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HAIL STORM
A MODEL FOR DETERMINING THE SURVIVABILITY OF FIXED-WING AIRCRAFT AGAINST SMALL ARMS

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ABSTRACT

In the past large caliber anti-aircraft guns were the only AAA considered important enough to model, but with the new focus on low-intensity conflict, small arms as air defense has risen in prominence. Hail Storm is the most resent attempt at modeling the survivability of aircraft against small arms. The model is a one-on-one deterministic duel of a small arms weapon and an aircraft. The results of the model supplies a probability of survival given an encounter and the expected number of hits.

1. INTRODUCTION

Small arms have brought down aircraft during every major conflict in which the United States has flown. In the latest conflict, Vietnam, the U.S. lost 410 aircraft to weapons of .51 caliber and below (O'Conner, 1987). Obviously, small arms are a threat to aircraft; but there are no models currently available which can accurately evaluate the survivability of fixed-winged aircraft against small arms. Hail Storm is the first survivability model built specifically for the small arms threat. The remainder of this paper is a brief explanation of the Hail Storm model. Complete details can be found in A Methodology for Determining the Survivability of Fixed-wing Aircraft against Small Arms (Grover, 1989).

2. ANTI-AIRCRAFT ARTILLERY (AAA) MODELS

The AAA models in use today were written for the larger caliber anti-aircraft guns, not small arms. These models are unsuitable for small arms for two basic reasons:

a. Flat trajectories. These trajectories assume there is no drop in the path of the bullet due to gravity. This assumption is reasonable for the larger high velocity rounds but small arms rounds experience drop after only a few hundred feet.

b. Single Shot Probability of Kill (PSSK). Small arms rely on volume fire for their effectiveness. The PSSK for any small arms round would be nearly zero, but how many times does an infantryman fire only one shot at an aircraft?

3. HAIL STORM METHODOLOGY

The purpose of the Hail Storm model is to provide an efficient and effective method of evaluating the survivability of a fixed-wing aircraft against small arms. The model estimates the probability an aircraft survives given and encounter with a small arms weapon and the expected number of hits the aircraft receives.

Scenario Limitations. There is an infinite number of situations and geometries in which small arms could be used to engage fixed-wing aircraft. To reduce the number of possible scenarios, limitations must be applied. The sce- nario is a 'worse case' for the aircraft. The aircraft is restricted to a constant velocity, altitude, and approach angle (i.e. no evasive maneuvers).

Also, no environmental effects are modeled.

Engagement Method. The Hail Storm model uses a lead point engagement method. In this method, the ground combatant picks a point in front of the aircraft and fires his weapon at that point until the aircraft passes it. To maximize the number of hits on the aircraft, it is assumed that the first bullet fired reaches the aim point before the aircraft and the last bullet passes after the aircraft.

Ballistic Trajectories. Ballistic equations are second order, coupled, nonlinear, differential equations. This interdependence of variables makes the integration of closed form solutions impossible. The most common approach used to avoid this problem is to assume it away by using flat trajectories. But, flat trajectories have one significant drawback. They are accurate for only a relatively short distance during ascent and since small arms trajectories curve down quickly, flat trajectories are not usable.

Hail Storm numerically estimates bullet trajectories by iterating basic ballistic equations. The idea is to assume that the coefficient of drag and bullet angle with respect to the ground are constants for very short periods of time. This assumption permits the velocity, angle, and location to be calculated at the end of each time period and the results to be used as the initial conditions for the next period. Using this method, it is possible to piece together the complete flight of a bullet with good accuracy.

Aircraft Representation. In most survivability models the aircraft is represented according to the number of aspect angles supplied, usually six or 26. As a first cut approximation, Hail Storm uses a sphere with the same total surface area, A(t) as the actual aircraft.

Coordinate System. The ground combatant is the origin of the system. The postive X-axis is the line formed on the ground between the combatant and the aim point. The positive Z-axis is the altitude above the ground. The Y-axis is the off range distance from the aim point. This coordinate system is represented in Figure 1.

