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RESPONSE SURFACE ANALYSIS OF STOCHASTIC NETWORK PERFORMANCE

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ABSTRACT

This paper describes a response surface approach to the stochastic network improvement problem. The network addressed in this study is acyclic, simple, and directed; and, is characterized by single commodity flow from multiple sources to multiple sinks. The network is stochastic in that the reliability of network components is described by a binary distribution of operative or failed states, where Ps_i is the ith component's probability of survival. A new class of control variables is introduced to reduce the variance of the maximal flow estimators.

1. INTRODUCTION

In this paper we consider the stochastic network improvement problem. We limit our study to acyclic, simple, directed networks which process a single commodity flow from multiple sources to multiple sinks. The networks are stochastic in that the components (arcs and nodes) are subject to failure. Individual components are modeled as binary random variables. The stochastic network improvement problem is to find those components whose feasible improvements yield optimum performance given budgetary constraints. Our research efforts are presented in the following manner. First, we discuss the class of networks used in this study, summarize recent literature pertinent to the problem, and state our performance measures. Next, we present our research methodology and demonstrate our approach on a sample network. We close the paper with a summary and provide recommendations for future research.

2. NETWORK FLOW AND RELIABILITY

A class of probabilistic networks applicable to this study are **stochastic binary networks** (SBN). Ball (1980) defines an SBN to be "a system that fails randomly as a function of the random failure of its components...(where) each component may take on either of two states: operative or failed and that the states of any two components are independent". Furthermore, he defines a **stochastic coherent binary network** (SCBN) as one where the pathset defines the minimal subset required for system operation and the cutset defines the minimal subset required for failure . The class of networks that this paper addresses fit Ball's definition with one exception: component failure is **not** necessarily independent. However, failure dependencies among network components are easily implemented in a Monte Carlo simulation.

Another important class of stochastic networks are randomly-capacitated networks (RCN). In an RCN, arc **capacity** varies over a range of values as a continuous function of a probability distribution. Arc capacity in a SBN/SCBN network, by contrast, is based solely on the binary (operative-failed) status of the arc; i.e., if the arc is operative, there is only one arc capacity. The networks investigated by this study are not part of the RCN category of stochastic networks. However, extension of this research to RCN systems would be straight-forward. For further explanation or research results in this class of networks, see Fishman (1987a), Somers (1982), and Evans (1976).

Fishman (1986) provides an overview of Monte Carlo methods for estimating network reliability. His article reviews four ways to calculate network reliability for an undirected graph version of a SCBN: (1) dagger sampling by Kumato and others (1980); (2) sequential destruction by Easton and others (1980); (3) bounds estimation by Fishman (1986); and, (4) estimation based on failure sets by Karp and Luby (1983). Karp and Luby's technique uses failure sets, (or equivalently cutsets) to estimate the graph's reliability, and is most closely related to this study's methodology. However, instead of sampling the entire cutset as we propose in this paper, Karp and Luby's Monte Carlo simulation procedure repeatedly samples single, randomly selected cuts to determine network reliability. Because our implementation of the max-flow min-cut algorithm (Ford and Fulkerson, 1962) evaluates the **entire** cutset for every replication in the simulation, Karp and Luby's technique is not used.

Fishman provides two papers that deal with Monte Carlo estimation of maximal flow on a network. The first paper develops an algorithm that offers both computational efficiency and reduced variance of an unbiased estimator of maximal flow. He models randomly decreasing arc capacities , using a cumulative process that describes the arc deterioration as normally distributed (Fishman, 1987a).

The second Fishman paper is more closely related to this study's efforts. It combines two methods of importance sampling in a Monte Carlo simulation to reduce the variance of the reliability estimators of communication networks typically described by an

SCBN (Fishman, 1987b). In this study, we investigated the effect of control variates, not importance sampling, in variance reduction. However, Fishman provides a proven approach to reducing the variance of the estimator. A comparison or synthesis of the two variance reduction techniques would be a useful continuation of this research.

In this paper we consider both expected maximum flow and source-to-sink reliability as network performance measures.

