Proceedings of the 1989 Winter Simulation Conference
E.A. MacNair, K.J. Musselman, P. Heidelberger (eds.)

OSCILLATION AMPLITUDE CONSIDERATIONS IN FREQUENCY DOMAIN EXPERIMENTS

Sheldon H. Jacobson
Dept. of Operations Research
Weatherhead School of Management
Case Western Reserve University
Cleveland, Ohio 44106

ABSTRACT

We present a discussion of the issues to consider when setting oscillation amplitudes in frequency domain experiments, with particular reference to gradient direction estimators.

1. INTRODUCTION

Frequency domain methodology (FDM) was first introduced as a screening tool for continuous input factors in discrete-event simulations (Schruben and Cogliano 1987). More recently, the approach has been extended to gradient direction estimation (Jacobson and Schruben 1988) and discrete input factor screening (Sanchez and Sanchez 1989).

Three questions which must be addressed when running frequency domain experiment are: how do we determine the unit of the experimental or oscillation index, how do we select the driving frequencies, and how do we set the oscillation amplitudes. The first two of these issues have been looked at by Jacobson, Morrice, and Schruben (1988) and Jacobson, Buss, and Schruben (1987) respectively. In this paper we look at the third issue, setting the oscillation amplitudes in frequency domain experiments, with a particular focus on gradient direction estimation.

The paper is organized as follows. In Section 2, the effect of changes in the oscillation amplitude on the power spectrum values are discussed. These effects are looked at with respect to input feasibility, system noise, and higher degree polynomial terms in the input/output representation of the system. In Section 3, the work is summarized.

2. OSCILLATION AMPLITUDE EFFECTS

Setting the oscillation amplitudes for frequency domain experiments can be a difficult task. We consider three factors which influence their selection. These factors are: feasibility, noise, and higher degree polynomial terms.

2.1 FEASIBILITY

Schruben and Cogliano (1987) suggested that when running frequency domain experiments, the simulation inputs should be varied over the widest range of values. Therefore the oscillation amplitudes should be set as large as possible (we always assume that the oscillation amplitudes are positive). This would also tend to make it easier to detect the resulting output signals. One constraint on these amplitudes is that the input values must remain feasible. To illustrate this, consider an M/M/1 queue with input processes $\{\mu(t)\}$ and $\{\lambda(t)\}$, where $t=0,1,\ldots$ is the customer number entering the system. If we vary $\mu(t)$ about $\mu(0)=1.0$ at frequency $\omega_1=.07$ with Φ_1 a uniform $(-\pi,\pi)$ phase shift (i.e. $\mu(t)=1+\alpha_1\sin(2\pi(.07)t+\Phi_1)$), we observe that the oscillation amplitude α_1 must be less than one, or else $\mu(t)$ can become negative or zero. If we vary $\lambda(t)$ about $\mu(0)=0.5$ at frequency $\omega_1=.28$ with Φ_2 a uniform $(-\pi,\pi)$ phase shift (i.e. $\lambda(t)=0.5+\alpha_2\sin(2\pi(.28)t+\Phi_2)$), we observe that the oscillation amplitude α_2 must be less than one half, or else $\lambda(t)$ can also become negative or zero. Note that the random phase shifts Φ_1 and Φ_2 are independent, and are needed to ensure stationarity of the input processes. In practice however, they are typically fixed at zero.

