shows that a vector length of less than 3 does not provide sufficient resolution to correctly identify the steady state.

Comparison of the 2-element and 3-element results reveals a further subtle source of error. The states of Table 3 could equally well apply to a continuous (undamped) sinusoidal oscillation. As explained earlier, this could be produced by removing the branch containing the damping coefficient B from the model. However, since we have explicitly included non-zero damping in our model, the qualitative results should exclude this case. Extending the vector length from 2 to 3 produces the required result, as can be seen in Table 4. The point of inflexion in the negative-going part of the sinusoid (State 4 in Table 4) occurs while the value is still positive, due to the effect of the exponential decay of the curve. For an undamped sinusoid, the point of inflexion would coincide with the zero axis, producing a state vector of [0 - 0]. This can be demonstrated easily by generating a corresponding state table for the case where $B = 0$.

One possible source of inaccuracy is therefore related to resolution. A description of a second-order system using only the level and its first derivative is certain to produce spurious behaviours of the type described above. A more adequate representation (such as Table 4 and Figure 5) produces results which are completely consistent with the results of conventional analysis, and therefore true to the model.

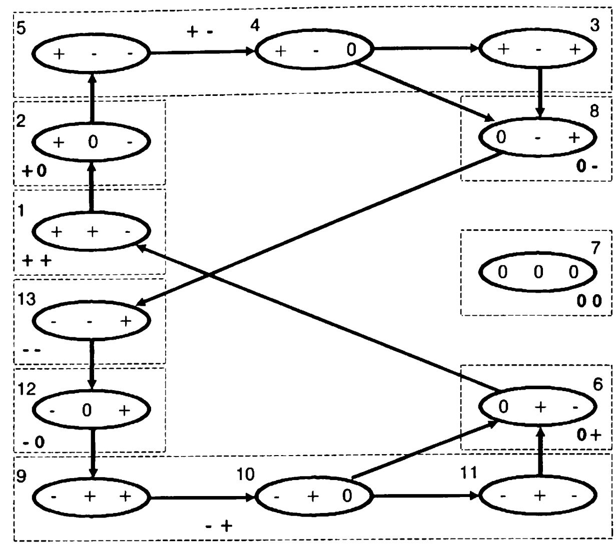


Figure 5. Transition Diagram for 3-element Vectors

A succession of states traced for [x] in Table 4 can be identified in Figure 3. For example, the first part of the d^0 curve corresponds to State 5, the first zero crossing to State 8, and the first negative peak to State 12.

The branches between different possible successor states in the tables and transition diagrams are not ambiguities introduced by qualitative reasoning. They represent behaviours corresponding to different combinations of system parameters. In fact, the state trajectory of the system can be used to indicate approximate values for components, as described in [Morgan 1990].

Table 5 shows the states and transitions for qualitative vectors of length 4, which can be seen to follow the same pattern. A transition diagram is not shown because the increasing numbers of states become difficult to follow when reproduced at a small scale.

Table 5. 4-element state vectors

state	x	successors
1	+ + - +	(4)
2	+ + - 0	(1 4)
3	+ + - -	(2)
4	+ 0 - +	(9)
5	+ - + +	(6 14 15)
6	+ - + 0	(7 15 16)
7	+ - + -	(16)
8	+ - 0 +	(5 14)
9	+ - - +	(8)
10	0 + - +	(1)
11	0 + - 0	(1 2 10)
12	0 + - -	(2 3 11)
13	0 0 0 0	( )
14	0 - + +	(15 23 24)
15	0 - + 0	(16 24 25)
16	0 - + -	(25)
17	- + + -	(18)
18	- + 0 -	(12 21)
19	- + - +	(10)
20	- + - 0	(10 11 19)
21	- + - -	(11 12 20)
22	- 0 + -	(17)
23	- - + +	(24)
24	- - + 0	(22 25)
25	- - + -	(22)

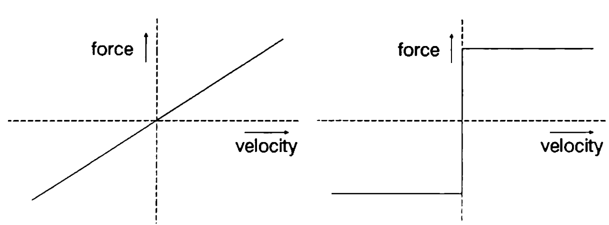
The maximum number of states possible with a vector of length n is 3^n. For a 2-element vector the theoretical maximum is then $3^2 = 9$ states, the same number produced by the process for generating consistent vectors described above. For a 3-element vector the number of generated states is 13; less than the maximum number of $3^3 = 27$. The reduction in the number of states generated is a consequence of the application of system constraints. In the 1-element and 2-element cases, the vector lengths are not sufficient for the constraints to apply, and so the internal structure of the system has no effect on the generated states. This also illustrates the effect of the conjecture above. A comparison of the numbers of states for vectors of length 1 to 5 is given in Table 6.