3. RESEARCH METHODOLOGY

The network improvement strategy employed three stages. First, a FORTRAN-based Monte Carlo simulation model was designed using a cut-set algorithm for estimating maximal flow and reliability. Relative to this stage we introduce a new class of control variables to reduce the variance of the maximal flow estimator. Second, response surface methodology was applied to derive a metamodel of expected network performance over the range of possible component improvements. This stage involved the sequential application of Plackett-Burman (1946) screening designs and full-factorial designs to derive an accurate polynomial approximation of network performance. Finally, the first-order response equations for maximal flow and reliability were used as objective functions in a linear programming formulation of the network improvement problem.

3.1 Simulation

The simulation program designed to calculate maximal flow and reliability is called MAXFLO (Bailey, 1988). Using an inversion technique described by Shier and Whited(1984), MAXFLO first derives the network cutset from the pathset. Once the proper cutset matrix is found, variations in maximum flow for each sample is modeled by using a one dimensional array representing the status of arcs. This state vector is based on the current replication's comparison of random number draws and the individual arcs' probability of survival, and is used by the maximum flow calculation routine in deciding which components in the cutset matrix to ignore in the current sample.

Variance reduction of the maximum flow estimator requires knowledge of a concomitant variable that has a known expectation. In the case of a SCBN, we offer as a general class of controls the **total number** of nodes that are up (or down) in a given subset. This control variate is an aggregate scalar measure of how many nodes in the subset are operative. For purposes of clarity, this class of control variates is referred to as **survival variables**.

Because of the stochastic binary nature of the network, the random variable X_i is defined as

$$X_i = 0 \text{ with probability of } P_i$$
$$X_i = 1 \text{ with probability of } 1 - P_i \qquad (1)$$

where P_i is the probability of survival (Ps) of component i. The control variate is defined as

$$SV = \sum_{i=1}^{N} X_i \qquad (2)$$

with expectation

$$E(\sum_{i=1}^{N} X_i) = \sum_{i=1}^{N} P_i = \mu_{SV} \qquad (3)$$

where N is the number of components in the subset. Therefore, the controlled estimate of the mean response (Y) is given by

$$\bar{Y}(\hat{\beta}) = \bar{Y} - \hat{\beta}(\overline{SV} - \mu_{SV}) \qquad (4)$$

where

$$\overline{SV} = \sum_{j=1}^{M} SV_j \qquad (5)$$

$\bar{Y}$ is the sample mean of the M samples, and $\hat{\beta}$ is the estimated control coefficient. Components may be nodes or arcs. For further explanation of variance reduction using control variates, see Bauer (1987), Kleijnen (1974), Lavenberg and Welch (1981), or Wilson (1984).

3.2 Response Surface Methodology

Once we have modeled the system we need a way to describe the performance of the system as a function of feasible component improvements. We applied response surface methodology to meet this end. Since both component survival probabilities and capacities influence expected maximum flow, N nodes and M arcs provide N + 2M possible factors requiring $2^{(N+2M)}$ experimental design points for a complete, 2-level factorial design. In the case of reliability, only the component parameter of survival probability affects network reliability, thus requiring $2^{(N+M)}$ design points. Obviously, in either case a reduction in the number of factors is necessary. We employed a Plackett-Burman screening design because of its small size and ability to detect mutually unaliased main effects (Box and Draper, 1987). From this initial screening, a reduced number of factors showing significant main effects was used to form the full first-order factorial design.

The sample network and component parameter list is given in Figure 1 and Table 1, respectively, on the following page. For this network, the following 19 factors from Figure 1 were selected for the Plackett-Burman design based on intuitive judgement of their influence on network performance: Ps for Nodes 8, 9, 10, 11, 13, 14 (N8p, N9p, N10p, N11p, N13p, and N14p); Ps and capacity for the four arcs that go directly from Node 8 to sink Nodes 15, 16, 30, and 31 (A8-15p, A8-16p, A8-30p, A8-31p, A8-15c, A8-16c, A8-30c, and A8-31c); and the capacities of arcs adjacent to Node 8 (A2-8c, A3-8c, A5-8c, A7-8c, and A8-9c). Capacity improvements are based on standard increments of 300, 1200, 2400, 9600, and 19200, while Ps