In addition to the above constraints on (α_1,α_2), it is desirable for $\rho(t)=\lambda(t)/\mu(t)$, the traffic intensity, to be less than one for all t, or on average less than one over all t. More explicitly, we have

$$\rho(t)=\frac{[0.5+\alpha_2\sin(2\pi(.28)t+\Phi_2)]}{[1.0+\alpha_1\sin(2\pi(.07)t+\Phi_1)]}\ . \tag{1}$$

From this equation, to ensure $\rho(t)<1$ for all t, we must have

$$(0.5+\alpha_2)/(1.0-\alpha_1)<1 \tag{2}$$

which gives us

$$\alpha_1+\alpha_2<0.5 \tag{3}$$

Therefore we could set $(\alpha_1,\alpha_2)=(.25-\Delta,.25-\Delta)$, $(.3-\Delta,.2-\Delta)$, or any such combination, for some $\Delta>0$ small. If we just want the traffic intensity to be less than one on *average*, we could choose α_1 and α_2 such that $\alpha_1+\alpha_2\geq 0.5$. Determining the exact range for α_1 and α_2 would require us to compute

$$r(\alpha_1,\alpha_2)=\lim_{T\to+\infty}\sum_{t=0}^{T}\frac{(.5+\alpha_2\sin(2\pi(.28)t+\Phi_2))}{(1.+\alpha_1\sin(2\pi(.07)t+\Phi_1))T} \tag{4}$$

and set α_1 and α_2 such that $r(\alpha_1,\alpha_2)<1$ as desired. Notice that for $\omega_1=.07$, $\omega_2=.28$, and any fixed phase shifts, $r(\alpha_1,\alpha_2)$ can be computed by taking the finite sum from 1 to 100, hence making it computationally tractable.

The above discussion focuses on general frequency domain experiments. All these issues also apply to frequency domain gradient direction estimation.

2.2 NOISE

Input/output representations of discrete-event simulations have been discussed in Sanchez and Buss (1987) and Jacobson (1988). A generalization of the *Hammerstein Model* (Narendra and Gallman 1966) was introduced as one such stochastic dynamical model (we will refer to this generalization simply as the Hammerstein model). This model is depicted in Figure 1.

Suppose we assume that the polynomial transform is quadratic (i.e. k=2). From Jacobson and Schruben (1988) we have the following result.

THEOREM 1: Let y(t) be a quadratic Hammerstein model $y(t)=\sum_{\substack{||\kappa||_1\leq 2\\ \kappa\geq 0}}(\int_0^{+\infty} x^{\kappa}(t-\tau)d\Gamma_{\kappa}(\tau))+\varepsilon(t)$ with associated steady state expected response Y(x). If the input x(t) is varied about x(0) in direction d at frequency ω with Φ independent and identically distributed uniform $(-\pi,\pi)$ phase shifts, then

$$\lim_{\omega\to 0} f(\omega)=(1/2)\langle\nabla Y(x(0)),d\rangle^2\delta(0)+f_{\varepsilon}(0) \tag{5}$$

where $f(\omega)$ is the output power spectrum value at ω, $f_{\varepsilon}(\omega)$ is the power spectrum of the noise process $\{\varepsilon(t)\}$ at frequency ω, $\kappa=(\kappa_1,\kappa_2,...,\kappa_p)$ is a vector exponent, and $\delta(\omega)$ is the dirac delta function.

The oscillation direction d completely defines the oscillation amplitude for each input, since $x(t)=x(0)+d\sin(2\pi\omega t+\Phi)$ can be rewritten as $x_i(t)=x_i(0)+d_i\sin(2\pi\omega t+\Phi_i)$ for $i=1,2,...,p$. Therefore we have $\alpha_i=d_i$ for $i=1,2,...,p$. From equation (5), the dirac delta function represents a discrete jump of size $(1/2)\langle\nabla Y(x(0)),d\rangle^2$ in the cumulative power spectrum $F(\omega)$, as $\omega\to 0$. We would like to set the oscillation direction such that this discrete jump is easily identifiable. From (5), it is clear that this can be achieved by increasing $||d||_1$. To demonstrate this, consider an M/M/1 queue as described in Section 2.1, with $\mu(0)=1.0$, $\lambda(0)=.5$, and output process $\{W(t)\}$, the customer waiting times in the system. We set the oscillation direction $d=\gamma(-1,1)$, the direction of the gradient of the steady state expected waiting time, where $\gamma=.01,.10,.25,.35$. We made four sets of five replications of a frequency domain simulation experiment, each set corresponding to a different γ and each replication involving one signal run (i.e the inputs are varied) and one control run (i.e. the inputs are fixed). The simulation run lengths were n=1000 with driving frequency $\omega=.10$ for the signal runs. We obtained $\bar{f}_\varepsilon(.10)=.0131$ using all twenty control runs. Table 1 shows how the average values for $\bar{f}(.10)$ and $\bar{f}(.10)/\bar{f}_\varepsilon(.10)$ change as γ is varied. These results give some indication of how the noise effect in a system can be reduced in the frequency domain by an increase in the oscillation amplitude.