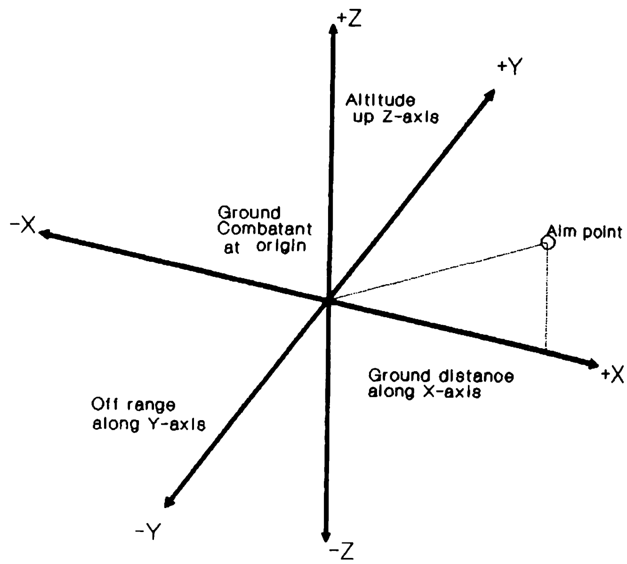

Fig. 1 Coordinate system

Aiming. The aim point is expressed in terms of the ground distance (GRD), off range distance (OFR), and altitude (ALT) from the combatant. This aim point is used to calculate the mean aiming azimuth and elevation angle. The mean azimuth angle always lies on the X-axis, therefore it is always zero. This also causes the OFR to always be zero. The mean elevation angle is simply the angle formed between the axis and the line of sight to the aim point.

Since there is an aiming error caused by the combatant's limitations and weapon inconsistencies between shots, the actual firing elevations and azimuths are described by normal distributions about the angle mean. The use of normal distributions implies that there is an infinite number of initial elevations and azimuths angles available for trajectory calculations and each angle has a zero probability of occuring. These characteristics of a continuous probability distribution makes the direct use of them in a deterministic model difficult. To remedy this problem, only fifteen angles on each side of the mean are considered. The angles are placed every 0.2 standard deviations within three standard deviations of the mean. By separating the normal distributions into descrete intervals, each of the initial angles will also have an associated probability of occurance. The probability of occurance can then be used as the percent of bullets fired at a given angle. In this manner an inherently continuous and stocastic phenomenon can be converted to a descrete and deterministic one, without a great loss of detail.

Trajectory Calculations. The positions and component velocities of the bullets' flight paths are first calculated for each elevation angle over the mean azimuth. Using the iterative method, the trajectories are plotted every 0.2 standard deviations within three standard deviations of the mean elevation angle. Once all trajectories are plotted, in the X,Z-plane, for the mean azimuth, the trajectories for the other azimuths can be plotted by rotating the mean azimuth trajectory plots every 0.2 standard deviations into the Y-plane.

Expected Hits. The expected number of hits E(h) on the target aircraft sphere is calculated for each of the 961 trajectories plotted. The first step is to find the points where the given trajectory cuts the horizontal planes tangent to the target sphere. The trajectory is then approximated by a straight line between these points. Each line defined by two points (X1,Y1,Z1) and (X2,Y2,Z2).

The second step is to transform the coordinate system to where all interaction between the aircraft sphere and the line segment representing the bullet trajectory can be given in 2-dimensions. The first transformation is a simple rotation of the aircraft angle of approach to zero degrees. The second transformation is another axis rotation. This transformation rotates the trajectory line segment to a vertical position in the Y',Z'-plane. The line segment formed by the bullets is now like the blade of a band saw and the aircraft a ball of wood. No matter where the blade cuts the sphere the intersection is a circle projected on the X'',Z''-plane. If the diameter of this cut circle is real, calculations continue. The circle is represented in Figure 2.

Step three is to find the points where the sliced circle first and last intersects the trajectory segment. The Z'' coordinates of these points are called I1 and I2. The next step is to calculate the length of time the bullets' path intersects the circle cut from the sphere. There are three mutually exclusive cases for the time of intersection calcualtion:

Case 1: The X'' component of the bullets' velocity is positive (i.e. the aircraft and bullets close on each other).

Case 2: The X'' component of the bullets velocity is negative and the component of the aircraft's velocity parallel to the bullets' velocity is greater than the bullets' velocity (i.e. the aircraft catches and hits the bullets).

Case 3: The X'' componet of the bullets' velocity is negative and the component of the aircraft's velocity parallel to the bullets' velocity is less than the bullets' velocity (i.e. the bullets catch and hit the aircraft).