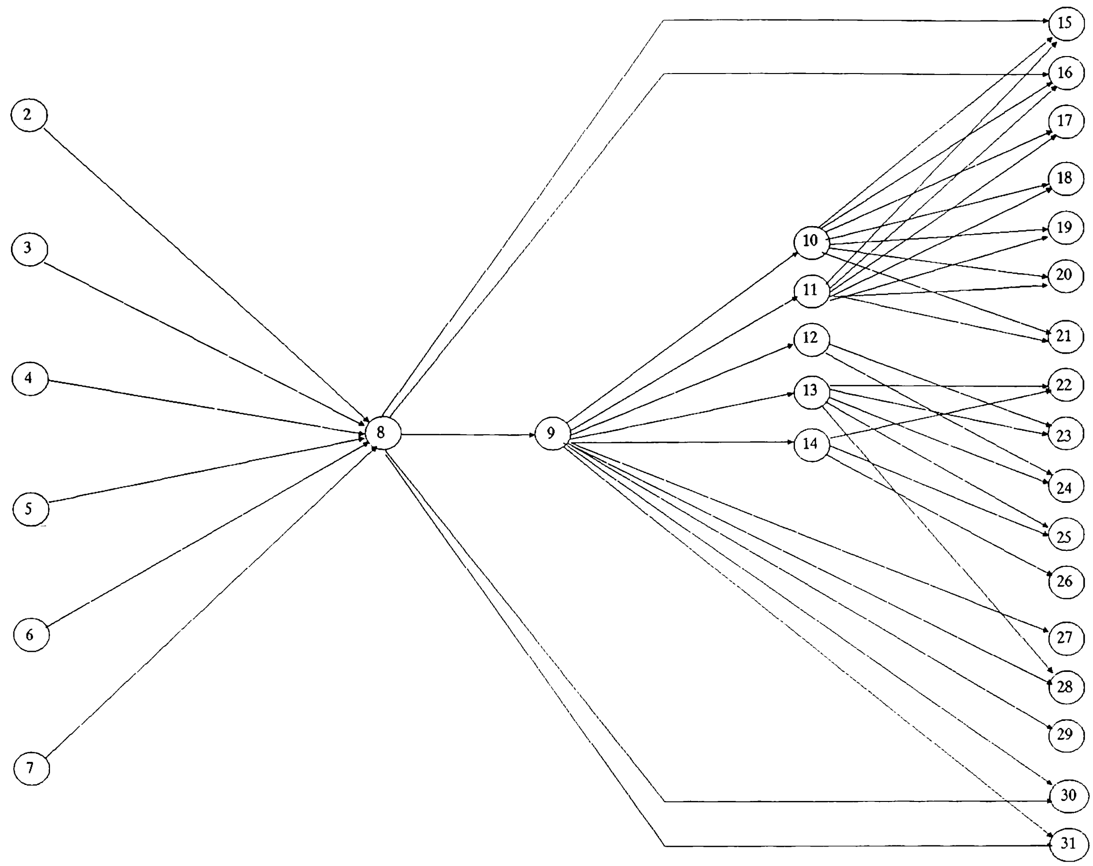

Figure 1: Network Diagram

Table1: Network Component Parameters

Component	Ps	Capacity	Component	Ps	Capacity	Component	Ps	Capacity
N2	.3	-	N27	1.0	-	A9-13	.638	1200
N3	.7	-	N28	.3	-	A10-15	.3	4800
N4	.5	-	N29	.7	-	A10-16	.6	4800
N5	.8	-	N30	.5	-	A10-17	.7	2400
N6	1.0	-	N31	.8	-	A10-18	.9	1200
N7	.3	-	A2-8	.9	1200	A10-19	1.0	1200
N8	.7	-	A3-8	1.0	1200	A10-20	1.0	1200
N9	.7	-	A4-8	.49	300	A10-21	.3	1200
N10	.5	-	A5-8	.6	300	A11-15	.6	4800
N11	.8	-	A6-8	.135	1200	A11-16	.3	4800
N12	1.0	-	A7-8	.48	300	A11-17	.6	2400
N13	.3	-	A8-9	.9	9600	A11-18	.7	1200
N14	.7	-	A8-15	.6	75	A11-19	.9	1200
N15	.5	-	A8-16	.3	75	A11-20	1.0	1200
N16	.8	-	A8-30	.6	1200	A11-21	1.0	1200
N17	1.0	-	A8-31	.7	1200	A12-23	.6	2400
N18	.3	-	A9-10	1.0	4800	A12-24	.3	1200
N19	.7	-	A9-11	1.0	4800	A13-22	.6	300
N20	.5	-	A9-12	.6	4800	A13-23	.7	2400
N21	.8	-	A9-13	.3	4800	A13-24	.9	1200
N22	1.0	-	A9-14	.6	4800	A13-25	1.0	300
N23	.3	-	A9-27	1.0	2400	A13-28	1.0	2400
N24	.7	-	A9-28	.3	2400	A14-22	.6	1200
N25	.5	-	A9-29	.252	600	A14-25	.3	300
N26	.8	-	A9-30	.245	1200	A14-26	.6	300

improvements are a uniform increase of .2. The design was run on a VAX 8650 under VMS 4.6 using 10000 replicates at each design point.

SAS PROC GLM was used to calculate the regression results, which appear in Table 2. The results show that of the 19 factors, only five account for a significant portion of the sums of squares for expected maximum flow: N8p, N9p, A2-8c, A3-8c, and A5-8c. Together, these five factors explain 95% of the variation of expected maximum flow. Table 2 also shows the regression results for the reliability response, with N8p accounting for the significant amount of the variation in reliability. Because this is a screening design, only the main effects are estimated (Plackett and Burman, 1946); however, the number of factors is reduced enough to allow for a full factorial design.