The relationship between oscillation amplitude and noise discussed above holds for systems with *additive* noise components. For systems with certain types of *non-additive* noise components, such as $x(t)\varepsilon(t)$ (see Jacobson 1989), this noise reduction effect will not necessarily be exhibited.

TABLE 1
M/M/1 Frequency Domain Experiments

γ	$\bar{f}(.10)$	$\bar{f}(.10)/\bar{f}_\varepsilon(.10)$
.01	.020	1.5
.10	.072	5.5
.25	.721	55.0
.35	1.597	121.9

2.3 HIGHER DEGREE POLYNOMIAL TERMS

In Section 2.2, we assumed that the degree of the polynomial transforms in the Hammerstein model were at most quadratic. This is a reasonable assumption to make if we are examining a simulation response locally. However, frequency domain methodology is often thought of as a *global* sensitivity analysis technique (Schruben and Cogliano 1987). We now discuss how cubic and higher degree terms can significantly distort input term screening and gradient direction information in the frequency domain.

Consider the cubic model $y(t)=x^3(t)$. Varying $x(t)$ according to the equation $x(t)=x(0)+\alpha\sin(2\pi\omega t)$ (Φ fixed at zero) gives us

$$y(t)=x^3(0)+3x^2(0)\alpha\sin(2\pi\omega t)+3x(0)\alpha^2\sin^2(2\pi\omega t)+\alpha^3\sin^3(2\pi\omega t) \qquad (6)$$

Using the trignometric relations $\sin^2(2\pi\omega t)=(1-\cos(2\pi(2\omega)t))/2$ and $\sin^3(2\pi\omega t)=(3/4)\sin(2\pi\omega t)-(1/4)\sin(2\pi(3\omega)t)$, equation (6) becomes

$$y(t)=x^3(0)+3x^2(0)\alpha\sin(2\pi\omega t)+3x(0)(\alpha^2/2)(1-\cos(2\pi(2\omega)t))+(\alpha^3/4)(3\sin(2\pi\omega t)-\sin(2\pi(3\omega)t)). \qquad (7)$$

Notice that the coefficient of $\sin(2\pi\omega t)$ is $3x^2(0)\alpha+3(\alpha^3/4)$. This coefficient is twice the output power spectrum value of $\{y(t)\}$ at frequency ω. At this frequency, we would like to identify $3x^2(0)\alpha$, the scaled (by α) derivative of $Y(x)=x^3$ evaluated at $x(0)$, rather than $3(\alpha^3/4)$. The presence of $3(\alpha^3/4)$ results in a *cubic* effect term at a *linear* indicator frequency. In fact, this cubic effect term will override the linear effect term at frequency ω if

$$3x^2(0)\alpha \leq 3(\alpha^3/4). \qquad (8)$$

This gives us the inequality

$$2|x(0)| \leq \alpha. \qquad (9)$$

Therefore for $|x(0)|$ small (near 0), α must be set sufficiently small ($\alpha << 2|x(0)|$, or more generally, α should be closer to zero than $|x(0)|$) to ensure that the cubic term does not dominate the power spectrum value at the linear indicator frequency. This puts an upper bound on the size of α. For $x(0) \in R^p$ and oscillation direction d, similar results can be derived which also tend to force $||d||_1$ to be close to zero for $x(0)$ near local optima.