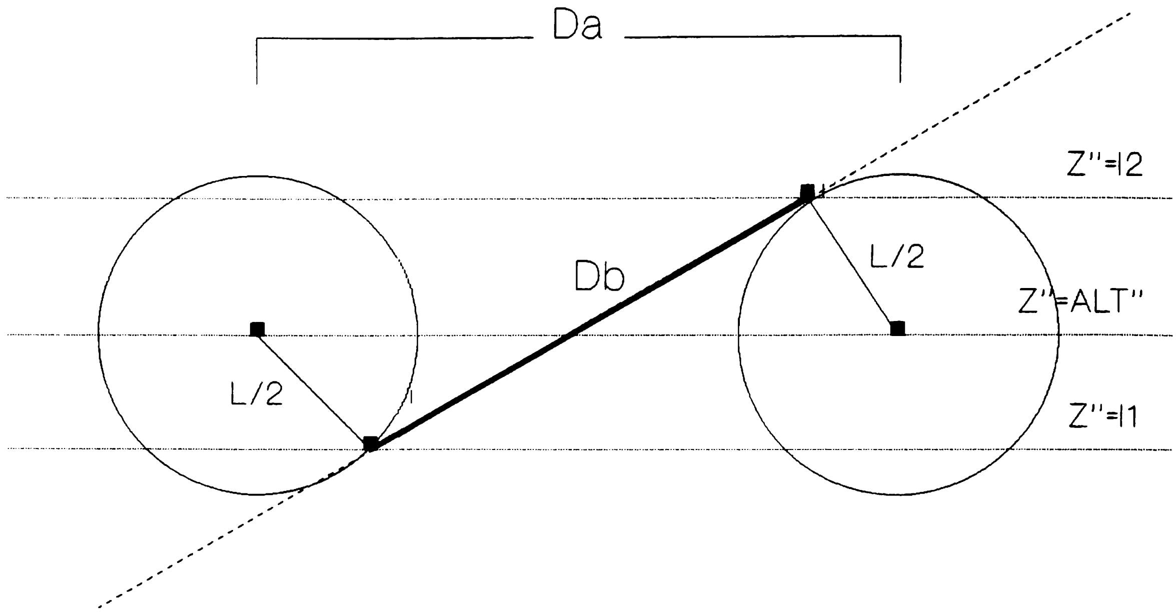

Fig. 2 X'',Z''-projection of circle sliced from sphere

For the three cases, the time of intersection is calculated as:

Case 1: ti = ta + tb

Case 2: ti = ta - tb

Case 3: ti = tb - ta

ta = Da / Va

tb = Db / Vb

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Da = [ (L/2)^2 - (I1-ALT'')^2 ]^.5 +
     [ (L/2)^2 - (I2-ALT'')^2 ]^.5 +
     Db * cos(Ai)

Db = ( I2 - I1 ) / sin(Ai)
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where

Da = distance covered by the aircraft during the intersection
Db = distance along the trajectory path which intersects the aircraft sphere
Va = velocity of aircraft
Vb = average velocity of bullets on line segment
L = diameter of the circle cut from the aircraft sphere
I1 = altitude of first intersection
I2 = altitude of last intersection
Ai = angle of trajectory as it passes through the aircraft sphere

The final calculation for the expected number of hits is:

E(hits) = ti * ROF * P(ele) * P(azi)

where

t_i = time period of intersection
ROF = weapon rate of fire
P(ele) = percent of bullets fired at the initial elevation (ele)
P(azi) = percent of bullets fired at the initial azimuth (azi)

Impact Velocity. The impact velocity is used to interpolate the appropriate probability of kill given a hit P(k/h) from a data base. The same three cases used for the time of intersection are also used for the impact velocity. The impact velocity (Vi) is:

Case 1: $Vi = Va * \cos(Ai) + Vb$

Case 2: $Vi = Va * \cos(Ai) - Vb$

Case 3: $Vi = Vb - Va * \cos(Ai)$

Probability of Survival. The probability of survival is simple to calculate once E(h) is calcualted and P(k/h) is interpolated for each trajectory segment the aircraft passes through. Using these two numbers the probability of survival against a single trajectory is:

$$P(sts) = EXP[-P(k/h) * E(h)]$$

The total probability of survival for the entire engagement is:

$$P(s/e) = \prod_{\substack{\text{for all}\\ \text{trajectory}\\ \text{segments}}} P(sts)$$

4. APPLICATIONS

There are two major uses for the Hail Storm model. The first is to provide an aid for survivability studies which analyze competing aircraft designs. The model provides P(s/e), but it also calculates the expected number of hits, E(hits), for battle damage repair models.

The other use is for wargaming. For high resolution combat models, Hail Storm could be incorporated directly. The model could also be used as a data generator for lower resolution wargames. For example, all memebers of an infantry platoon and their various small arms could be evaluated separately and the results databased for a larger model.

5. CONCLUSION

The Hail Storm model was developed to provide a link between established bullet ballistics equations and high resolution combat scenarios using small arms as air defense. Being one of the first to provide such a link, it should prove to be a useful tool for the analysis of aircraft survivability against small arms.

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