Since there are five remaining factors, a full factorial design requires only 32 design points (2^5). Additional design centerpoints are also required to test for second order effects, and to form the basis of a second order design. Since 10 centerpoints are required if we expand to a 2^5 central composite, uniform precision design (Montgomery, 1984), all 10 are simulated in addition to the required 32 design points. These centerpoints also provide a good statistical sampling for second order effects. Again, 10000 replicates were taken at each design point. Tables 3 and 4 show the results of the full factorial design and regression.

Table 2: Sums of Squares of Screening Design

	Sums of Squares	
Source	Maximum Flow	Reliability
MODEL	3299594.387	1705.785
N8p	1249935.001	1673.718
N9p	196741.382	14.416
N10p	246.837	0.808
N11p	3649.321	0.007
N13p	2223.266	0.255
N14p	168.386	0.001
A8-15c	30.076	0.002
A8-15p	3943.274	0.481
A8-16c	359.484	0.421
A8-16p	5383.399	3.715
A8-30c	261.075	0.318
A8-30p	3749.855	2.113
A8-31c	80347.080	0.648
A8-31p	25760.694	5.429
A2-8c	153612.938	0.662
A3-8c	1203365.268	1.270
A5-8c	339612.880	0.392
A7-8c	17895.632	0.592
A8-9c	2308.539	0.538

The first-order model has an R-Square value of .988, indicating a high degree of fit of this model to the data. (Small, but statistically significant, two-way interactions are also present; however, they are ignored because of their practical insignificance). Furthermore, an additional check for second order effects was calculated using the centerpoint data from runs 33 through 42 in Table 3. The resulting