Table 6. Numbers of states for vector lengths up to 5.

length	max. (3^n)	generated
1	3	3
2	9	9
3	27	13
4	81	25
5	243	45

7. EXTENSION TO A NON-LINEAR CASE

The linear system described in the preceding section included viscous friction. This is a close representation of the operation of many real components, such as the dampers used as a component of automobile suspension systems. The damping coefficient B relates the force through the damper to the relative velocity of its two ends. This is a linear relationship, as shown in Figure 6a. The more general applicability of the qualitative vector representation is shown here by the substitution of Coulomb friction for viscous friction. Coulomb friction (sometimes known as static friction or "stiction") has a non-linear characteristic. It describes the phenomenon of "sticking" until the applied force reaches a certain level, as shown in Figure 6b.



(a) Viscous friction (b) Coulomb friction

Figure 6. Two Types of Damping Coefficient

The effect of Coulomb friction is reflected in the system equation:

$$M.\frac{d^2x}{dt^2} + C(\text{sign}\frac{dx}{dt}) + K.x = 0$$

where C is the constant associated with the Coulomb friction. Comparison with the viscous example shows that the damping term is no longer proportional to the velocity, but is a term which is "switched" to a fixed value according to the sign of the velocity. In the qualitative case, this is equivalent to transforming the qualitative vector for velocity into a qualitative scalar.

The block diagram of the system must be modified from the original linear system to reflect the revised characteristics of the damping. This entails the replacement of the damping coefficient block (labelled B in Figure 2) with another block which switches a fixed positive or negative value, depending on the sign of the velocity v.

Analytic results for this case can be derived along the following lines. A new variable, y, can be defined so that:

$$y = x + \frac{C}{K}(\text{sign}\frac{dx}{dt}).$$

Substituting in the system equation gives:

$$M\frac{d^2y}{dt^2} + C(\text{sign}\frac{dx}{dt}) + K[y - \frac{C}{K}(\text{sign}\frac{dx}{dt})] = 0.$$

$$\text{So: } M\frac{d^2y}{dt^2} + Ky = 0$$

Integrating this gives:

$$\frac{M}{2}(\frac{dy}{dt})^2 + \frac{Ky^2}{2} = \text{constant},$$

$$\text{and so: } (\frac{dx}{dt})^2 + \frac{Ky^2}{M} = \text{constant}$$

Substituting for y gives:

$$v^2 + \frac{K}{M}(x + \frac{C}{K})^2 = C1 \text{ for } v > 0$$

$$v^2 + \frac{K}{M}(x - \frac{C}{K})^2 = C2 \text{ for } v < 0$$

In terms of coordinates v, $x(K/M)^{1/2}$, these two equations describe circles centred at:

$v = 0$, $x = -C/(K.M)^{1/2}$ for $v > 0$, and
$v = 0$, $x = C/(K.M)^{1/2}$ for $v < 0$.

It is convenient to visualise the free response for this system on the *phase plane* – a graph of v against $x(K/M)^{1/2}$. The system response follows a trajectory consisting of semi-circles in each half-plane towards the origin of the v, $x(K/M)^{1/2}$ plane. The switch between the two circle equations occurs on the horizontal axis of the phase plane, which of course is the $v = 0$ line. The trajectories continue until the $v = 0$ line is reached between one of the two circle centres ($x = -C/(K.M)^{1/2}$ or $x = C/(K.M)^{1/2}$) and the origin (where $v = 0$ and $x = 0$). The reason for the trajectory halting at this point is that the forces due to the mass and the spring are no longer adequate to overcome the Coulomb friction, and so the system "sticks". Typical phase plane trajectories for the free response are shown in Figure 7. Two observations follow immediately.

- Unlike the case of viscous friction, the system can reach steady-state conditions in finite time.
- The steady-state position finally reached may have a positive, negative, or zero value of displacement x, depending on the sticking position.