3. SUMMARY

We have identified and discussed three factors which influence the size of the oscillation amplitudes for frequency domain experiments. We noted that to ensure input feasibility and reduce higher degree term effects, the amplitudes should be set as small as possible. Furthermore, to reduce the noise effects the amplitudes should be set as large as possible.

It may not be possible to set oscillation amplitudes which satisfactorily address all three constraints. This situation will tend to arise when experiments are run with the inputs close to a local optimum. For such situations, the priority of these constraints should be feasibility, higher degree terms, and noise effect. This leads us to conclude that we should set the oscillation amplitudes as large as possible such that the inputs remain feasible and higher degree term effects do not dominate the power spectrum values at linear term indicator frequencies.

REFERENCES

Jacobson, S.H. 1988. Discrete-Event Computer Simulation Response Optimization in the Frequency Domain. Ph.D. Dissertation. School of Operations Research and Industrial Engineering. Cornell University.

Jacobson, S.H. 1989. Input/Output Models for Simulation Response Gradient Direction Estimation in the Frequency Domain. Working Paper. Department of Operations Research. Case Western Reserve University. Cleveland, Ohio.

Jacobson, S.H., A.H. Buss, and L.W. Schruben. 1987. Driving Frequency Selection for Frequency Domain Simulation Experiments. *Operations Research*. (Submitted for Publication).

Jacobson, S.H., Morrice, D.J., Schruben, L.W. 1988. The Global Simulation Clock as the Frequency Domain Experiment Index. *Proceedings of the 1988 Winter Simulation Conference*. 558-563.

Jacobson, S.H., and L.W. Schruben. 1988. Simulation Response Gradient Direction Estimation in the Frequency Domain. *Operations Research* (Submitted for Publication).

Narendra, K.S., and P.G. Gallman. 1966. An Iterative Method for the Identification of Non-Linear Systems Using a Hammerstein Model. *IEEE Transactions on Automatic Control*. AC-11. 546-550.

Priestley, M.B. 1981. *Spectral Analysis and Time Series*. Volumes 1 and 2. Academic Press. London.

Sanchez, P.J., Buss, A.H. 1987. A Model for Frequency Domain Experiments. *Proceedings of 1987 Winter Simulation Conference*. 424-427.

Sanchez, P.J., Sanchez, S.M. 1989. Spectral Design of Experiments for Discrete-Valued Factors. Working Paper. Systems and Industrial Engineering Department. University of Arizona.

Schruben, L.W., and V.J. Cogliano. 1987. An Experimental Procedure for Simulation Response Surface Model Identification. *Communications of the Association for Computing Machinery*. Volume 30. 716-730.

ACKNOWLEDGEMENT

This research was partially support by an Ohio Regents Board Initiation Grant and an Ohio Supercomputer Facility Grant.

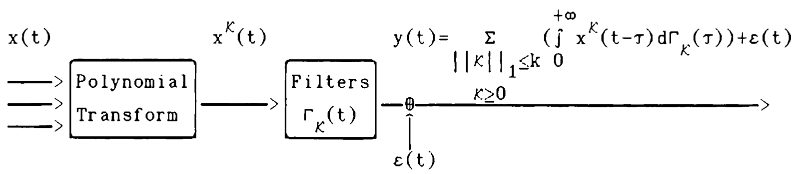

FIGURE 1: The Hammerstein Model

AUTHOR'S BIOGRAPHY

SHELDON H. JACOBSON is an Assistant Professor of Operations Research in the Weatherhead School of Management of Case Western Reserve University in Cleveland, Ohio. His research interests include simulation optimization, spectral methods, just-in-time manufacturing system analysis, and computational complexity issues in optimization algorithms.

Sheldon H. Jacobson
Department of Operations Research
Case Western Reserve University
Cleveland, Ohio 44106
(216) 368-4778
JACOBSON@CWRU.CWRU.EDU
JACOBSON@CWRU.BITNET