Table 3: 2^5 Experimental Design for Sample Network

Run	N8p	N9p	A2-8c	A3-8c	A5-8c	Est. Max Flow	Est. Rel.
1	-	-	-	-	-	1169.152	62.78
2	-	-	-	-	+	1376.310	"
3	-	-	-	+	-	1548.608	"
4	-	-	-	+	+	1750.167	"
5	-	-	+	-	-	1310.505	"
6	-	-	+	-	+	1516.162	"
7	-	-	+	+	-	1687.208	"
8	-	-	+	+	+	1886.432	"
9	-	+	-	-	-	1288.522	64.21
10	-	+	-	-	+	1527.113	"
11	-	+	-	+	-	1743.144	"
12	-	+	-	+	+	1974.992	"
13	-	+	+	-	-	1464.682	"
14	-	+	+	-	+	1700.581	"
15	-	+	+	+	-	1915.268	"
16	-	+	+	+	+	2144.053	"
17	+	-	-	-	-	1434.863	77.33
18	+	-	-	-	+	1679.648	"
19	+	-	-	+	-	1889.871	"
20	+	-	-	+	+	2127.018	"
21	+	-	+	-	-	1614.300	"
22	+	-	+	-	+	1857.175	"
23	+	-	+	+	-	2065.141	"
24	+	-	+	+	+	2298.823	"
25	+	+	-	-	-	1573.297	79.52
26	+	+	-	-	+	1865.037	"
27	+	+	-	+	-	2129.271	"
28	+	+	-	+	+	2414.532	"
29	+	+	+	-	-	1781.073	"
30	+	+	+	-	+	2070.851	"
31	+	+	+	+	-	2332.181	"
32	+	+	+	+	+	2614.178	"
33	0	0	0	0	0	1801.424	71.37
34	0	0	0	0	0	1833.608	71.02
35	0	0	0	0	0	1820.961	71.29
36	0	0	0	0	0	1820.931	71.43
37	0	0	0	0	0	1816.268	71.20
38	0	0	0	0	0	1815.133	70.86
39	0	0	0	0	0	1803.838	70.43
40	0	0	0	0	0	1797.171	70.91
41	0	0	0	0	0	1779.531	70.25
42	0	0	0	0	0	1816.184	70.92

Coded Value	Uncoded Value				
	N8p	N9p	A2-8c	A3-8c	A5-8c
-	.70	.70	1200	1200	300
+	.86	.86	2400	2400	1200
0	.78	.78	1800	1800	750

Table 4: ANOVA and Parameter Estimates of 2^5 Design

Source	DF	Sum of Squares	F-Value
Model	5	3742567.705	428.800
N8p	1	1031177.244	590.73
N9p	1	345985.548	198.21
A2-8c	1	239270.791	137.07
A3-8c	1	1661489.497	951.82
A5-8c	1	464644.626	266.18
Error	26	45385.368	
Total	31	3787953.074	

Parameter	Estimate	Param. = 0 T Value	Std. Error of Parameter
Intercept	1804.692	244.35	7.386
N8p	179.511	24.30	7.386
N9p	103.981	14.08	7.386
A2-8c	86.471	11.71	7.386
A3-8c	227.863	30.85	7.386
A5-8c	120.500	16.32	7.386

F-statistic is 1.1017, considerably lower than the $F_{.05,1,9}$ value of 5.12, thus indicating no significant second-order effects have been detected. Based on the results in Table 4, the response of maximum expected flow for the coded variables is described by the first-order polynomial

$$Y = 1804.692 + 179.511(N8p) + 103.981(N9p) + 86.471(A2\text{-}8c) + 227.863(A3\text{-}8c) + 120.5(A5\text{-}8c). \quad (6)$$

A more useful version of Eq (6), using the uncoded values, is found by converting the coefficients. For this example, the uncoded version is

$$Y = -2{,}103.19 + 2243.389(N8p) + 1299.763(N9p) + .144(A2\text{-}8c) + .380(A3\text{-}8c) + .268(A5\text{-}8c). \quad (7)$$

Both equations are valid only for the region of the response surface defined by the input domain of Table 3. Tests were conducted to confirm the statistical assumptions of constant variance and normal distribution of the residuals.

3.3 Optimization of Response Surface

Given that Eqs (6) and (7) accurately describe the response surface of maximum flow, several insights into this network's performance are available.

First, any improvement in network maximum flow should focus on getting more information from the source nodes to Node 8. This is demonstrated by the fact that three of the five significant parameters are the capacities of arcs incident to the source nodes. Furthermore, this occurs in spite of the fact that four arcs from Node 8 to the sink nodes were screened for both capacity and survival rate. Apparently, network flow is diverse enough after Nodes 8 and 9 to insure that some flow will get through.