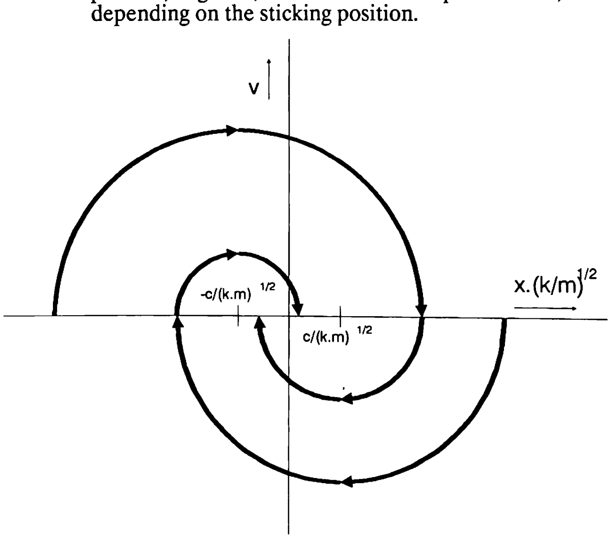


Figure 7. Phase Plane Portrait for Coulomb Friction

A qualitative analysis of the Coulomb friction case produces rather more states than the same vector length in the viscous case. This is because the non-linear system is not able to provide such strong constraints as the equivalent linear system. The generated qualitative states for a system vector length of n = 3 are shown in Table 7 and as a transition diagram in Figure 8. The states shown in the diagram and the table are produced by exactly the same method as the previous viscous friction examples, by simply changing the damping block in Figure 2. The generated Coulomb friction states can be contrasted with the viscous friction case (viscous states are listed in Table 4 and transition diagram shown in Figure 5).

Table 7. 3-element vectors with Coulomb friction.

state	[x]	successors
1	+ + -	(4)
2	+ 0 +	(3)
3	+ 0 0	(2 4)
4	+ 0 -	(3 7)
5	+ - +	(2 12)
6	+ - 0	(5 12)
7	+ - -	(6)
8	0 + -	(1 11)
9	0 0 +	(10)
10	0 0 0	(9 11)
11	0 0 -	(10)
12	0 - +	(9 19)
13	- + +	(14)
14	- + 0	(8 15)
15	- + -	(8 18)
16	-0 +	(13 17)
17	-0 0	(16 18)
18	-0 -	(17)
19	- - +	(16)

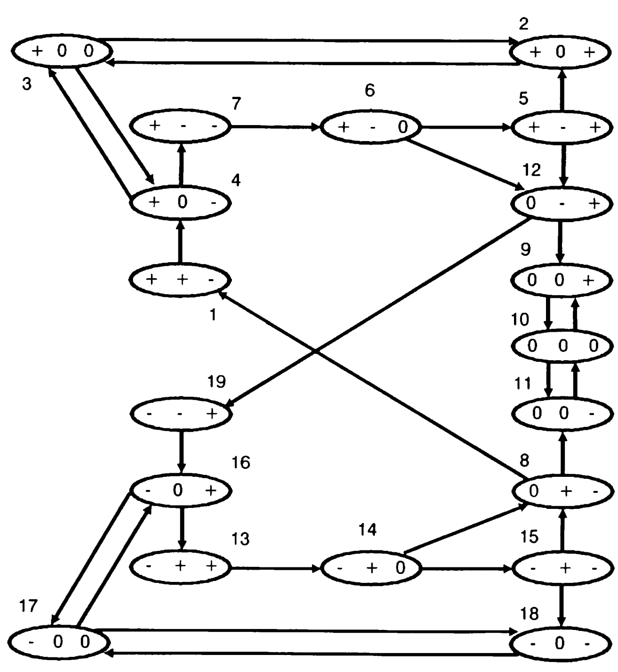


Figure 8. Transition Diagram for Coulomb Friction

The correct identification of the three possible Coulomb steady states (states 3, 10, and 17), and their reachability from other states is immediately apparent. A detailed simulation of the system's time response shows the expected oscillation, decaying in a slightly different way to the damping due to viscous friction, with the oscillations ceasing when their amplitude becomes too low to overcome the frictional constraints. Again, this illustrates a correspondence between the conventional and qualitative solutions.

8. CONCLUSIONS

The discussion in this paper has illustrated some aspects of the relationship between conventional and qualitative approaches to system dynamics. In both cases there is a need for care in defining the model to be used, and in understanding the limitations of the model and the corresponding impact on results. The relationship between conventional and qualitative results is important, because it touches on the issue of accuracy. There is no reason why qualitative reasoning *per se* should not be completely accurate, within the limits imposed by the relatively coarse resolution, and the examples in this paper have shown one way of achieving this. Qualitative vectors prove to be a convenient and expressive tool in representing system behaviours. In particular, they preserve, and, in a way, even reinforce the correspondence between conventional and qualitative mathematics. This provides a welcome degree of confidence in assuring the quality of qualitative descriptions of system behaviour.

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