A second useful observation is obtained by comparing the response surface of expected maximum flow to that of network reliability. Following the same procedure used for finding Eqs (6) and (7), the uncoded version of the network reliability response surface (in percentages) is given by the first-order polynomial

$$Y = 62.84 + 9.4(N8p) + .94(N9p) + .71(A8\text{-}31p) \quad (8)$$

The insight provided by this response surface is the dominant influence of Node 8 on network reliability (which is probably due to the node's position in the network). Apparently, flow from the source nodes arrives often enough that if Node 8 survives, then at least one of the sink nodes will receive flow as well. Since Node 8 is also the second most influential component in the maximum flow response surface, any improvement of it's survivability will increase network performance in both areas.

The response surfaces described above can also be used directly to solve the network improvement problem. For example, assume we wish to maximize the expected maximum flow of the sample network as described by Eq (7), subject to the following constraints:

1. The cost of hardening nodes 8 and 9 is $10k per .1 unit of P_S. The total cost of hardening cannot exceed $15k.

2. The cost of increasing arc capacity for A2-8c, A3-8c, and A5-8c is $5k per 100 units. The total cost of increased capacity cannot exceed $150k.

3. The total cost of improvement cannot exceed $160k.

4. Eq (7) is valid only for the region of space defined by the experimental design. Therefore, the five components' values are implicitly bound by the uncoded values given in Table 3.

Let the **improvement variables** H_8 and H_9 represent the amount of hardening for nodes 8 and 9; and, $C_{2\text{-}8}$, $C_{3\text{-}8}$, and $C_{5\text{-}8}$ the increase of capacity for arcs A2-8c, A3-8c, and A5-8c, respectively. Since the coefficients of Eq (7) are applicable to both the original, uncoded variables and the improvement variables, the objective function can be re-written for just improvement variables (minus the intercept term). Thus, a linear programming formulation that maximizes expected maximum flow subject to the listed constraints is

$$\text{Maximize } Z = 2234.389(H_8) + 1299.763(H_9) + .144(C_{2\text{-}8}) + .380(C_{3\text{-}8}) + .268(C_{5\text{-}8}) \quad (9)$$

subject to

$$H_8 + H_9 <= .15$$

$$C_{2\text{-}8} + C_{3\text{-}8} + C_{5\text{-}8} <= 3000$$

$$100(H_8) + 100(H_9) + .05(C_{2\text{-}8}) + .05(C_{3\text{-}8}) + .05(C_{5\text{-}8}) <= 160 \quad (10)$$

and

$$0 <= H_8 <= .16 \quad 0 <= H_9 <= .16$$

$$0 <= C_{2\text{-}8} <= 1200 \quad 0 <= C_{3\text{-}8} <= 1200$$

$$0 <= C_{5\text{-}8} <= 900. \quad (11)$$

The three inequalities in Eq (10) formulate the cost restrictions of Items 1, 2, and 3 respectively, while the constraints in Eq (11) reflect the implicit bounds of the design space mentioned in Item 4.

Using standard linear programming techniques, the optimal solution for this sample problem is 1147.558, where $H_8 = .15$, $H_9 = 0.0$, $C_{2\text{-}8} = 800$, $C_{3\text{-}8} = 1200$, and $C_{5\text{-}8} = 900$. Adding the intercept to the optimal flow improvement gives an estimated maximum flow of the improved network of 2235.12. This represents an increase of 1065.968 over the unimproved estimated maximum flow of 1169.152. As a further enhancement, multiple optimization is possible by using Eqs (7) and (8) in the constraints of a goal programming formulation.

4. CONCLUSIONS

This paper has shown one approach to applying response surface methodology to the stochastic network improvement problem. Using expected maximum flow and source-to-sink reliability as the measures of network performance, we demonstrated how the resultant first-order polynomial metamodels can be applied in a linear programming formulation for optimal network improvement. For further research, we suggest combining other techniques, such as importance sampling, with survival variables in reducing the variance of the estimators of maximal flow and reliability.

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